DESIGN OF CONTINUOUS HIGHWAY BRIDGES WITH PRECAST, PRESTRESSED CONCRETE GIRDERS

Clifford L. Freyermuth
Design Research Section
Portland Cement Association
Skokie, Illinois

Continuous highway bridges with precast, prestressed girders have been built by a number of states. Examples of bridges of this type built by the states of Tennessee and California are presented in Figs. 1 and 2, respectively. The Big Sandy River Bridges in Tennessee were built in 1963-64. Performance has been excellent. The 639-ft. and 703-ft. continuous lengths of deck slab on these bridges are believed to be the longest built to date supported on precast, prestressed girders. The Los Penasquitos Bridge in California was a 1966 PCI award winner.

A primary reason for using continuity with precast, prestressed girders is the elimination of the maintenance costs associated with bridge deck joints and deck drainage onto the substructure. Continuity also improves the appearance and the riding qualities of this type of bridge. Due to the structural economy of continuous designs and the elimination of the deck joint details, some initial economic advantage may also be obtained.

The construction sequence for the type of bridge under consideration is shown in Fig. 3. Continuity is

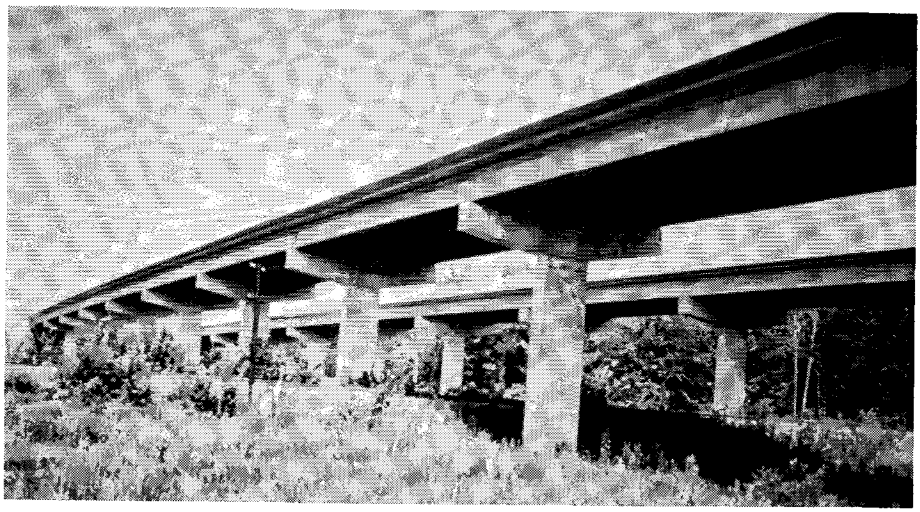


Fig. 1. Big Sandy River Bridges, Tennessee

To achieve continuity in a bridge using precast, prestressed single span units requires design considerations that also include the effects of creep and shrinkage. A complete design example illustrates these considerations following a general discussion of the principles.

achieved for live load plus impact moments by use of non-prestressed reinforcement in the deck slab and in the diaphragms over piers. An extensive research program on this type of bridge was conducted by the Research and Development Laboratories of the Portland Cement Association during 1960-61. The results of this research, which provide most of the background for this publication, have been made available in a series of six PCA Development Department Bulletins[1-6]. While the above research and the applications of this procedure to date have been directed primarily towards bridges built with I-beams, the procedure applies equally well to bridges with box beams, tee beams, or other available shapes.

The first portion of this report contains a general description of the design features peculiar to this type of bridge and the background material necessary to make design calculations. The second portion consists of a design example.

Part 1. Design Procedure

PRESTRESSING REINFORCEMENT

Except for the determination of the governing live load plus impact moment, the design of the prestressing reinforcement for continuous bridges follows the well-known procedure used for simple-span bridges[7]. The use of continuity permits a reduction of 5 to 15 percent in the required prestress force when compared to simple-span designs. The larger reductions occur in shorter span bridges where the live load plus impact moments are a large portion of the total design moment. Illustrative calculations for determining the governing live load plus impact moments and the required prestressing reinforcement are presented in the design example.

For bridges made up of a series of equal spans, the governing live load plus impact moment occurs at about the 0.4 or 0.5 point of the end spans, with the moment at the mid-point of interior spans being considerably less. Therefore, in some cases it may pay to make separate designs for the prestressing reinforcement in the interior and exterior spans. Where

