Control of Horizontal Cracking in the Ends of Pretensioned Prestressed Concrete Girders

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SYNOPSIS

This paper describes a limited investigation of the stresses which occur in the ends of pretensioned prestressed concrete girders at the time of transfer of prestress, and which can result in the formation of horizontal cracks in the ends of such girders. Also reported is a study of the stresses set up in vertical stirrup reinforcement near the ends of pretensioned prestressed girders when horizontal end cracking does occur. On the basis of experimental data obtained in this study, an equation is proposed for the design of vertical stirrup reinforcement necessary to restrict the size of any horizontal end cracks which may occur in a pretensioned prestressed concrete girder.

Introduction

Horizontal cracks in the ends of pretensioned prestressed concrete girders have been reported on numerous occasions in recent years. These cracks have been observed mainly in beams having an I-shaped cross-section, but they have also been noted in inverted tee and rectangular section girders.

A typical example of horizontal end cracking is shown in Fig. 1. The cracks usually occur near the centroidal axis, and often close to the junction of the web and the lower flange in the case of I- and inverted tee-shaped girders. As would be expected, this type of cracking occurs more commonly in I-shaped or inverted tee-shaped members than in rectangular girders. This is due to the smaller horizontal cross-section of the concrete available to resist the tensile forces set up.

In a recent survey(1) of prestressed concrete highway bridges in the United States, horizontal end cracking was noted in 25 out of 41 pretensioned prestressed bridges examined. This type of cracking occured with the greatest frequency in the case of girders having draped strands, where the strands were concentrated in two groups at the ends. The provision of end blocks did not appear to prevent the formation of these cracks.

Horizontal end cracks occur as a result of the high tensile stresses set up at the end face of girders between the groups of strand. The maximum tensile stress usually occurs near the centroidal axis, and the cracks usually form in this location.

When an adequate amount of vertical stirrup reinforcement is provided in the end regions of pretensioned prestressed girders, the

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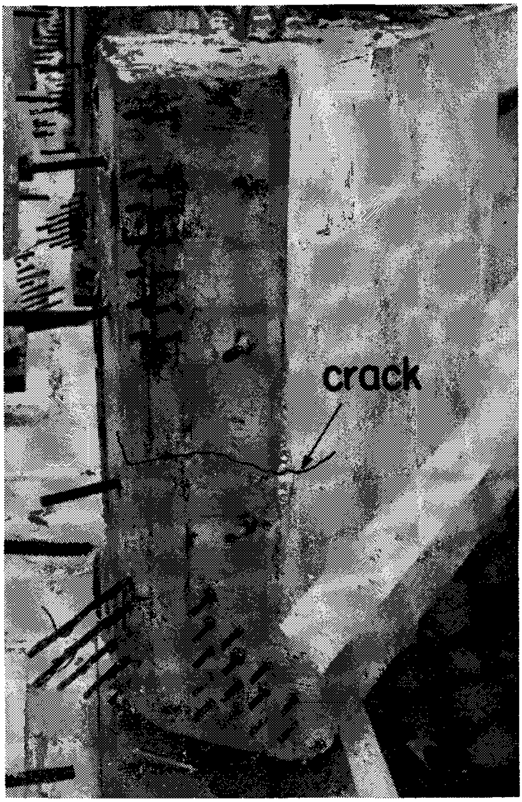

Fig. 1—Example of a Horizontal Crack in the end of a Pretensioned Prestressed Bridge Girder. (Crack marked to emphasize location)

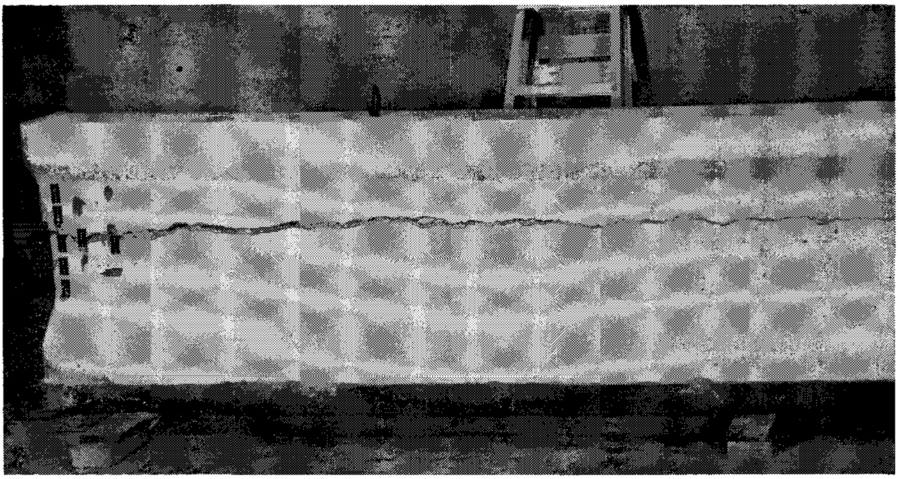

Fig. 2—Severe Horizontal Cracking in a Girder without Vertical Stirrup Reinforcement.

development of the horizontal cracks is restricted. In these cases, the cracks are fine; usually their width is about one hundredth of an inch or less, and they penetrate only a matter of inches into the end of the member. However, it should be noted that if vertical stirrup reinforcement is not provided, the horizontal crack may widen and spread along the girder as may be seen in Fig. 2. In the extreme case such a crack may split the girder from end to end. This behavior was observed during the test program carried out at the PCA Laboratories, and also occurred a few times in plants before vertical stirrups came into general use.

If sufficient vertical stirrup reinforcement is provided, so that the horizontal cracks are restricted to a few inches in length, and in width to one hundredth of an inch or less, then these cracks would not be detrimental to the performance of girders either at service load level or at ultimate strength. The cracks are caused primarily by the concentration of prestressing forces. They will not lengthen or widen as a result of the application of additional loads to the member, since in a pretensioned prestressed concrete girder the tension in the prestressing strand at the ends of the girder does not appreciably increase when loads are applied to the girder. If restricted as described above, the cracks would not be greater in width than those which have occurred in ordinary reinforced concrete members for over half a century.

The field survey(1) referred to earlier has shown that the provision of end blocks does not ensure the absence of horizontal cracking at the ends of pretensioned prestressed concrete girders. In addition, end blocks cannot restrict the growth of a horizontal end crack once cracking has occurred, and they also add appreciably to the formwork costs for a prestressed girder. It therefore appears sound both from economic and structural considerations to omit end blocks from pretensioned prestressed concrete girders, and to substitute in their place an adequate amount of vertical stirrup reinforcement. The primary outcome of this investigation is a proposal for design criteria for this vertical stirrup reinforcement.

