

# Design of Anchorage-Zone Reinforcement in Prestressed Concrete Beams

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## NOTATION

The symbols as used in the text are defined as follows:

- $p$  — uniform stress on cross section due to prestress
- $c$  — distance to the longitudinal crack from the bottom of beam
- $e$  — distance to the applied force from the bottom of beam
- $P$  — applied force
- $L$  — distance from the end of the beam to a cross section where the stresses are linear due to  $P$
- $M$  — unbalanced moment on a free body formed by a longitudinal crack
- $T$  — total tensile force to be supplied by the stirrups
- $C$  — vertical compression force (on the free body formed by a horizontal crack)
- $h$  — height of the anchorage zone (or beam)
- $f'_c$  — cylinder strength of concrete at the time of testing
- $F$  — force in one reinforcing bar
- $A$  — area of one reinforcing bar
- $s$  — slip of one reinforcing bar
- $w$  — crack width
- $f_s$  — stress in stirrups

- $E$  — modulus of elasticity of steel
- $m$  — numerical factor in bond-slip relationship

## INTRODUCTION

The object of this paper is to introduce a simple method for the design of transverse reinforcement to restrain anchorage-zone cracks and to present a series of test results to confirm the validity of the method.

In prestressed beams, the prestressing force is introduced into the concrete section either at a single plane, as in the case of beams with mechanical anchorages, or over a short length of the tendons, as in the case of pretensioned bonded beams. In either case, the stress distribution over the section becomes linear, and conforms to that dictated by the over-all eccentricity of the applied forces, within a distance less than the height of the beam from the point of application of the force. As the applied force, concentrated at a certain level in the section, flows in curved patterns to conform to the linear stress pattern, it sets up transverse tensile stresses. These stresses may lead to longitudinal cracks which can extend throughout the beam span, thus causing failure. Similar stress conditions exist in bonded members where the prestressing force is transmitted to the concrete over a finite length.

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Calculated contours<sup>(1)</sup> of equal transverse stresses in a rectangular prism loaded uniformly over a distance equal to  $h/8$  at an eccentricity of  $3h/8$  (where  $h$  is the depth of the section) are shown in Fig. 1. It is seen that the tensile stresses can be categorized in two groups. The tensile stresses along the load axis reach a maximum a short distance from the plane at which the load is applied. These stresses, to be referred to as "bursting" stresses, are generally associated with bearing failures. The transverse tensile stresses away from the load axis reach a maximum at the loaded face. These, to be referred to as "spalling" stresses, are most often associated with anchorage-zone cracking.

Various methods have been proposed by different authors to calculate the transverse stresses. The relevant studies are reviewed in References 1, 2 and 3.

Disagreement was noted among the results of some of the most widely used methods. Finite differences were used in this investigation to obtain the elastic distribution. The results are reported in Reference 1; however, these mathematical solutions are not discussed in this paper.

The usefulness of the methods dealing with elastic stresses has to be examined in view of the structural performance of the member. One may be primarily interested in the magnitude and the position of the maximum transverse stress in order to predict crack initiation. However, the stress distribution can be obtained, at best, only under idealized conditions. The initiation of a crack can be estimated only if the tensile strength of the concrete, under the complex conditions of stress, is known. This is especially important

in the case of the spalling stresses that are confined to a small area. Also, frequently there are initial (mainly shrinkage) cracks or differential shrinkage stresses in the member that invalidate the "elastic" analysis. For these reasons, the usefulness of the purely elastic stress approach is limited in practice.

Hence, of the two important questions: "What starts a crack?" and "What stops a crack?", the first one cannot be answered with sufficient ease and accuracy. The new approach, used in this investigation, was more concerned with the second question. It attempted to find the effects and the propagation of the anchorage zone cracks. This led to the examination of the role of the transverse reinforcement in confining the crack. The method is direct and it has resulted in a conservative design procedure.

#### THE EFFECTS OF TRANSVERSE REINFORCEMENT ON CRACKING

The method of analysis of the effect of transverse reinforcement advanced in this paper is based on the equilibrium conditions of the cracked anchorage zone. The crack, admitted a priori, is to be restrained by the stirrup to a prescribed width.