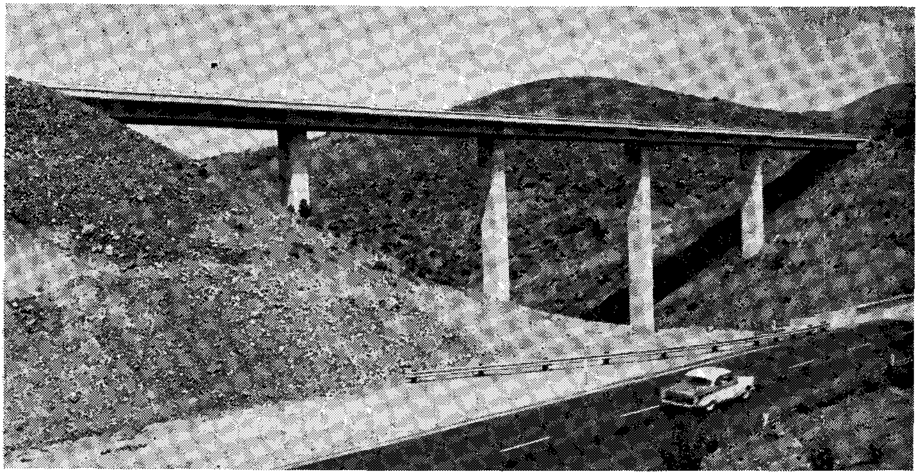


Fig. 2. Las Penasquitos Bridge, California

the designer has freedom in choosing the span layout, another alternative is to shorten the exterior spans to obtain the same amount of prestressing reinforcement in interior and exterior spans.

The positive restraint moments over piers, as described in the following section, also increase the positive moments at mid-span sections. When a section at mid-span is subject to the maximum service load moment plus the limiting value of the positive restraint moment at that section, it is recommended that the stress at the bottom face of the girder be limited to about 80 percent of the 28-day modulus of rupture of the concrete. The modulus of rupture is usually taken as $7.5\sqrt{f'_c}$. This requirement will usually be satisfied by the prestressing reinforcement supplied for gravity loads calculated as described in the paragraphs above.

POSITIVE MOMENTS AT PIERS

One of the unique features of the type of design under consideration is that positive moments develop over piers due to creep in the prestressed girders, as well as due to the effect of live loads in remote spans. These positive moments are partially counteracted by negative moments resulting from differential shrinkage between the cast-in-place deck slab and the precast girders. The positive live load plus impact moments over piers due to loads in remote spans are calculated by conventional procedures.

The effect of creep. The deformations and restraint moments induced in a two-span continuous beam by creep due to prestress force are illustrated in Fig. 4. Since creep is time dependent, with the more rapid deformation occurring during the early stages of loading, the amount of positive restraint moment induced by the prestress force depends on the time when the continuity connection is made, as well as on the inherent creep potential of the concrete mix, the exposure conditions, and the volume to surface ratio of the prestressed member.

The most accurate method of obtaining creep data is by extrapolation from a number of laboratory samples prepared in advance from the actual mix to be used in the girders. In instances where many beams are to be produced, it may be possible to take observations on the initial beams produced, and to develop the necessary creep data from those observations. In most design work, however, it will suffice to rely on available research data. Such research[8,9] indicates that the basic creep value for loading at 28 days can be predicted from the elastic modulus according to the curves in Fig. 5. For design purposes, the 20-year creep curve can be regarded as the ultimate creep.

The ultimate creep value for loading at 28 days, from Fig. 5, must be adjusted to account for the age when the girders are prestressed and for the volume/surface ratio of the girders. The effects of reinforcement on creep can usually be ignored for a prestressed beam. The variation of creep with age at loading is given by the curve in Fig. 6[8,10]. For loading when the concrete is one day old, for example, creep would be 1.8 times the value from the 20-year curve in Fig. 5. The variation of creep with the volume/surface ratio is shown in Fig. 7[8,11]. The volume/surface ratio of the AASHO-PCI I-beams, and the related volume/surface ratio creep correction factor, are listed in Table 1. The amount of creep which will take place after the continuity connection is made can be found from Fig. 8[8,12]. For example, for a connection made at 28 days, about 40 percent of the creep strains has taken place, leaving the remaining 60 percent to develop moments in the connection.

It has been shown[5] that the effects of creep under prestress and dead load can be evaluated by an elastic analysis assuming that the girder and slab were cast and pre-