Previous Investigations

As far as can be ascertained, no experimental investigations have been carried out previously which had as their primary purpose the measurement of those stresses which cause horizontal cracking in the ends of pretensioned prestressed girders. However, during an investigation of the transmission length of wires in pretensioned prestressed concrete, Base(2) made measurements of strains due to vertical tension in the end zones of three girders.

Of particular interest are the data from Base's second and third girders. Both these girders were of an inverted tee cross-section, 22 in. deep, 20 in. wide, and with a 3-in. thick web. The girders were prestressed by straight 0.20-in. diameter wires arranged in two groups, 62 wires at the bottom of the section and 10 wires at the top. Ten ⅜-in. square twisted bars placed vertically in the end nine inches of the second girder prevented the formation of visible cracks, although a maximum vertical tension strain of 0.00035 was measured in the web at the end face.

The vertical tension strains decreased to zero at about 12 in. from the end face of the girder.

The third girder was provided with eight ⅜-in. square twisted bars in the end 22 inches. In this case a visible horizontal crack occurred in the lower part of the web, and the maximum vertical tension strain recorded was in excess of 0.0015. Zero vertical strain did not occur until about 20 in. from the end face of the girder. The longitudinal strains measured in the bottom flange reached a peak 12 in. from the end face of the girder in this case, as against a distance of about 48 in. in the case of the second girder, indicating a much shorter transfer length for the third girder than for the second. The more extensive cracking of the third girder was therefore probably due to a combination of the effects of the shorter transfer length and of the smaller concentration of vertical stirrup reinforcement close to the end of the girder.

Base did not attempt to analyze these results but commented as follows: "Considerable work would be necessary to produce and check any theoretical method of forecasting the peak stress, but too great a concentration of wires should obviously be avoided."

The results obtained by Base appear to indicate that the maximum vertical tension stress probably depends on the arrangement of the prestressing wires at the end of the girder, and more specifically that *division of the prestressing wires* into concentrated groups leads to higher vertical tension stresses. The magnitude of these stresses is also apparently affected by the transmission length of the wire, shorter transmission lengths leading to higher stresses. The results further sug-

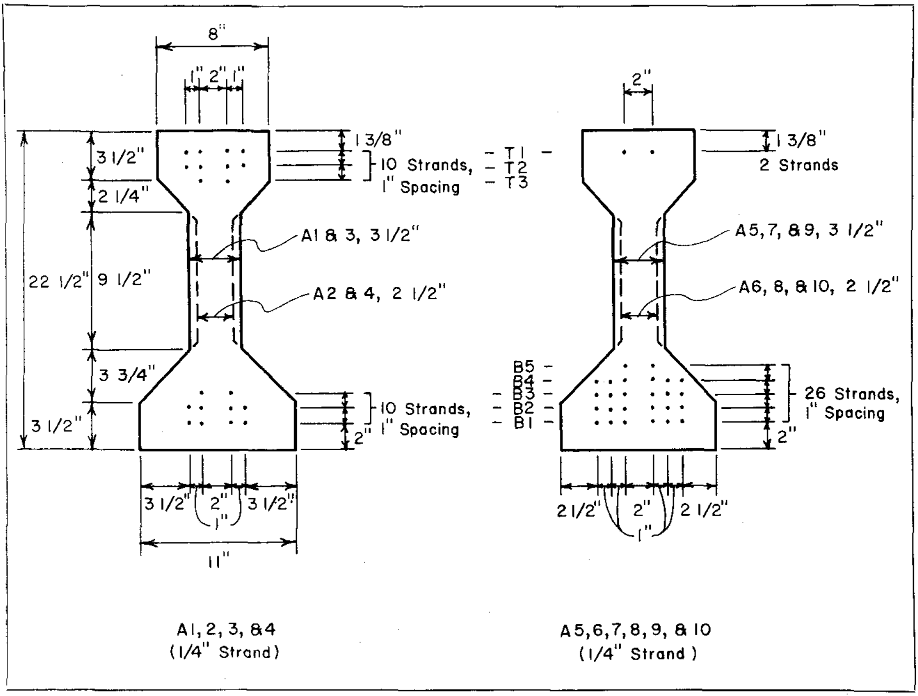

Fig. 3—Series A Girders, A1 through A10.

gest that in order to be effective in restraining the growth of cracks, vertical stirrup reinforcement must be concentrated close to the end face of the girder.

Experimental Investigation

Scope of Investigation

Test Series A concerned measurement of the concrete stresses in the end zones of pretensioned prestressed girders at the time of transfer of prestress to the concrete. Ten short girders were tested; all had basically the same I-shaped cross-section and were prestressed by the same size strand. The variables included in the test series were: web thickness, arrangement of prestressing strand, and surface condition of the strand.

Test Series B concerned measurement of the stresses set up at prestress transfer in the vertical stirrup reinforcement provided near the ends of pretensioned prestressed girders. From the results obtained in these tests, it was sought to develop an empirical method for the design of end zone reinforcement. Twenty five girder specimens were tested, having two basic cross-sections and containing two sizes of vertical stirrup reinforcement. The other variables included in the tests were the size and location of the prestressing strands. and the magnitude of the prestressing force.

Test Girders

All test specimens were 10 ft long; their cross-sections are shown in Figs. 3, 4, and 5. Since this investigation was concerned only with the measurements of stresses in the end zones of pretensioned prestressed girders at the time of transfer of prestress, the length of the test girders was governed by the requirement that the theoretical linear distribution of prestress in the girder should be achieved over at least the middle third of the girder length. It was considered that if this condition was achieved, then any further lengthening of the test specimen would not significantly affect the stresses at the ends of the specimens at transfer. Measurement of longitudinal strains along the length of the specimens indicated that in every case the desired condition was achieved.

The stirrups used had two legs and were made of No. 2 or No. 3 bars as indicated on Figs. 4 and 5. The cross piece was welded to the two legs at the points of intersection. It should be noted that the stirrups had hooks pointing along the axis of the girder at the bottom of each leg to ensure satisfactory anchorage. The stirrups were spaced as shown in Fig. 6.

Materials

The prestressing steel was seven-wire, stress-relieved strand of ¼, ⅜, or ½ in. diameter, having the properties indicated in Table 1. Except for the strand used in specimens A3, A4, A7, and A8, all strand was free of rust, and was cleaned of surface oil before tensioning. The ¼-in. diameter strand used for specimens A3, A4, A7, and A8 was purposely rusted by placing it in a moist curing room for 8 days prior to use.

The stirrups were either No. 2 deformed bar with a yield point of 49.9 ksi, or No. 3 deformed bar with a yield point of 44.4 ksi. The No. 3 deformed bar conformed to the requirements of ASTM A305 for deformations, and the No. 2 bar used was similarly deformed.