The common method of designing reinforcement for end blocks is to compute the tensile stresses and forces according to some elastic solution and then to provide steel at an arbitrary working stress to carry the total tensile force or part of it. This approach ignores some important aspects of the behavior of end blocks. There is inelastic action at relatively low loads that changes the stress distribution. The concrete must be cracked before the reinforcement comes into action. The formation of a crack invalidates the elastic stress distribution. An initial

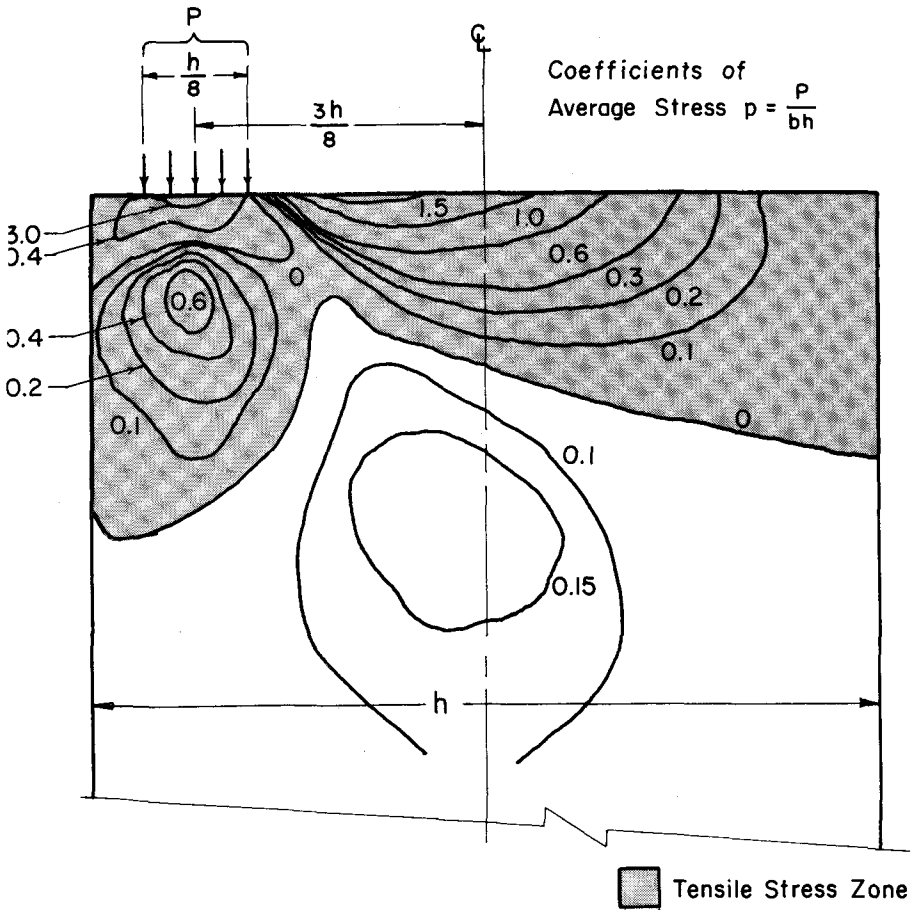


Fig. 1—Contours of Equal Transverse Stresses

crack (for example, at the junction of the web and the flange, as found in one of the test specimens of the present investigation) also modifies the elastic conditions. Even for an assumed elastic case, there is no generally accepted solution. In addition, the tensile strength of the concrete under complex conditions is not known sufficiently.

The analysis presented in this section investigates the conditions in a cracked end block in order to limit the length and width of the crack. A method is presented for estimating the position of the first crack. The equilibrium of the free body

bounded by the crack is investigated in order to estimate the internal forces. The relationship between the width and length of the crack and the stirrup force is also examined. The admitted longitudinal crack and the inside end face of the anchorage zone delineate the free body. The following quantities enter the analysis: applied prestressing force, stirrup force, the length and width of the crack, and the dimensions of the end block.

The forces acting on the free body having a rectangular section are shown in Fig. 2. The contribution of the tensile strength of the concrete

is ignored. The crack and the applied load are at distances of  $c$  and  $e$  from the bottom of the end block. The sketch on the top part of the figure illustrates the beam with the free body marked in full lines. The prism is shown enlarged in the bottom part of the figure.

The applied force  $P$  produces a linear stress distribution at a distance  $L$  from the end. To maintain equilibrium, there must be a moment  $M$  and a shearing force acting on the top part of the prism. The moment is to be supplied by the tensile force  $T$  in the reinforcement and by the compression  $C$  in the concrete. The height of the free body (that is, the position of the crack) will be determined from the condition that on that longitudinal section the moment will be the largest.

The moment on the longitudinal section is:

$$M = P \left[ c - e - \left( \frac{c}{h} \right)^2 \left( 2h - 3e - c + \frac{2ec}{h} \right) \right] \quad (1)$$

This moment (and, hence,  $T$  and

$C$ ) changes with the height of the free body,  $c$ . The moment takes extreme values for the following two values of  $c$ :

$$c_1 = \frac{h^2}{3(h-2e)} \quad \text{and} \quad c_2 = h \quad (2)$$

The first of these gives the maximum moment, the second gives the zero moment on the top surface. The magnitude of the former is:

$$M_{max} = P \left[ \frac{h^2}{27} \frac{4h - 9e}{(h - 2e)^2} - e \right] \quad (3)$$