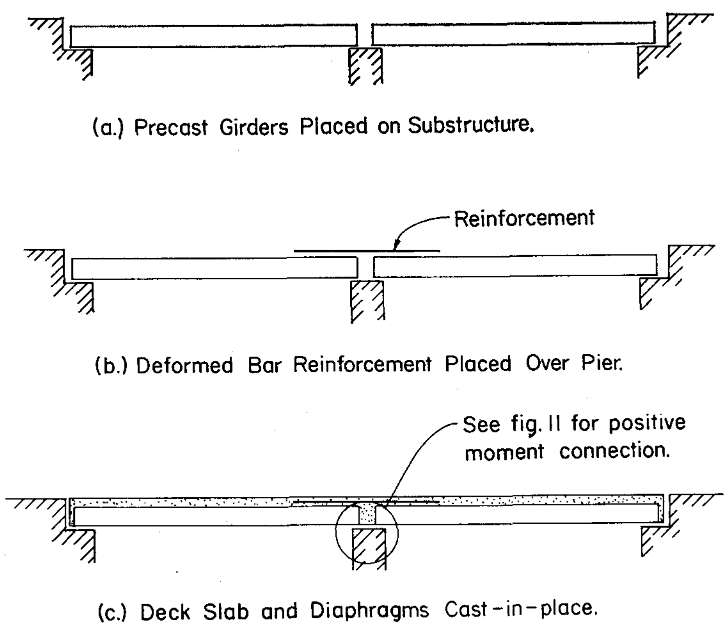
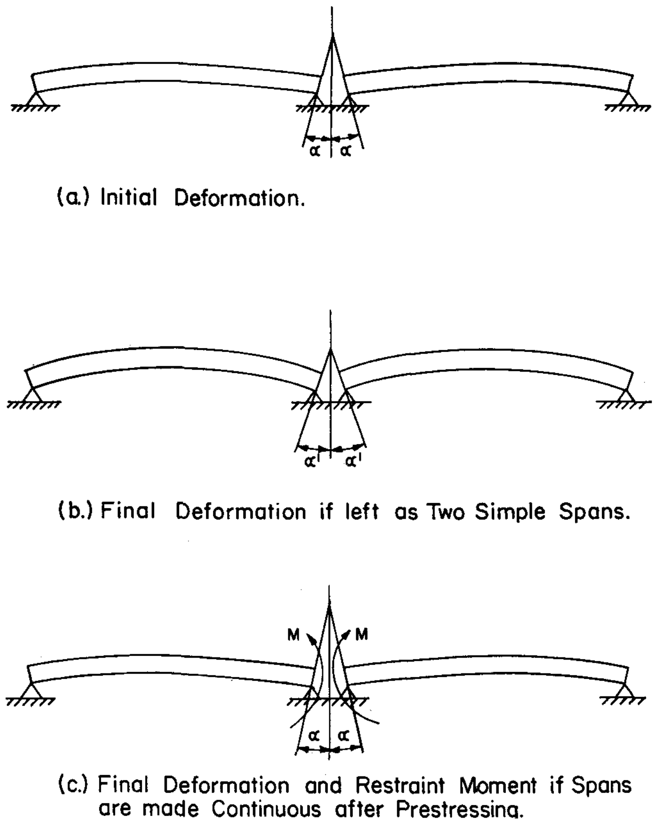


Fig. 3. Construction sequence for a two-span bridge with precast, prestressed girders made continuous for live loads

stressed as a monolithic continuous girder. The moments obtained from this analysis are multiplied by a factor $(1-e^{-\phi})$ to allow for the effect of creep. The factor ϕ is the ratio of creep to elastic strain (ϵ creep/ϵ elastic). This factor can be evaluated as follows:

1. For a Type VI AASHO-PCI I-beam with an elastic modulus of 4×10^6 psi at release of strands, the creep for loading at 28 days can be obtained from the 20-year loading curve in Fig. 5 as 0.34×10^{-6} in./in./psi.
2. Assuming strand release two days after casting, the multiplication factor from Fig. 6 is about 1.7, and the adjusted creep becomes: $1.7 \times 0.34 \times 10^{-6} = 0.58 \times 10^{-6}$ in./in./psi.
3. The volume/surface ratio for a Type VI AASHO-PCI beam is 4.4 and the correction factor from Fig. 7 is 1.18. Therefore, the further adjusted creep becomes $1.18 \times 0.58 \times 10^{-6} = 0.67 \times 10^{-6}$ in./in./psi.
4. Assuming the moment connection is made at 28 days, Fig. 8 indicates that 40 percent of the ultimate creep will have oc-



(a.) Initial Deformation.

(b.) Final Deformation if left as Two Simple Spans.

(c.) Final Deformation and Restraint Moment if Spans are made Continuous after Prestressing.

Fig. 4. Deformations and restraint moments in a two-span continuous beam caused by creep under prestress force

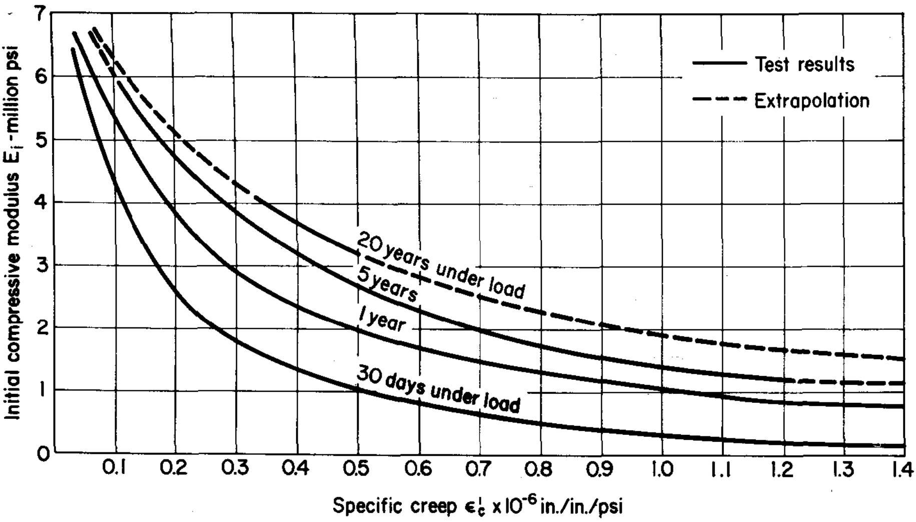


Fig. 5. Prediction of basic creep from elastic modulus

curred at the time the connection is made. The connection will have to be capable of developing moments that result from the remaining 60 percent, or, $0.60 \times 0.67 \times 10^{-6}$ in./in./psi $= 0.40 \times 10^{-6}$ in./in./psi.