The concrete used in the fabrication of the specimens was made with Type III portland cement and ¾-in. maximum size aggregate. The slump

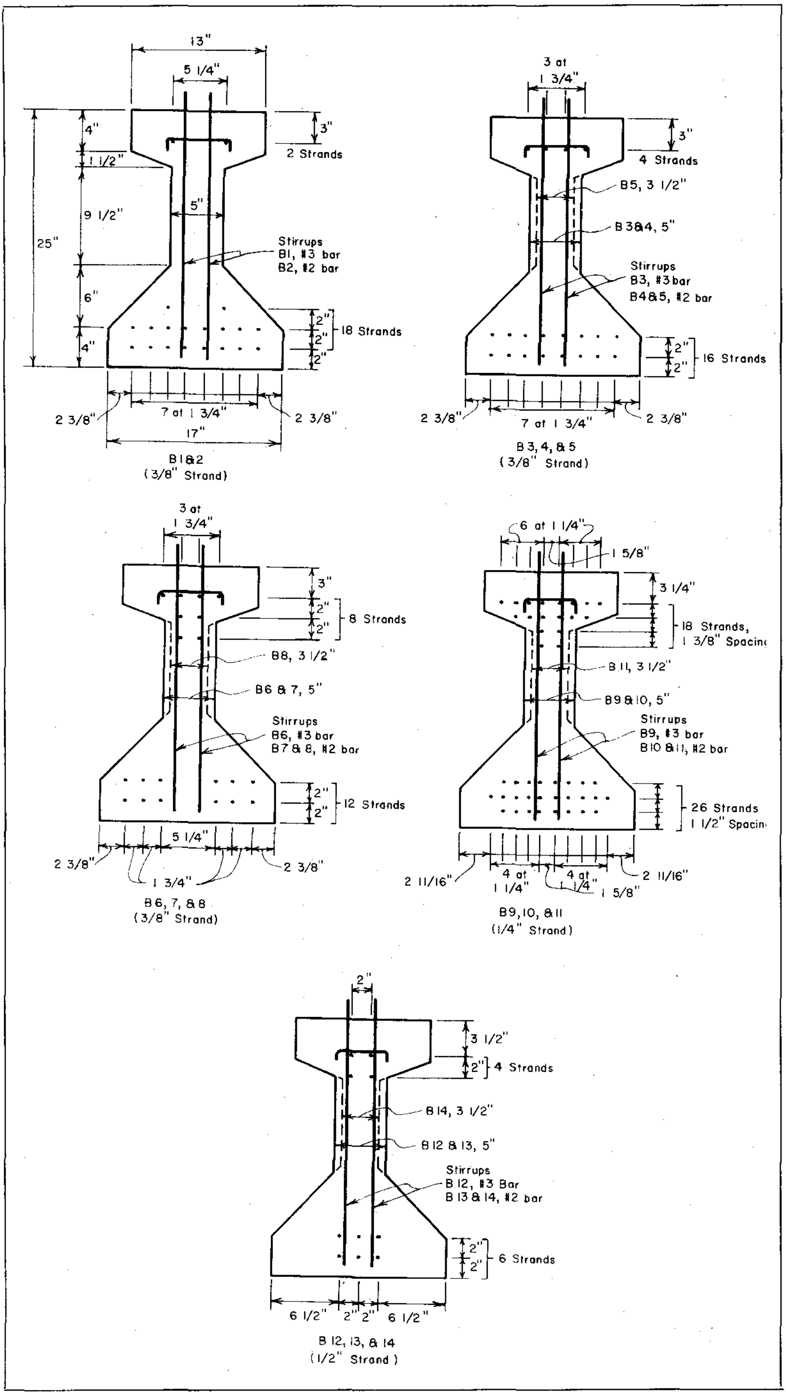

Fig. 4—Cross-sections, Girders B1 Through B14.

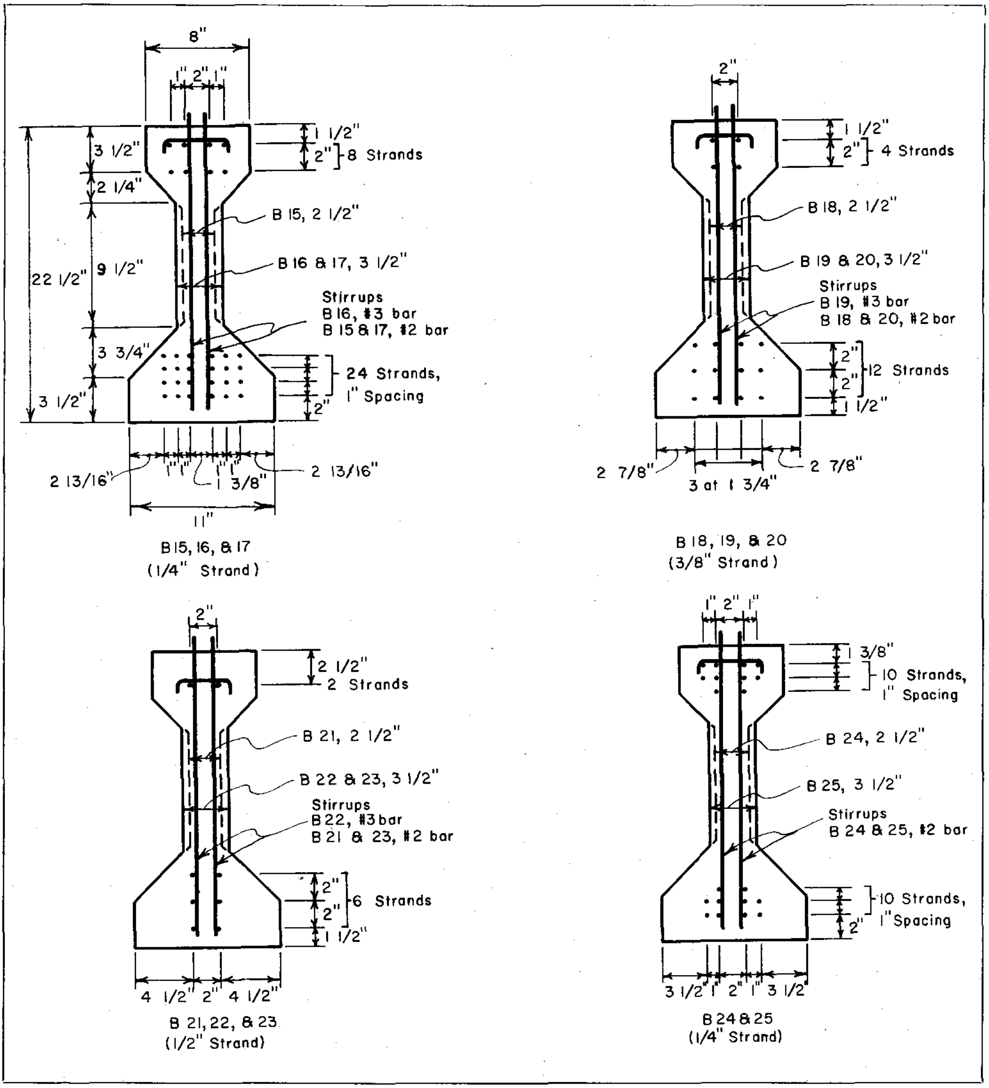

Fig. 5—Cross-sections, Girders B15 Through B25.