The maximum moment may also occur along the line of the load. This moment can be obtained from the general expression of the moment by setting  $e = c$ . This yields

$$M_e = 2P \frac{e^2}{h^3} (h - e)^2 \quad (4)$$

Knowing the moment, the forces can be calculated if the distance between the forces can be estimated. The position of the tensile force is given by the center of gravity of the stirrup forces; the position of the centroid of the compressive force is not known. It is somewhere between the end of the crack and the

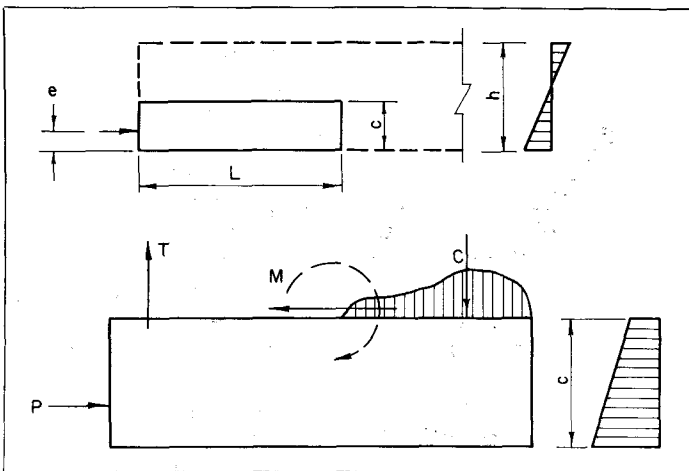


Fig. 2—Forces on the Free Body

plane where the normal-stress distribution becomes linear. If the position of the compressive force is assumed to be at the end of the crack, the design will be on the safe side. Thus, the length of the crack must be known or assumed. In addition to the relationship between the moment on the free body, the stirrup force, and the crack length, the stirrup force and the crack width must also be related in order to establish a crack width limitation. Force-slip relations can be obtained from bond tests. The crack width will be twice the slip if the bar is anchored similarly on both sides of the crack. The elongation of the steel between the surfaces of the crack is compared with the slip. However, the usual bond values cannot be applied directly here. The stirrup forces and the amounts of slip dealt with in this problem are small, while the customary bond-slip relationships are usually used near the ultimate conditions. For this reason, the uniform bond stress used to approximate the actual bond-slip distribution is smaller than the values generally used. The difference is illustrated in Fig. 3, where a uniform bond of  $3\sqrt{f'_c}$  (in force/length) was used as an approximation at smaller forces, while  $6\sqrt{f'_c}$  approximates the actual curve at high forces. This modification, corroborated by test results, is used in the design specification presented in this paper.

#### COMPARISON WITH TEST RESULTS

The analysis presented in the previous section is based on simplifying assumptions regarding the interaction of forces and the development of cracks. An experimental investigation was conducted to confirm the assumptions and to investigate possible new approaches to the

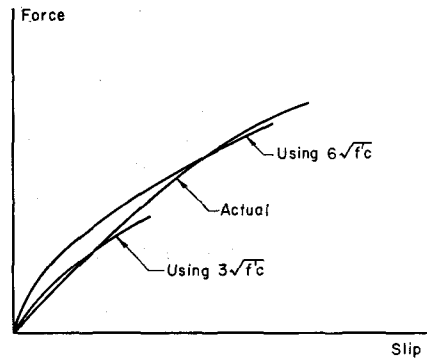


Fig. 3—Approximations to the Force-Slip Curve

solution.

The following factors and relationships were examined in the experimental investigation reported here: the position of the crack, the effects of the amount of reinforcement, the differences in the behavior of end zones with rectangular and I-shaped cross sections, the effect of the size of the loading plate, the variation of the forces and cracks with the applied load, and the relationship between the stirrup force and the crack width.

The position of the crack in the spalling zone is calculated from Equation 2. The expression yields 5.3 in. from the bottom for the rectangular beams used in this investigation. The crack was observed to occur at about 5.5 in. from the bottom in seven of the nine test specimens without reinforcement (not reported in this paper). The same calculated distance is 5.5 in. for the I-beams.

The measured relationship between the stirrup strain at the crack and the applied load was linear up to the yielding of the reinforcement. The smaller reinforcement (No. 7 USSWG) carried only little less force than the larger bar (No. 3 deformed bar). The crack width, how-

ever, was much larger in the case of the smaller size reinforcement.

When two reinforcing bars were placed in the specimen, they shared about equal portions of the total force. The bars were placed relatively close (0.5 and 2 in.) to the end of the beams. The force in each of the bars was about equal to the force in single bars, under equal loads.

The propagation of the cracks was carefully noted. The larger amount of reinforcement restricted the cracks more effectively. The rate of increase of the crack width and crack length with the applied load depends on the amount of reinforcement. With little reinforcement the crack width increased much faster than the crack length. With sufficient amount of reinforcement the crack length increased faster than the crack width.