$$\phi = \frac{\epsilon \text{ creep}}{\epsilon \text{ elastic}}$$

$$E = \frac{1}{\epsilon \text{ elastic}}$$

$$\phi = \epsilon \text{ creep} \times E$$

$$\phi = 0.40 \times 10^{-6} \times 4.0 \times 10^{6} = 1.60$$

Thus, for a connection at 28 days after prestressing, the creep strains still to occur are 1.60 times the original elastic strain.

5. Having evaluated ϕ, the value of $(1\text{-}e^{-\phi})$ can be taken directly from Fig. 9. This figure also contains a graph of the factor $(1\text{-}e^{-\phi})/\phi$ which is used to allow for the effect of creep in calculating the negative restraint moment due shrinkage.

Dead load creep partially counteracts prestress creep because dead load causes tension in the precompressed bottom fibers. The amount of creep bending that occurs depends on the residual compressive stress gradient under dead load. For this reason, more creep bending is likely to occur on shorter span bridges where a larger percentage of the precompression remains in the bottom of the beam under dead load.

The effect of shrinkage. Shrinkage of the deck slab with respect to the girders causes negative restraint moments over piers that reduce the creep restraint moment. As shown in a PCA Development Department Bulletin[(5)], the restraint moments at the piers due to shrinkage are calculated as if the differential shrinkage moment were applied to the continuous composite girder along its entire length, assuming the girder to behave elastically. The differential shrinkage moment applied to the girder along its entire length is given by:

$$M_s = \epsilon_s E_b A_b \left(e'_2 + \frac{t}{2}\right)$$

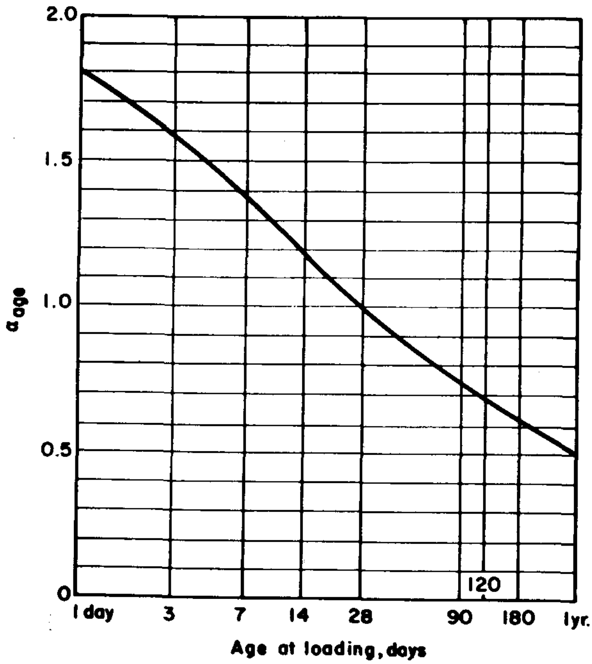


Fig. 6. Creep vs. age at loading

where ϵ_s = differential shrinkage strain (as described below)

E_b = elastic modulus for the deck slab concrete

A_b = cross-sectional area of deck slab

$\left(e'_2 + \frac{t}{2}\right)$ = distance between the mid depth of the slab and centroid of composite section

t = slab thickness

The most reliable means of estimating shrinkage values is from observations on specimens made from the mix to be used in the structure. With such observations, the final values of shrinkage can be projected by assuming the shrinkage-time curve can be represented by the following equations[10]:

$$\epsilon_s = \frac{\epsilon_{su} T}{N_s + T}$$

where ϵ_s = measured shrinkage strain at any time T in days

ϵ_{su} = ultimate shrinkage strain at T = infinity

$N_s = 26.0\ e^{0.36V/S}$

V/S is the volume/surface ratio of the member under consideration (see Table 1 for V/S ratios of the AASHO-PCI I-beams).

When test data are not available, the ultimate shrinkage under exposure at 50 percent relative humidity can be estimated[12,13] as 0.600 x 10^{-3}. This value should be corrected for very humid or very dry environments by multiplying by a humidity correction factor as presented in Fig. 10. The differential shrinkage strain

Table 1. Creep data for AASHO-PCI I-beams

AASHO-PCI girder type	Volume/surface ratio in.	Creep volume/surface ratio correction factor (from Fig. 7)
I	3.0	1.28
II	3.4	1.25
III	4.0	1.20
IV	4.7	1.16
V	4.4	1.18
VI	4.4	1.18

between slab and girder for a given time lapse between castings can then be obtained by using the time-shrinkage relationship in Fig. 8. For example, with a 35-day lapse between casting of the girder and the slab, Fig. 8 indicates that about 0.42 of the girder shrinkage has taken place. Thus a slab exposed to 50 percent relative humidity would have $0.42 \times 0.600 \times 10^{-3}$ in./in. or 0.252×10^{-3} in./in. more shrinkage remaining than the girder. The value of 0.252×10^{-3} would be used for ϵ_s in the above equation for M_s. This value checks very closely the value of 0.245×10^{-3} in./in. obtained in the PCA research[5] using 10 ft. long free shrinkage specimens of the slab and girder and a time lag between cast-