TABLE 1—PROPERTIES OF PRESTRESSING STRAND

Nominal Strand Diameter, in.	Cross-sectional Area, sq. in.	Stress at 1% Extension, ksi	Ultimate Strength, ksi
¼	0.0356	251	280
⅜	0.0799	259	286
½	0.1438	231	254

TABLE 2—CONCRETE STRENGTHS AT TRANSFER OF PRESTRESS

Girder Number	Cylinder Strength, f'_c, psi	Girder Number	Cylinder Strength, f'_c, psi	Girder Number	Cylinder Strength, f'_c, psi
A1	5050	B3	7090	B15	4030
A2	4145	B4	7090	B16	3715
A3	4625	B5	7190	B17	3715
A4	4145	B6	4855	B18	4450
A5	4900	B7	4855	B19	4575
A6	4560	B8	4995	B20	4575
A7	5095	B9	4290	B21	4090
A8	4560	B10	4290	B22	4975
A9	4520	B11	4475	B23	4975
A10	4520	B12	4415	B24	4660
B1	4580	B13	4415	B25	4660
B2	4580	B14	4200	..	....

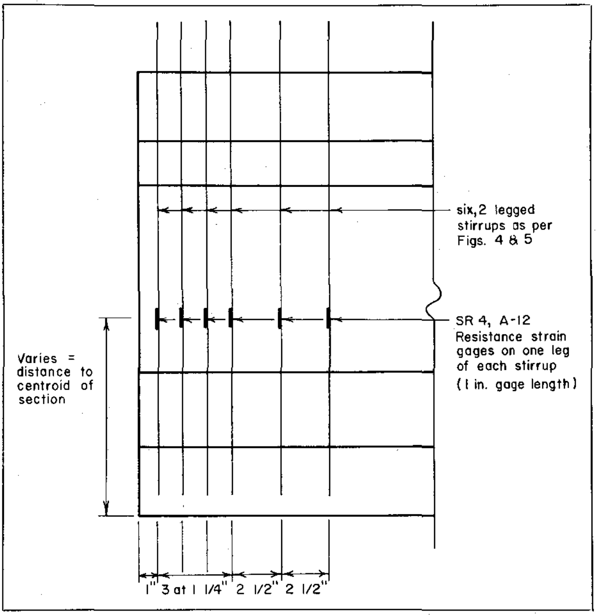

Fig. 6—Spacing of Vertical Stirrups in Ends of Series B Girders.

was 2 in. for Series A and 3 in. for Series B. The specimens were moist cured under plastic sheets at 70°F for the first three days after casting, and subsequently were stored at 70°F and 50% relative humidity until transfer of prestress at an age of seven days. The concrete cylinder strengths at transfer are listed in Table 2. These concrete strengths are in each case the average of three 6 by 12-in. cylinders taken from batches of concrete placed in the webs of the test specimens. The cylinders were cured alongside the girder specimens.

Fabrication and Test Procedure

The specimens were fabricated and tested in sets of from one to three girders. They were produced in a prestressing bed set up on the laboratory test floor(3) with a clear distance of up to 38 ft between anchorage blocks. The strands were tensioned individually using a center-hole ram with a 20-in. stroke. The tension in the strand was measured by a load cell placed between the hydraulic ram and the temporary anchorage used to grip the strand during the tensioning operation. The strands were over-tensioned by an amount sufficient to compensate for the loss in prestress due to the "draw in" of the permanent anchorages. The prestress remaining after permanent anchorage was measured for five strands, using load cells placed between the permanent anchorages and the anchorage cross head. By these methods of control the initial prestress was kept very close to the chosen value of 175,000 psi.

The day after prestressing, the stirrups (when used) and form-work were positioned, and the concrete was cast. The girders were moist cured for three days, and the forms were then stripped. Two days later SR-4 electrical resistance strain gages and mechanical strain gage points were mounted on the girders as shown in Fig. 7, or in a similar pattern. Seven days after casting the prestress was transferred to the specimens by torch-cutting the strand.

Readings of all gages were taken before cutting of the strand was commenced, together with readings from the load cells behind the strand anchorages. The strands were cut in predetermined groups, initially adjacent to each of the anchorage blocks, and subsequently between the specimens. Readings of all gages and load cells were taken after the cutting of each group of strands. In this way the prestress forces applied to the specimens and the strains produced were measured for each stage in the transfer process.

Test Results

Discussion of Series A Results

The distribution of strains in the ends of the girders without stirrups was similar for all specimens. Figs. 8 and 9 show typical examples of the distribution of vertical tension strains over the depth of the web at the end of the girder, and along a horizontal plane through the centroidal axis of the section. It can be seen that the maximum vertical tension stress existed only in a region near mid-depth of the web and that it died away rapidly on moving into the girder from the end face. In all cases in which cracking did not occur the vertical tension strains became zero at a distance from the end face not greater than about one third of the girder depth, and in most cases at a distance of about one quarter the depth. These results were thought to indicate that if vertical stirrup reinforcement were

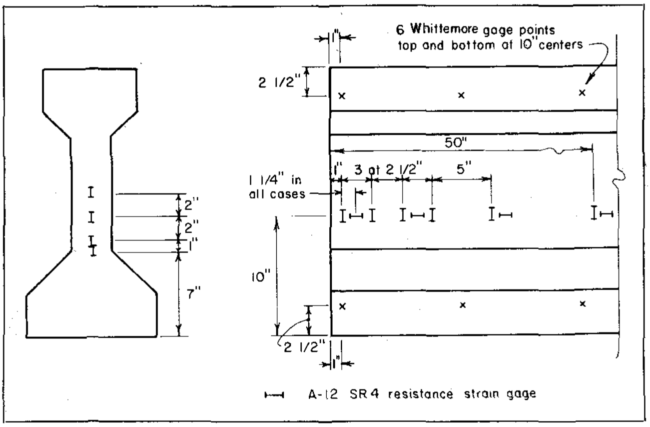

Fig. 7—Typical Distribution of Strain Gages.