According to the analysis, the unbalanced moment is smaller in the I-beams than on the rectangular beams. This results from the smaller moment arm between the applied load and the resultant of the longitudinal stresses on the free body. The ratio of the moments, for the sections used in this investigation, is 1.8. The comparison of the behavior of the rectangular and I-beams confirmed this interesting difference. The stirrup strains (measured at the crack in the spalling zone) were larger in rectangular beams (about 1.5 times in the case of the smallest amount of reinforcement) than in I-beams. Also, the cracks were wider and longer in the rectangular beams. These differences result from the different flow of forces in the two sections. In an I-beam with the force applied to the flange, a large portion of the force is confined to the flange. The force trajectories have smaller curvature. Consequently, the trans-

verse stresses are smaller in the end zones with I-shaped cross sections. In the rectangular beams the flow of stresses reaches more readily the "web" portion of the beam, with correspondingly larger stresses.

The ratio of measured stirrup forces in the rectangular and in the I-beams for specimens with adequate reinforcement (two No. 2 bars) was 1.7, that agrees well with the ratio of 1.8 given by the analysis. For insufficient amounts of reinforcement, the crack opens and the top part of the beam participates less and, hence, the ratio is smaller (1.5 for one No. 2 bar and 1.1 for one No. 7 USSWG). The ratio of measured crack widths in rectangular beam to that in I-beams was 1.4 for equal applied loads. The ratio of crack lengths was about 2.5 for specimens reinforced with two No. 2 bars.

The agreement between the predicted and measured stirrup forces was good for both I-beams and rectangular beams except for rectangular beams with little reinforcement. In this case the crack width would limit the stirrup force in the design method. To illustrate agreement, at an applied load of 20 kips the predicted stirrup force in an I-beam with one No. 2 bar was 0.75 kips (using a moment arm equal to the height of the beam less the distance between the stirrup and the end of the beam). The measured force was 0.63 kips. In the case of two No. 2 bars in I-beams the calculated force was 0.84 kips while the measured force was 0.88 kips. The corresponding values were 1.50 kips versus 1.54 kips for rectangular beams.

The effect of the size of the loading plate on the spalling stresses and the stirrup forces was small. This supports the equilibrium analysis. If the eccentricity of the ap-

plied load is small, the effect of the size of the loading plate is larger and may then be easily included in the calculation of the unbalanced moment, or it can safely be neglected.

For the relationship between the stirrup force and the crack width, an average bond (in force/length) was used in the form of  $m\sqrt{f'_c}$ , where  $m$  is a numerical coefficient. This results in the following expression

$$s = \frac{F^2}{2mEA\sqrt{f'_c}} \quad (5)$$

where  $F$  is the force in one bar,  $A$  is the area of one bar,  $s$  is the slip of the bar. This relationship represents a second degree parabola and since the actual variation has smaller curvature (is closer to being linear), the above approximation is good at certain intervals only, as it was explained in the previous section. For low stirrup forces (No. 2 or No. 3 bars)  $m = 4$  can be used. This value is reasonable if the limiting crack width is about 0.005 in.

The comparison between the experimental and analytical results indicates the feasibility of a design method based on the approach advanced above.

#### DESIGN RECOMMENDATIONS

The design method presented in this paper is based on the method of analysis outlined above. In essence, it admits the presence of a longitudinal crack in the anchorage zone and is concerned with the equilibrium of the beam portions on either side of the crack. Transverse reinforcement is provided to satisfy equilibrium for any possible position of the longitudinal crack. The force in the reinforcement is calculated from the maximum of the unbalanced moments, using an assumed moment

arm. The steel stress is controlled by a limiting crack width through an approximate force-slip relationship. The pivotal assumption is that there is a longitudinal crack in the anchorage zone. The prime role of the reinforcement is to confine the crack.

The method is best suited for the design of anchorage-zone reinforcement for loads of high eccentricity where the conditions of anchorage do not influence the forces in the spalling zone. If the load has small eccentricity, the size of the loading plate influences the forces along the line of the load, though the results given by this method are on the safe side. In the case of pretensioned tendons, the method can be used by arbitrarily concentrating the force transferred at a finite number of points within the anchorage length.

The approach used results in a straightforward method suited for use in the design office. First, the maximum moment acting on a longitudinal section is calculated, as described above. In order to estimate the moment arm of the stirrup forces and the compressive force of the concrete, the crack length has to be known. There is no practical way of estimating this quantity. Since linear stress distribution is reached at a distance of  $h$  (or less) from the end face, this value is taken as the upper limit for the distance between the compressive force and the end face. The stirrup force can be calculated by dividing the known moment by this assumed moment arm to obtain the total stirrup force. Bond-slip relationships are used to check if the crack width is less than a prescribed limit. If the actual moment arm is smaller than the one defined above, the stirrups will be overstressed. Then the limitation on the crack width will modify the design. The

method is illustrated below.