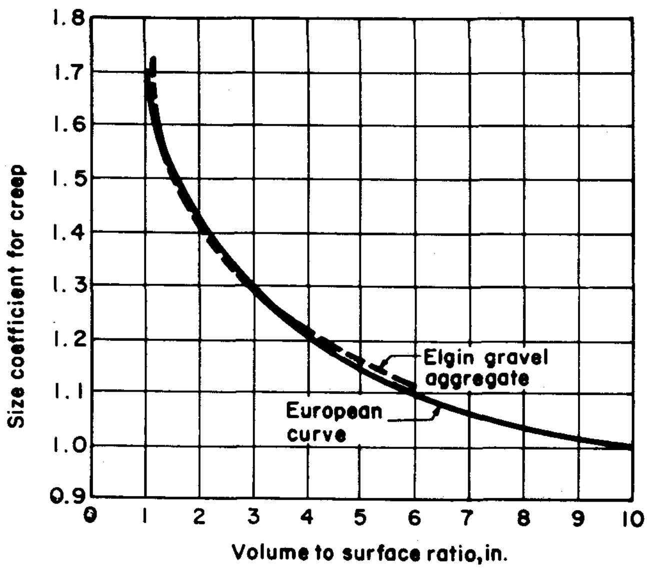


Fig. 7. Creep vs. volume-to-surface ratio

ings of 35 days.

After calculating the negative elastic moments at the piers due to shrinkage, they must be modified for the effect of creep by multiplying by the factor $(1\text{-}e^{-\phi})/\phi$. A graph of this factor with respect to ϕ is presented in Fig. 9.

Summation of restraint moments. The final positive restraint moment over piers is calculated as:

$$M_r = (Y_c - Y_{\mathrm{DL}})(1 - e^{-\phi}) - Y_s\left(\frac{1 - e^{-\phi}}{\phi}\right) + Y_{\mathrm{LL}}$$

where M_r = final restraint moment

Y_c = restraint moment at a pier due to creep under prestress force

Y_{DL} = restraint moment at a pier due to creep under dead load

Y_s = restraint moment at a pier due to differential shrinkage between the slab and girder

Y_{LL} = positive live load plus impact moment.

Details of the positive moment connection. In the PCA Development Department Bulletin on this subject[3], various connection details to resist the positive restraint moment were evaluated. The detail considered most practical is shown in Fig. 11.

In the tests run on this connection, the hooked bars were bent essentially at right angles and a relatively short horizontal projection was used to conserve on the width of the diaphragm. Although the connections developed substantial moments under static test loads, under fatigue testing (with a stress range of 20,000 psi) most of the connection bars failed after about 670,000 applications of the load. The bars fractured in a brittle manner at the knee of the hook. To avoid this type of failure, the live load plus impact stress range in the bars *at the point where the bend starts* should be limited to 50