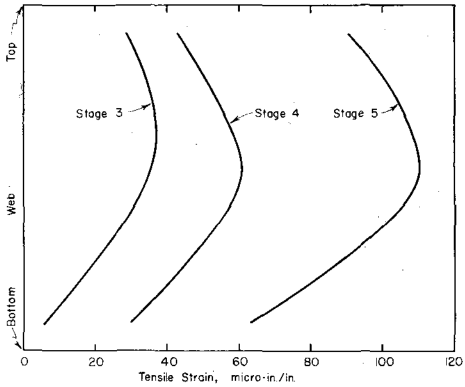

Fig. 8—Distribution of Vertical Tension Strains Over the Depth of the Web at the End of a Typical A-Series Girder, at Various Stages of Cutting the Strands.

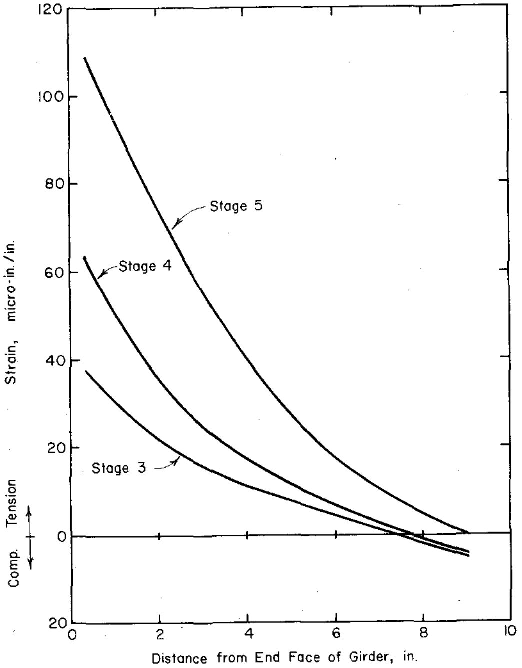

Fig. 9—Distribution of Vertical Tension Strain at the Level of the Central Axis in the End Zone of a Typical A-Series Girder, at Various Stages of Cutting the Strands.

to be used to control cracking resulting from these vertical tension stresses, then in order to be most effective it should be concentrated as close to the end face of the girder as is practicable.

The values of the maximum vertical tension stress for each test specimen at various stages of transfer were calculated from the measured maximum vertical tension strains, using for the modulus of elasticity $(60{,}000\sqrt{f'_c})$ psi. An attempt was made to correlate these measured stresses with the maximum stresses calculated using an adaptation of Sievers'(4) theory for the distribution of stress in the end zones of post-tensioned prestressed girders. The theoretical and measured stresses were in reasonable agreement for the girders having equal groups of strand in both top and bottom flanges. However, for the girders in which the majority of the strands were in the bottom flange the analytical approach grossly under-estimated the maximum stresses. It is thought that this may be due to the fact that a horizontal shear of considerable magnitude will exist at the level of the centroid of the section for this arrangement of prestressing strand, and that the influence of such a shear on the maximum vertical tensile stress in the end zone was not taken into account in the development of the expression for vertical tensile stress. To resolve this matter completely would require a somewhat extensive experimental investigation. At this stage of the present investigation, however, it was decided to direct attention to the stresses in stirrup reinforcement after cracking, rather than to continue the experimental study of the stresses existing before cracking.

Discussion of Series B Results

Twenty five specimens with end stirrups were tested in this series; seven of these did not crack, and in four girders the cracks occurred remote from the location of the strain gages on the stirrups. Useful information was therefore obtained from 14 of the 25 specimens. The behavior and relevant characteristics of the test specimens are recorded in Table 3.

Similarity of behavior was observed in 14 of the 18 girders which cracked. In these girders the crack occurred in the lower part of the

TABLE 3—BEHAVIOR OF SERIES B GIRDERS

Girder Number	Strand Diameter, in.	Percent of Strand at Top of Section	Behavior*	Girder Number	Strand Diameter, in.	Percent of Strand at Top of Section	Behavior*
B1	3/8	10	N	B14	1/2	40	C
B2	3/8	10	N	B15	1/4	25	N
B3	3/8	20	C	B16	1/4	25	C
B4	3/8	20	C	B17	1/4	25	C
B5	3/8	20	C	B18	3/8	25	C+
B6	3/8	40	C+	B19	3/8	25	C+
B7	3/8	40	C	B20	3/8	25	N
B8	3/8	40	C	B21	1/2	25	N
B9	1/4	40	C	B22	1/2	25	N
B10	1/4	40	C	B23	1/2	25	N
B11	1/4	40	C	B24	1/4	50	C
B12	1/2	40	C	B25	1/4	50	C+
B13	1/2	40	C				

*C = cracking, C+ = cracking remote from gages on stirrups, N = no cracking.

web close to the centroidal axis of the section, and resulted in a distribution of tension strains in the vertical stirrup reinforcement similar to those shown in Fig. 10. In the other four girders which cracked, the crack occurred in the upper part of the web remote from the strain gages on the stirrups; the distribution of stirrup strains for these four girders is therefore not known. In all cases the width of the crack at the end face of the girder was between 0.001 and 0.004 in. To the naked eye the cracks appeared to travel from two to four inches into the web of the girder from the end face.

The total tension force in the vertical stirrup reinforcement in each girder that cracked near the centroidal axis was calculated from the measured stirrup strains, and is listed in Table 4, together with the measured effective prestress force after transfer. The effective prestress was taken as the prestress just before transfer, as measured by the load cells at the strand anchorages, less the loss due to the measured longitudinal shortening of the concrete at transfer. Also listed in Table 4 are the measured maximum stirrup stress, the length of the horizontal crack, and the ratio of the depth of the girder to the transfer length of the prestress strand.

Since the cracks were very fine it was almost impossible to determine their exact length by visual observation. It was therefore decided to determine their length by reference to the measured vertical stirrup strains. If a value of $(7.5\sqrt{f'_c})$ psi is assumed for the limiting tensile stress of the concrete, then the strain at the end of the crack will be given by:

$$\begin{aligned}\epsilon_c &= (7.5\sqrt{f'_c})/E_c \\ &= (7.5\sqrt{f'_c})/(60{,}000\sqrt{f'_c}) \\ &= 125 \text{ micro in./in.}\end{aligned}$$

The point at which the vertical tension strain became 125 micro in./in. was therefore taken to be the point to which the crack extended. Examples of the use of this criterion are shown in Fig. 10. It can also