An approximate expression for the limit on the stirrup stresses can be derived from the bond-slip relationship presented above (Equation 5). It can be written as:

$$F^2 = 4EA\sqrt{f'_c}w \quad (6)$$

$$\text{or } f_s^2 = \frac{4E\sqrt{f'_c}w}{A} \quad (7)$$

where  $w$  is the crack width, and  $f_s$  is the stress in the stirrups. For  $f'_c = 5000$  psi, a representative concrete strength at release for prestressed concrete, the value of the square root of  $4E\sqrt{f'_c}$  is  $0.92 \times 10^5$ , which can be taken conservatively as  $1.0 \times 10^5$ . Therefore,

$$f_s = 10^5 \sqrt{\frac{w}{A}} \text{ psi} \quad (8)$$

Naturally,  $f_s$  should also be less than some allowable stress. This limiting stress, with the equilibrium analysis described above, constitutes the design procedure.

The determination of the maximum unbalanced moment is simple and can be obtained by using algebraic expressions. For I-shaped end zones the expressions are complicated and a trial and error procedure is necessary. However, for I-sections of common proportion, the use of Table I (see Appendix) will facilitate calculation of the position and the magnitude of the maximum unbalanced moment. The table is also applicable in the case of rectangular beams.

The maximum unbalanced moment is given by the moment of the forces shown in Fig. 4, as follows.

$$M_{max} = P_B(x - c_B) - (x - a_R)F_R - (x - a_T - t)F_T - (x - a_W)F_W \quad (9)$$

where  $P_B$  is the resultant of the bottom group of prestressing forces. The other quantities are shown in the figure, and are defined by the following equations:

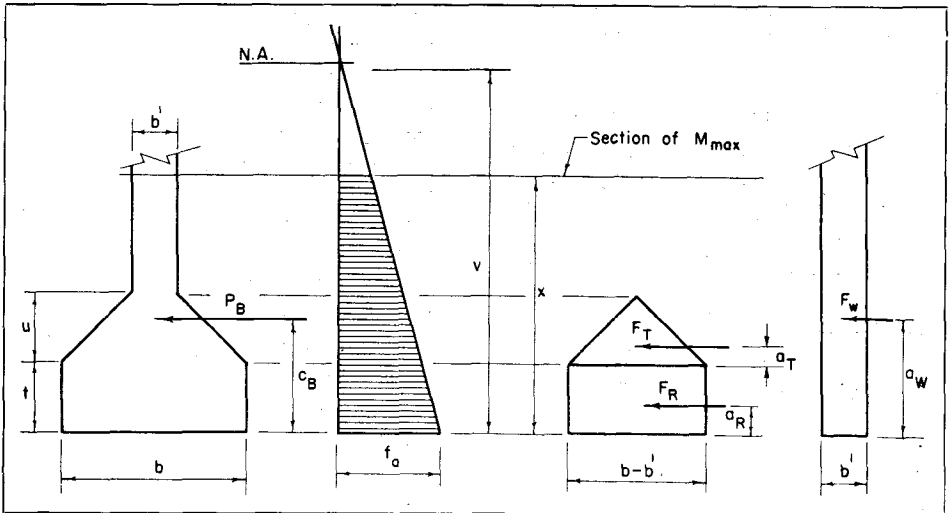


Fig. 4—Definition of Symbols for Table I



$$F_R = \left(1 - \frac{1}{2v/t}\right) t(b - b') f_a \quad (10a)$$

$$F_T = \frac{u}{6} \left(\frac{3v - u - 3t}{v}\right) (b - b') f_a \quad (10b)$$

$$F_W = \frac{x b' f_a}{2} \left(\frac{2v - x}{v}\right) \quad (10c)$$

$$a_R = \frac{t}{3} \left(\frac{3v - 2t}{2v - t}\right) \quad (11a)$$

$$a_T = \frac{u}{2} \left(\frac{2v - u - 2t}{3v - u - 3t}\right) \quad (11b)$$

$$a_W = \frac{x}{3} \left(\frac{3f - 2x}{2v - x}\right) \quad (11c)$$

$$x = v - \sqrt{v^2 - 2v c_B}$$

$$= v (1 - \sqrt{1 - 2c_B/v}) \quad (12a)$$

$$c_B = \frac{P_B - F_R - F_T}{b' f_a} \quad (12b)$$

Table I gives  $F_R$ ,  $a_R$ ,  $F_T$ ,  $a_T$  for various  $v/t$  and  $u/t$  values. Then only  $x$  and  $c_B$  have to be determined from the above equations for the use in the expression for  $M_{max}$ . The letter  $v$  designates the distance to the neutral axis of the stresses due to the total prestressing force (using  $\sigma =$

$P/A \pm Mc/I$ ) from the bottom of the beam. The maximum moment occurs either on a plane given by  $x$ , or along the line of the resultant force if  $v$  is large.