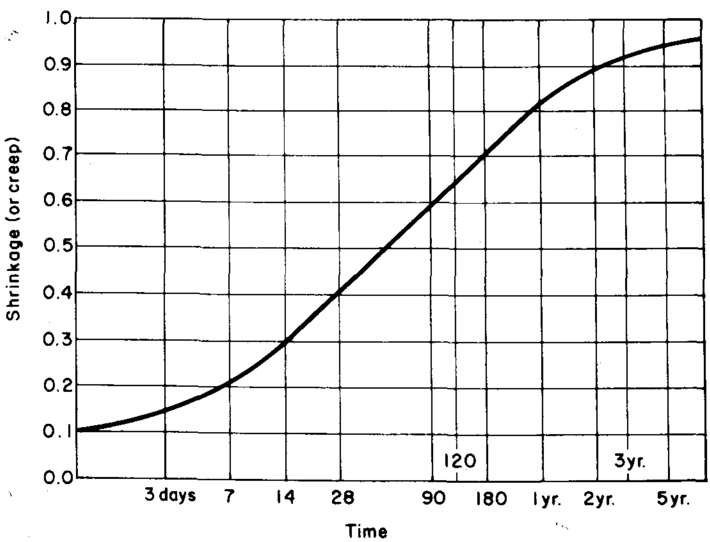


Fig. 8. Proportion of final shrinkage or creep vs. time

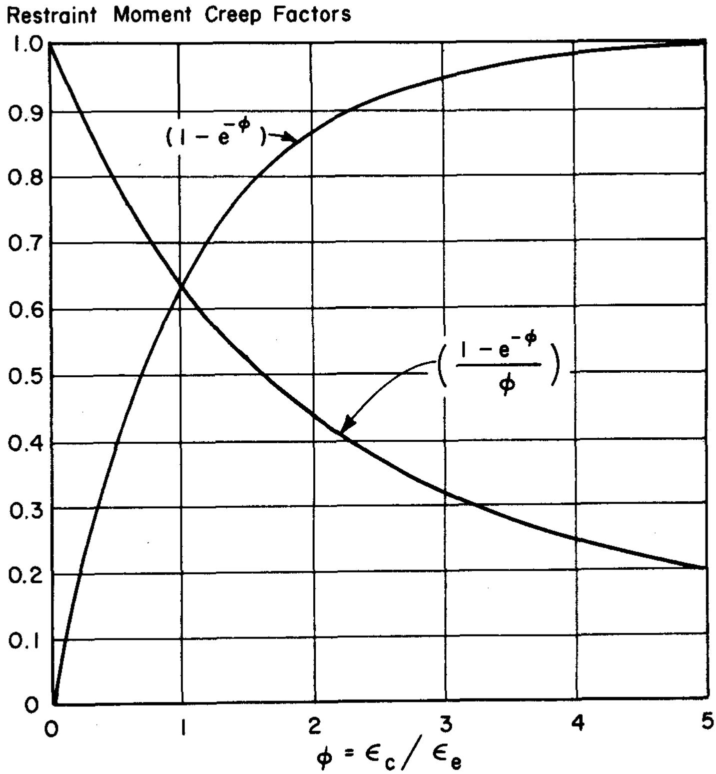


Fig. 9. Variations in restraint moment creep effect factors for both creep and shrinkage

percent of the fatigue strength[14]. A conservative value for this stress range is 12,000 psi[15]. Connections designed in accordance with the recommendations in the following paragraphs will usually conform to this stress range restriction.

The total length of embedment required for the hooked connection bars can be calculated considering bond to develop uniformly from the face of the beam around the bend to the end of the bar[16]. The unit bond stress should be 0.10 f'_c (maximum 350 psi) as prescribed in the AASHO Specifications[19]. The distance from the inside face of the precast girder to the inside face of the hook should be at least 12 times the bar diameter. The minimum bend radius should conform to Table 2.

Table 2. Minimum bar bend radii

Bar Size	Minimum Radii
#3, #4, #5	2½ bar diameters
#6, #7, #8	3 bar diameters

As determined in the tests at the PCA Laboratories, the restraint moment over the piers does not affect the factor of safety of the structure. For this reason, it was suggested that

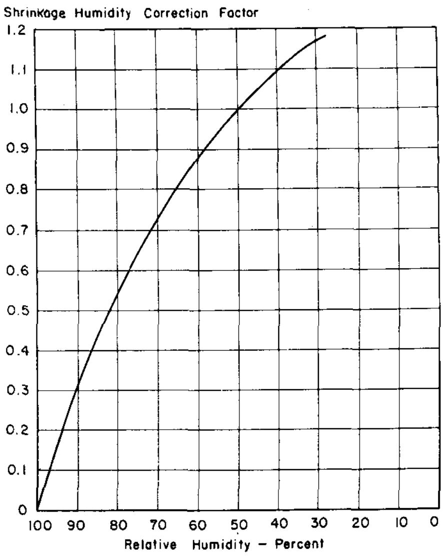


Fig. 10. Shrinkage humidity correction factor

the positive moment continuity reinforcement be designed to work at 75 percent of its yield point when subject to the limiting value of the restraint moment due to creep and differential shrinkage together with the maximum positive moment due to live load plus impact in remote spans. However, when general research data are used to estimate creep and shrinkage coefficients, it is recommended that the design be based on the lower stress of 0.6 times the yield strength. This lower stress will reduce the live load plus impact stress range, and it will also provide more assurance against the possibility of diaphragm cracking.

Due to the amount of calculation involved in the design of a positive moment connection, as described above, it is suggested that standard details be developed for the various prestressed sections commonly used. A practical consideration in designing the positive moment connection is to detail it to facilitate the removal of the end girder form after prestressing. This may require that the connection bars be cast in the beam straight and then field bent after removal of the end form. However, in some areas, end forms have been developed to accommodate shop bent bars. If field bending is re-

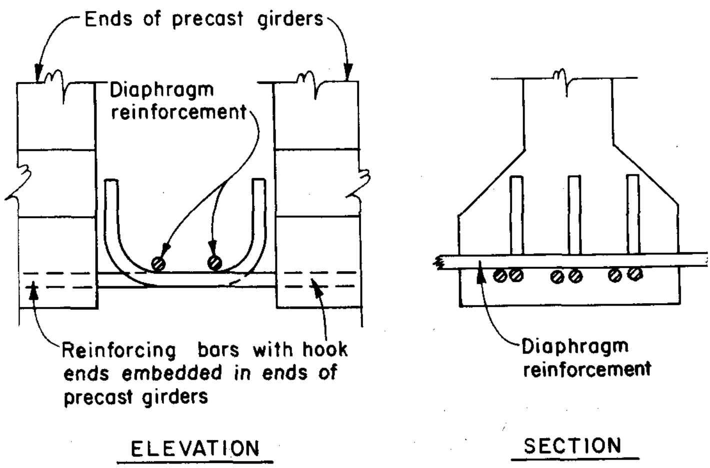


Fig. 11. Positive restraint moment connection details

quired, No. 6 connection bars should probably be the maximum size used. Consultation with local prestressed concrete fabricators is recommended to develop practical and economical positive moment connection details for use in a particular area. The connection and bearing details for the Big Sandy River Bridges are shown in Fig. 12.