TABLE 4—EFFECTIVE PRESTRESS FORCE AND TOTAL VERTICAL STIRRUP FORCE AT TRANSFER

Girder Number	Effective Prestress Force, T, kips	Total Stirrup Force, S kips	$\frac{S}{T}$	Size of Stirrup Bar	Maximum Stirrup Stress, ksi	Length of Crack, in.	$\frac{h^*}{l_t}$
B3	246	5.17	.0210	3	11.32	3.6	1.28
B4	246	3.57	.0145	2	13.12	5.0	1.28
B5	246	3.28	.0134	2	12.80	4.5	1.28
B7	246	3.99	.0162	2	13.90	5.4	1.28
B8	246	3.28	.0133	2	12.30	4.2	1.28
B9	261	9.00	.0344	3	18.27	4.3	1.76
B10	261	5.26	.0202	2	22.50	4.9	1.76
B11	261	5.36	.0205	2	23.20	4.9	1.76
B12	214	4.04	.0189	3	9.00	2.6	0.97
B13	214	2.20	.0103	2	10.80	3.2	0.97
B14	214	1.82	.0085	2	9.00	2.7	0.97
B16	186	4.71	.0253	3	11.30	2.9	1.63
B17	186	2.61	.0140	2	13.50	3.2	1.63
B24	116	1.86	.0161	2	8.10	3.3	1.63

*h = girder depth; l_t = strand transfer length calculated assuming an average transfer bond stress of 400 psi(5).

be seen in Fig. 10 that within the nominal crack length the stirrup strains vary in a linear manner, but that beyond the crack length there is considerable departure from this linear variation. This behavior was observed in all the girders which cracked and is considered to support the criteria used to determine the crack length.

It can be seen in Table 4 and Fig. 10 that for girders which are identical except for the stirrup reinforcement, a higher total stirrup force was measured in the No. 3 bar stirrups than in the No. 2 bar stirrups. This is apparently due to the fact that the heavier stirrups restrained the opening of the crack to a greater degree and hence had to supply a greater force. Comparison of results for girders identical except for the stirrup reinforcement shows that the maximum stress in the No. 2 bar stirrups was approximately 20 per cent in excess of the maximum stress in the No. 3 bar stirrups, and also that the average length of crack in girders reinforced with No. 2 bar stirrups was about 20 per cent more than in girders reinforced with No. 3 bar stirrups. It appears that an increase of 120 per cent in the cross-section of stirrups resulted in a decrease in the length and width of the horizontal cracks of only about 17 per cent.

Since the behavior of the girders with No. 2 bar stirrups was considered satisfactory with respect to cracking, it was decided to use the results from the tests of these girders as a basis for the derivation of a formula for the amount of stirrup reinforcement necessary for satisfactory end zone performance in practice.

The analytical study showed that, for a given cross-section, the magnitude of the vertical tensile stresses before cracking is a function of the strand transfer length and the prestress force. The set of girders B6 through B14 were tested to determine whether these same parameters also influenced the tension in the stirrup reinforcement after cracking. The results obtained in the tests of these girders are plotted in Fig. 11. It can be seen that, for the range of transfer lengths covered by these tests, the ratio of the total stirrup force to the prestressing force could be taken as inversely proportional to the strand transfer length for a given cross-section and distribution of prestressing strand. This relationship is confirmed by the behavior of both the girders with No. 2 bar stirrups and the girders with No. 3 bar stirrups.

The transfer length is a measure of the abruptness with which the prestress is transferred from the strand to the concrete. The shorter the transfer length, the more abruptly is the prestress transferred to the concrete and the higher are the vertical tension stresses in the end zone. The analytical study indicated that for the more general case of girders of different size, the relative abruptness of transfer of prestress, expressed as a function of both the transfer length and the depth of the girder, is a more correct parameter to consider than the transfer length alone.

The set of girders B1 through B8 were tested with the intention of determining the influence on the stirrup stresses of the way in which the strands were distributed across the section. Girders B1 and B2 did not crack. Examination of the stirrup forces measured in girders B3 through B8 did not reveal a significant increase in stirrup force when the amount of strand in the

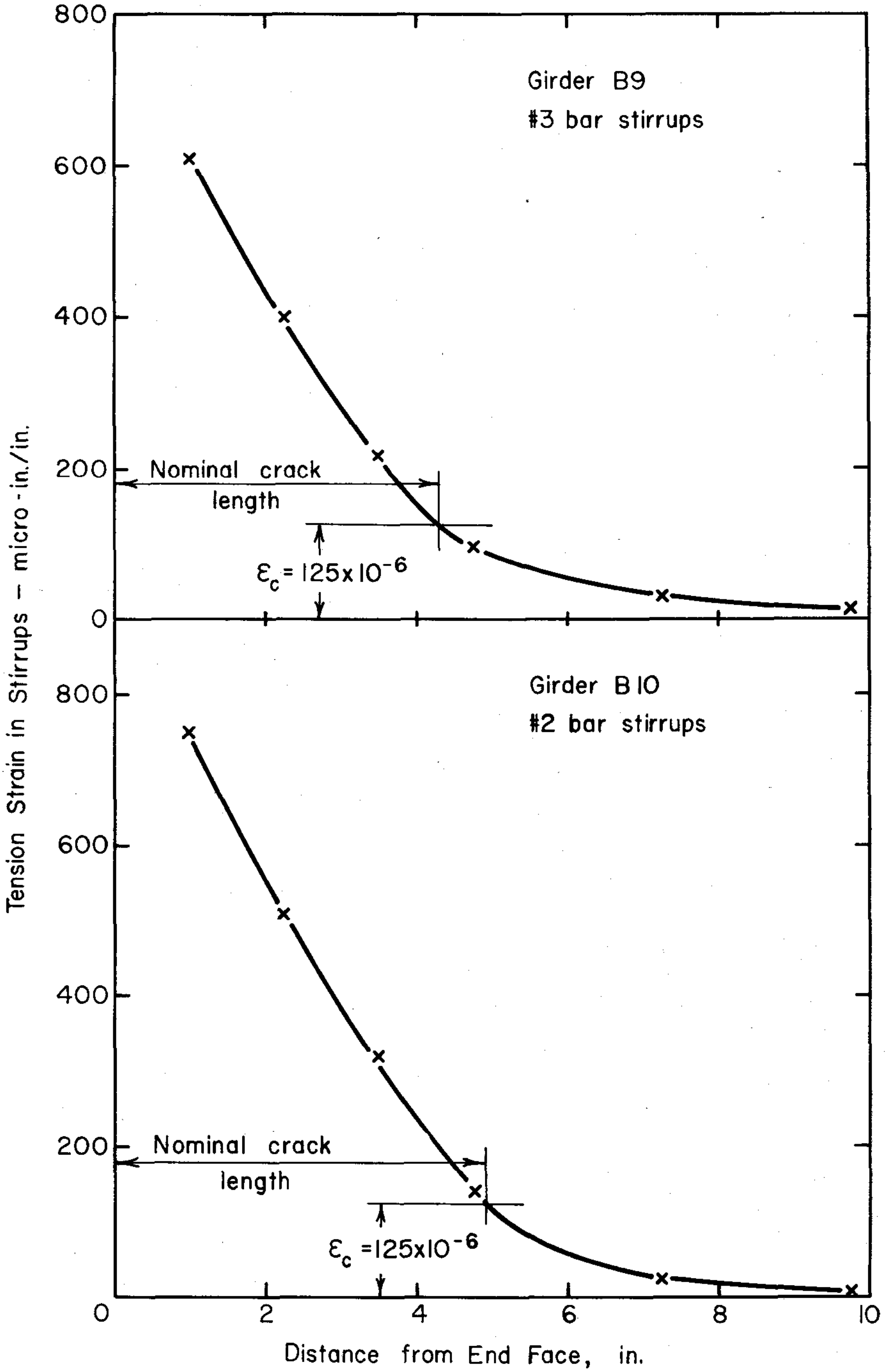

Fig. 10—Typical Variation of Stirrup Strain with Distance from End Face of Girder.