*Example:* The transverse reinforcement will be designed for the AASHO standard beam No. 1, shown in Fig. 5. The area and the moment of inertia of the section are 276 in.<sup>2</sup> and 22,750 in.<sup>4</sup>, respectively. The prestressing force is applied by 22 strands in the bottom part of the section and by two strands at the top of it and anchored at the end of the beam. The center of gravity of all the forces (a total of 336 kips) is at 6.17 in. from the bottom, while the similar distance for the resultant of the bottom strands is 4.37 in. The center of gravity of the cross section is 12.59 in. from the bottom. The bending stresses due to all prestressing forces are 2.41 ksi compression and 0.24 ksi tension. Hence:

$$v = 25.4 \text{ in.}, u = 5 \text{ in.}, t = 5 \text{ in.},$$

$$b = 16 \text{ in.}, b' = 6 \text{ in.}$$

$$f_a = 2.41 \text{ ksi}$$

$$(b - b') f_a = 24.1 \text{ k/in.}$$

$$u/t = 1, v/t = 5.1$$

Table I—Coefficients for the Determination of Forces and Their Position

$\frac{v}{t}$	$\frac{F_R}{(b - b') t f_a}$	$\frac{a_R}{t}$	$\frac{F_T}{u(b - b') f_a}$				$\frac{a_T}{u}$				
			$\frac{u}{t}$	0.9	1.0	1.1	1.2	0.9	1.0	1.1	1.2
3	.833	.465		.284	.278	.272	.267	.304	.300	.296	.292
3.5	.856	.472		.314	.310	.305	.300	.310	.308	.305	.302
4	.875	.476		.338	.333	.329	.325	.315	.312	.310	.308
4.5	.889	.479		.356	.352	.348	.346	.318	.316	.314	.312
5	.900	.482		.370	.367	.363	.360	.320	.318	.316	.315
5.5	.909	.483		.382	.379	.376	.373	.322	.320	.318	.317
6	.917	.485		.392	.389	.386	.383	.323	.322	.320	.319
6.5	.923	.486		.400	.398	.395	.392	.324	.323	.322	.321
7	.929	.487		.407	.405	.402	.400	.325	.324	.323	.322
7.5	.933	.488		.413	.411	.409	.407	.325	.324	.324	.323
8	.938	.488		.419	.417	.415	.412	.326	.325	.324	.324

therefore, using the proper values in Table I:

$$\begin{aligned}
 F_R &= 0.901 \times 5 \times 24.1 = 108.5 \text{ k.} \\
 a_R &= 0.482 \times 5 = 2.41 \text{ in.} \\
 F_T &= 0.368 \times 24.1 \times 5 = 44.3 \text{ k.} \\
 a_T &= 0.318 \times 5 = 1.59 \text{ in.}
 \end{aligned}$$

$$c_B = \frac{P_B - F_R - F_T}{b' f_a} = 10.74 \text{ in.}$$

$$\begin{aligned}
 x &= 25.4 - \sqrt{25.4^2 - 2 \times 25.4 \times 10.74} \\
 &= 15.8 \text{ in.}
 \end{aligned}$$

(the position of the plane of the maximum moment)

$$F_w = \frac{xb' f_a}{2} \left( \frac{2v - x}{v} \right) = 157 \text{ kips}$$

$$a_w = \frac{x}{3} \left( \frac{3v - 2x}{2v - x} \right) = 6.70 \text{ in.,}$$

hence the maximum moment is:

$$\begin{aligned}
 M_{max} &= 308 (15.8 - 4.37) \\
 &\quad - 108.5 (15.8 - 2.41) \\
 &\quad - 44.3 (15.8 - 1.59 - 5) \\
 &\quad - 157 (15.8 - 6.7) \\
 &= 237 \text{ k-in.}
 \end{aligned}$$

The resultant of the stirrup forces is placed at 5 in. from the end. Hence, the total stirrup force is  $237/23=10.3$  kips. Try No. 3 bars, area 0.11 in.<sup>2</sup> The allowable stress is

$$f_s = 10^5 \sqrt{\frac{0.005}{0.11}} = 21,400 \text{ psi,}$$

using an allowable crack width of 0.005 in. If this is less than a limiting steel stress (say 30,000 psi), the total stirrup area required is:

$$\frac{10.3}{21.4} = 0.48 \text{ in.}^2$$

Use three No. 3 two-legged stirrups, with a total area of  $2 \times 3 \times 0.11 = 0.66 \text{ in.}^2$

The first stirrup should be placed as close as possible to the end of the beam. Also, careful mixing and plac-

ing of concrete is very important. For draped pretensioned strands the point of action of the resultant prestressing forces can be taken at 25 diameters along the strands from the end. The stirrups should be well anchored at the top of the beams and should enclose all prestressing tendons. It is advisable not to have stirrup spacing larger than  $h/5$  over a distance of  $h$  from the end of the beam. (Hence, one more stirrup may be used in the above example.)