NEGATIVE MOMENT REINFORCEMENT

Since most of the dead load moments are carried by the beam acting as a simple span, the negative design moments over piers are those due to live load plus impact. In some designs, the dead load applied after continuity is achieved may also be considered in the negative design moment. The recommended design procedure for proportioning negative moment reinforcement is to use ultimate strength procedures with a load factor of 2.5 on live load and 1.5 on dead load. The effect of initial precompression due to prestress in the precast girders may be neglected in the negative moment computation of ultimate strength if the maximum precompression stress is less than 0.4 f'_c and the continuity reinforcement is less than 1.5 percent[1]. A design aid to facilitate determination of the required steel percentage is presented in Fig. 13[17]. Use of this aid is illustrated in the design example.

It will usually be found that the depth of the compression block will be less than the thickness of the bottom flange of the precast girder. For this reason, the negative moment reinforcement required can be determined by assuming the beam to be a rectangular section with a width equal to the bottom flange width of the girder. Due to the lateral restraint of the diaphragm concrete, ultimate negative compression fail-

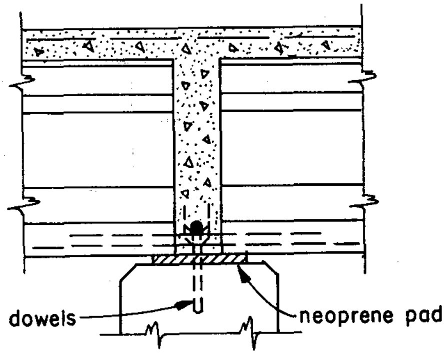


Fig. 12. Bearing and connection details at piers, Big Sandy River Bridges, Tennessee

ure in the PCA tests[1] always occurred in the girders, even though the diaphragm concrete strength was about 2000 psi less than that of the girder concrete. For this reason, it is recommended that the negative moment reinforcement be designed using the compressive strength of the girder concrete.

COMPRESSIVE STRESS IN GIRDERS AT PIERS

A point that concerns some designers is the high compression stresses that might theoretically occur due to addition of stresses due to prestress and negative live load bending over the supports. The PCA studies indicate that these stresses do not affect the ultimate strength for the following reasons:

1. Stress relief is obtained through inelastic strains.
2. Full compression does not occur at beam ends due to the strand transfer length.
3. Diaphragms strengthen the area by providing lateral restraint.

In addition, the effect of the negative live load plus impact moment is reduced by the positive creep restraint moment. However, when con-