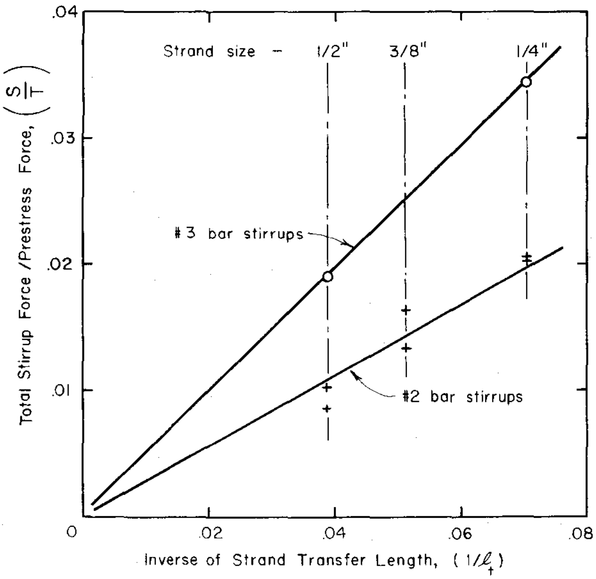

Fig. 11—Variation of Stirrup Force with Strand Transfer Length-Girders B7 Through B14.

top of the section was increased from 20 to 40 per cent of the total.

Ignoring therefore, for the time being, the influence of the manner of distribution of the strand, the force in the stirrups is seen to be a function of prestress force, strand transfer length, and depth of section. We may write:

$$S :: T^x l_t^y h^z \qquad (1)$$

If F and L are the fundamental units of force and length, then the corresponding dimensional equation is:

$$[F] = [F]^x \, [L]^y \, [L]^z \ldots\ldots(2)$$

which is dimensionally consistent for $x = 1$ and $z = -y$. Now the test data plotted in Fig. 11 demonstrate that the tensile force in the stirrups is inversely proportional to the transfer length for the range of transfer lengths considered, hence $y = -1$ and therefore $z = 1$, and we may write:

$$S = KT \; \frac{h}{l^t} \quad \ldots\ldots\ldots\ldots(3)$$

where K is an empirical constant.

In order to determine the value of K, the results of tests of girders containing No. 2 bar stirrups have been plotted in Fig. 12 in terms of the ratio of the total stirrup force to the effective prestress force, S/T, and of the ratio of the depth of the girder to the transfer length of the prestressing strand h/l_t. It can be seen that the relationship between S/T and h/l_t can be idealized as the straight line

$$S/T = 0.0106\,(h/l^t)\ldots\ldots(4)$$

that is, the value of K is 0.0106. Using this equation, the average ratio of S/T(test) to S/T (calc) is 1.00, and the standard deviation is 0.119.

Verification of the form of Eq. (4) is obtained from a consideration of the behaviour of two hypothetical girders which are similar in all respects. Let all dimensions of the first girder be N times the corresponding dimensions of the second girder.

i.e. $h_1 = N \cdot h_2$; $D_1 = N \cdot D_2$ etc.

The number of strands and stirrups will be the same in each girder.

The theory of structural similitude shows that if the two girders are made from materials having the same stress-strain relationships, and if concentrated loads P_1 and P_2 are applied to the first and second girders, respectively, such that

$$P_1 = N^2 P_2$$

then the stresses at corresponding locations within the girders will be equal.

Now the diameter of the prestressing strand in the first girder, D_1, is N times the diameter of the strand in the second girder, D_2. Therefore, if the strand in both girders is tensioned to the same stress, it follows that the prestress force in the first girder, T_1, is N^2 times the prestress force in the second girder, T_2, and hence the corresponding stresses set up in the stirrups of both girders will be equal. However, since the diameter of the stirrups in the first girder is N times the diameter of those in the second girder, the total stirrup force, S_1, in the first girder will be N^2 times the total stirrup force, S_2, in the second girder. Hence,

$$\frac{S_1}{T_1} = \frac{N^2\,S_2}{N^2\,T_2} = \frac{S_2}{T_2}$$

This result is also achieved by use of Eq. (4) if we make the reasonable assumption that the transfer length, l_t, is given by a constant, K', times the strand diameter, D, for similarly manufactured strand. Then:

$$\frac{S_1}{T_1} = 0.0106\frac{h_1}{K'D_1} = 0.0106\frac{N\,h_2}{K'\,N\,D_2}$$

$$= 0.0106\frac{h_2}{K'\,D_2} = \frac{S_2}{T_2}$$

Hence Eq. (4) yields results for girders of different size which are in agreement with those obtained by use of the theory of similitude. This result is seen as giving support to the form of Eq. (4) even though the effect of varying the depth of girder, h, was not investigated ex-

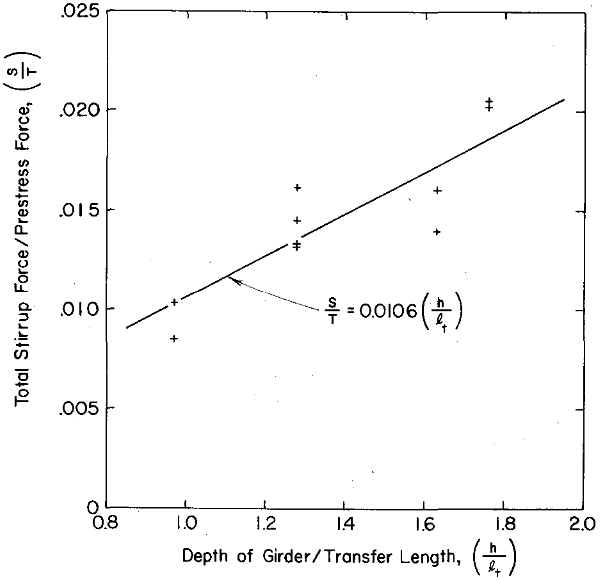

Fig. 12—Relationship of Stirrup Force and the Ratio of Girder Depth to Strand Transfer Length.

perimentally.