A similar procedure may be used to investigate the cracking parallel to the sides of the beam. Vertical cracks may appear in wide beams with prestressing forces anchored close to the side edges of the end face.

### SUMMARY

The transverse stresses created by the prestressing forces may cause

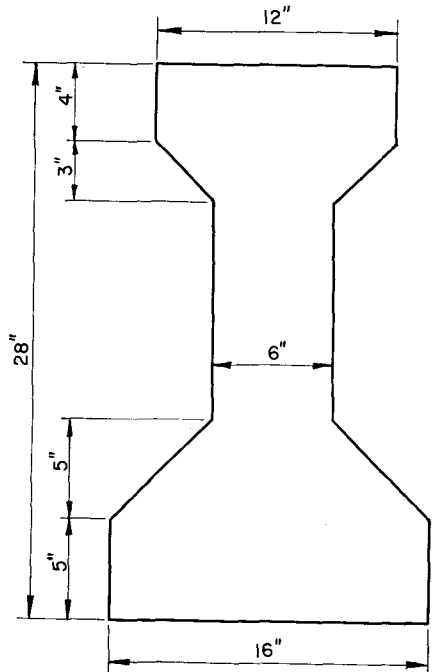


Fig. 5—AASHTO Standard Beam No. 1

longitudinal cracks in the end zones of beams. Many uncertainties make the prediction of the initiation of the first crack and the rate of propagation of existing cracks difficult. Also, no simple measures are available to prevent cracking even if the conditions were known.

The analysis of the equilibrium of the cracked end zone yielded expressions for the location of the crack and the magnitude of the unbalanced moment which creates the tension forces in the transverse steel. A simple procedure resulted for the design of stirrups to restrain the crack.

One series of tests confirmed the proposed method, the prediction of the location of the crack and the forces in the reinforcement. A second series of tests resulted in a force-slip relationship that was used to develop an expression to limit the width of the crack. Both test series are briefly described in the Appendix.

The investigation brought out the importance of the spalling stresses, transverse tensile stresses away from the line of the load, and showed that for eccentric prestressing forces these stresses are larger for rectangular than for I-shaped end blocks.

A table is included to simplify the design of stirrups based on the approach developed in the paper.

#### ACKNOWLEDGMENTS

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opinions, findings, and conclusions expressed in this publication are those of the authors and not necessarily those of the State of Illinois, Division of Highways, or the Bureau of Public Roads.

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#### APPENDIX EXPERIMENTAL INVESTIGATION

The experimental investigation, described briefly below consisted of two parts. The first part included tests on unreinforced beams (10 specimens) and beams with transverse reinforcement (25 specimens). Only the latter are reported here. The second part of the experimental investigation attempted to establish empirical bond-slip relationships for the use in the design method presented in this paper.

For both parts the concrete was made of Marquette brand Type III Portland cement, and of aggregate with maximum size of  $\frac{3}{8}$  in. The strength of the concrete ranged from 4500 to 5900 psi. Compression and split cylinder tensile strength tests were made for every batch.

Two kinds of transverse reinforcement were used: No. 2 deformed bars and No. 7 USSWG wires (with cross-sectional areas of 0.05 and 0.025 in<sup>2</sup>, respectively).

*(a) Beams with transverse reinforcement*

The dimensions of the beams are shown in Fig. A1. The specimens had either rectangular or I-shaped cross sections.

The loading of the specimens presented some difficulties. It was excessively troublesome to keep an external load aligned. After various loading systems had been tried, the one shown in Fig. A2 was adopted. A T-1 steel rod was cast unbonded in the concrete. It was lubricated to prevent bond. Paper wrapping was also used in some cases but that was later found to be unnecessary. On one end of the specimen a steel bearing block with a bearing area of 6 x 3 in. was used. A 50-ton center hole jack pressed on the steel block with the reaction supplied by a nut on the end of the rod. On the other end, the bearing area was 6 x 1.5 in. A dynamometer was placed between this block and the nut.

The instrumentation consisted of electric strain gages on the concrete,

placed transversely along longitudinal lines, strain gages on the transverse reinforcement and 0.0001 in. dials glued across the crack to measure crack width. (In some cases mechanical gages, Whittemore plugs, were also used to measure transverse deformations.)

In order to measure the stirrup strains at the crack, the gages had to be placed at the proper positions. Since the approximate location of the crack was known, the gage was placed at the corresponding elevation, and a pre-crack was made by placing a piece of plastic strip at the center of the gage to ensure cracking there.

The development of cracks was noted during testing. The loading was applied in about 15 increments to failure. Each test took less than two hours.