A further check was made on the possible influence on the stirrup forces of the manner of distribution of the strand. In Fig. 13 the product $(S/T)(l_t/h)$ is plotted against the percentage of strand in the top of the section for all girders reinforced with No. 2 bar stirrups. Also plotted is the average value of the product, 0.0106. The values of the product appear to be scattered in a fairly random manner about the average value and the diagram shows no significant trend. It appears therefore that Eq. (4) can be used to calculate the total stirrup force regardless of the percentage of prestressing strand present at the top of the girder end section.

If f_s is the maximum allowable stress in the stirrups, then the average stress will be very nearly $f_s/2$, since the stirrup stresses can be assumed to vary linearly from a maximum close to the end face of the girder to zero near the end of the crack. The total cross-sectional area of stirrups necessary, A_t, will then be given by Eq. (4) as

$$A_t = \frac{S}{(f_s/2)} = 0.021\frac{T}{f_s}\cdot\frac{h}{l_t}\ldots\ldots\ldots(5)$$

The amount of stirrup reinforcement calculated using Eq. (5) should be distributed uniformly over a length equal to one fifth of the girder depth, measured from the end face of the girder. For most efficient crack control the first stirrup should be placed as close to the end face of the girder as possible, since surface crack width is to some extent controlled by concrete cover(6). It is suggested that for design purposes l_t may be assumed to be 50 times the strand diameter.

It should be noted that cracking due to restraint of shrinkage and thermal contraction of the concrete by formwork can occur in end zones which would otherwise not crack due to vertical tension stresses set up at transfer of prestress. However, once the horizontal cracks have been initiated, from whatever cause, the local stresses due to anchorage of the strands will tend to open them, and stirrup reinforcement is then necessary. Since the amount of re-

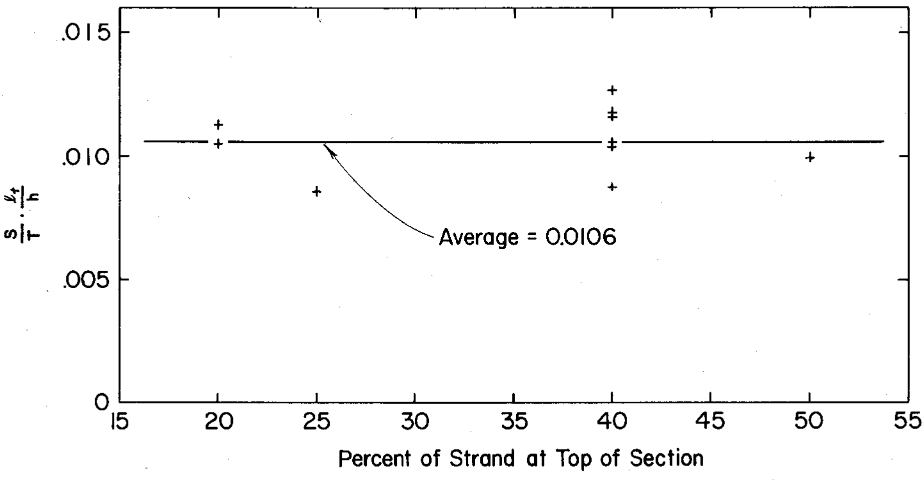

Fig. 13—Variation of Stirrup Force with Manner of Distribution of Strand Across the Section.

inforcement involved is relatively small, it is strongly recommended that suitably designed stirrup reinforcement should be placed in the end zones of *all* pretensioned prestressed concrete girders, even when the maximum vertical tension stress occurring at the time of transfer is relatively low.

Range of Applicability of Design Equation

Equation (5) is based on a simplified linear relationship between the ratio of the stirrup force to the prestressing force, (S/T), and the ratio of the depth of section to the strand transfer length, (h/l_t). This simplified linear relationship has been justified experimentally only for values of (h/l_t) of up to about 2. It is considered that as (h/l_t) increases beyond 2 the design equation will tend to become conservative, the degree of conservatism increasing as (h/l_t) increases. This is apparent if the limiting case of a post-tensioned girder is considered. In this case l_t is zero and Eq. (5) would yield infinity for the cross-sectional area of the stirrups. The curve representing the true relationship between (S/T) and (h/l_t) could therefore be expected to become asymptotic to the value of (S/T) corresponding to the case of a post-tensioned girder.

It should further be noted that the design equation has been developed from measurements made on the end zones of girders in which the strand was divided into two groups located respectively in the top and bottom of the section. If the groups were placed closer together, or if the strand were distributed uniformly over the end face of the girder, then it appears likely that the stirrup forces set up would be somewhat less than those occurring in the case considered in this experimental investigation. The design Eq. (5) will therefore tend to be conservative for girders in which the groups of strand are spaced more closely, or in which the strand is distributed uniformly over the end of the grider.

Design Example

To illustrate the use of design Eq. (5), the case will be considered of a Type IV AASHO-PCI standard bridge girder prestressed by 48 half-inch diameter strands, 16 of which are draped. Each strand will be assumed to be tensioned to 25.2 kips.

Depth of Type IV section, $h = 54$ in.

Transfer length of ½-in. strand,

$l_t = 50 \times ½ = 25$ in.

Total prestress force,

$T = 48 \times 25.2 \times = 1209.6$ kips

Maximum reinforcement working stress, $f_s = 20$ ksi

Area of stirrups,

$$A_t = 0.021\frac{T}{f_s} \cdot \frac{h}{l_t}$$

$$= 0.021\frac{1209.6 \times 54}{20 \times 25}$$

$$= 2.74 \text{ sq. in.}$$

Use three No. 6 bar two-legged stirrups in the end 10½ in. of the girder, which yields $A_t = 2.64$ sq. in.

Additional stirrup reinforcement should be provided in the remainder of the end zone of the girder. Determination of the amount of this reinforcement depends on shear force due to load and is outside the scope of this investigation.

Concluding Remarks

As a result of field observations(1) and of the observation of the behavior of laboratory specimens it is concluded that end blocks are not necessary for the satisfactory performance of the end zones of pretensioned prestressed concrete girders. A relatively small amount of vertical stirrup reinforcement placed close to the end face will ensure satisfactory performance of the end zone of a pretensioned prestressed concrete girder; any horizontal cracks which occur will be fine and short and will not affect the service performance of the girder.

It is considered desirable that vertical stirrup reinforcement be placed near the ends of all girders. Restraint of shrinkage and thermal contraction of the web concrete by formwork can lead to cracking. Cracks originated in this manner will tend to open under the action of the prestress forces, and hence adequate vertical stirrup reinforcement should always be provided.

Acknowledgments

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