*(b) Bond tests*

Two kinds of bond tests were made. The simple pull-out specimens were 6-in. cubes with a single bar or wire protruding at the center of one

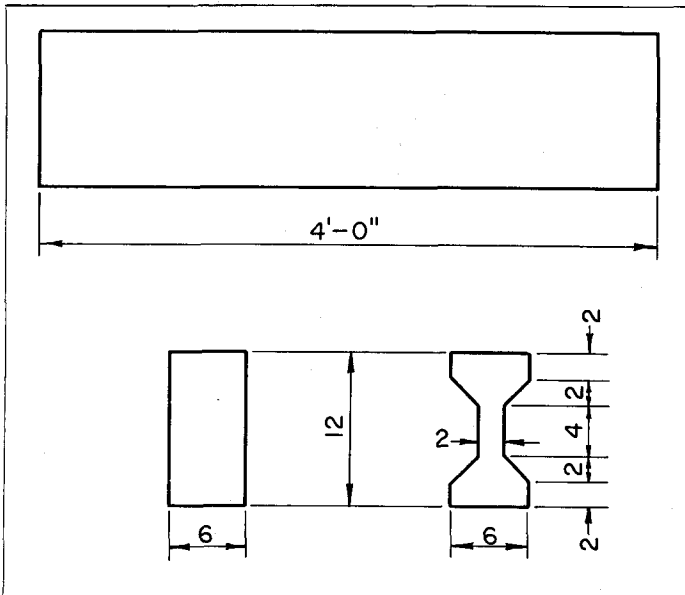


Fig. A1—Nominal Dimensions of Test Specimens

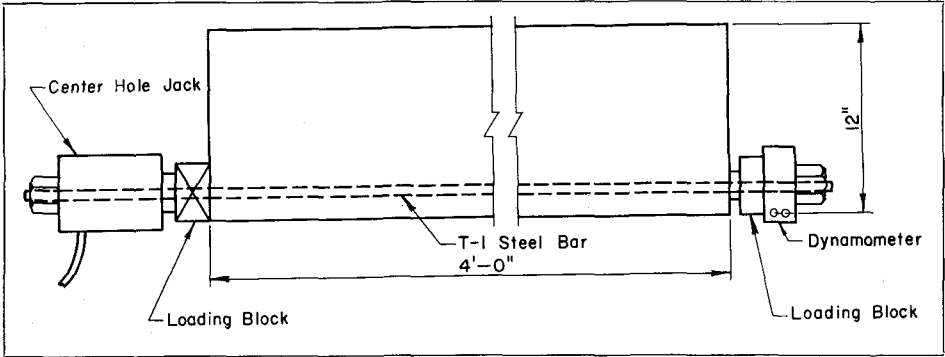


Fig. A2—Test Setup

face. To simulate the conditions in the end block, twin pull-out specimens were also made. In this case there was no pressure on the concrete around the bar that would produce confinement. Two symmetrical placed bars were pulled at the same time. The twin pull-out tests are illustrated in Fig. A3.

The single pull-out tests were made in the usual manner. The twin

pull-out specimens were loaded by a small jack pressing against the center of the block on an area of 4 x 4 in. on one end and against a dynamometer on its other end.

The slip of the bars was measured with a traveling microscope. An electric strain gage on one of the bars in the twin pull-out specimens served as a check on the loads in the symmetrically placed bars.

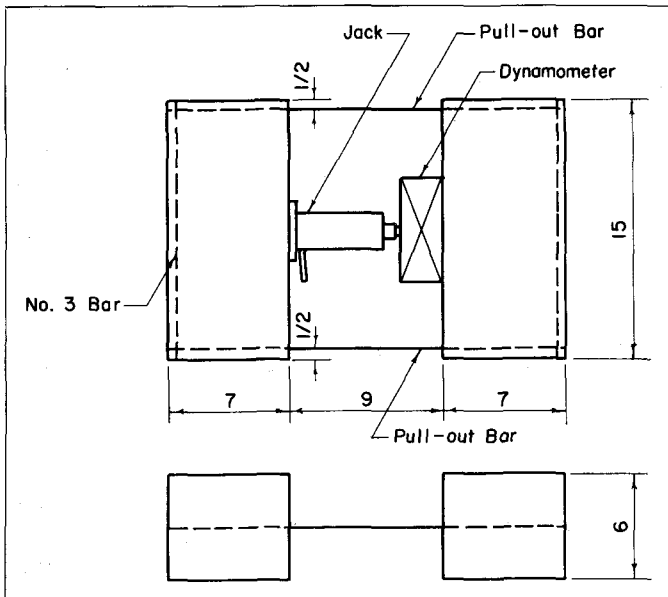


Fig. A3—Twin Pull-Out Specimens

Discussion of this paper is invited. Please forward your Discussion to PCI Headquarters before July 1 to permit publication in the October 1967 issue of the PCI JOURNAL.