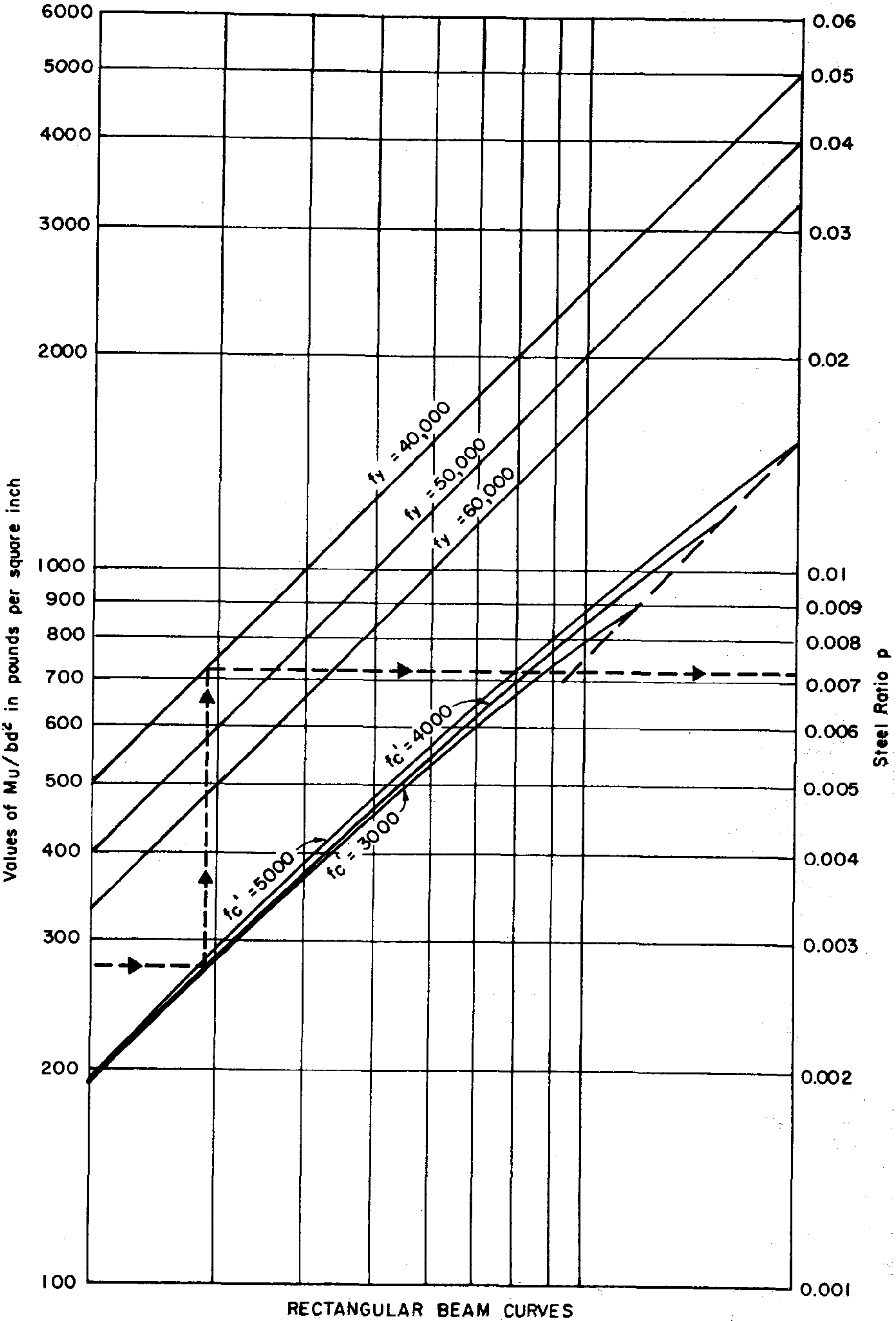


Fig. 13. Design aid for negative moment reinforcement

sidered essential, the theoretical precompression can be reduced by raising the strand center of gravity and/or by blanketing some of the strands at the girder ends. Tests of girders with blanketed prestressing strands have been made at the PCA Laboratories[18].

SHEAR

The shear tests of continuous beams conducted at the PCA Laboratories[4] revealed that the present shear provisions in the American Association of State Highway Officials (AASHO) Specifications for Highway Bridges[19] provide a conservative estimate of the shear capacity of a continuous beam. However, on continuous beams the formulas must be applied over the full length of the beams rather than only over the middle half of the spans, as is usually the case for simply supported girders.

The horizontal shear stress between the cast-in-place slab and the precast girder was also investigated at the PCA Laboratories[4,5]. These tests found the horizontal shear stresses between slab and beam permitted by the AASHO Specifications to be very conservative.

Part 2. Design Example

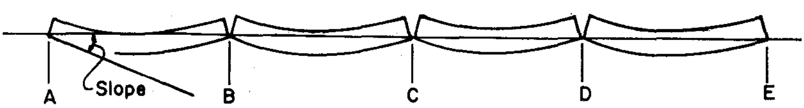
DERIVATION OF GENERAL FORMULAS—FOUR EQUAL CONTINUOUS SPANS

The design example is for a bridge with four equal spans of 130 ft. Prior to the detailed calculations, development of general formulas for restraint moments due to shrinkage and creep is presented. Similar developments for bridges with any number of equal or unequal spans can be made by use of the same procedure or any other consistent procedure for the analysis of indeterminant structures. The conjugate beam theory is used to calculate the various fixed-end restraint moments. The final restraint moments are then obtained by moment distribution.

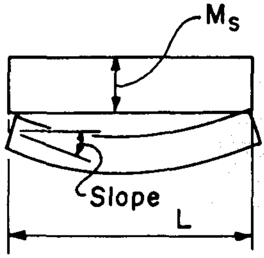
Shrinkage restraint moments. As previously discussed, differential shrinkage between the deck slab and the prestressed beam causes the following moment on the composite section:

$$M_s = \epsilon_s E_b A_b \left(e_2' + \frac{t}{2} \right)$$

If the bridge were composed of 4 equal simple spans, this moment would cause bending as shown below:



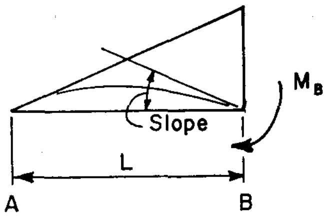
The slope at the ends of the simple-span beams may be obtained as the reaction of the ML/EI diagram:



$$\text{Slope} = \frac{M_s L}{2\,EI}$$

Apply moments at B, C and D to return the beam ends to a horizontal position:

End Span



$$\text{Slope at B} = -\frac{M_B L}{2\,E\,I} \times \frac{2}{3}$$

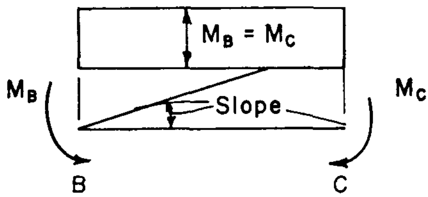
$$= -\frac{M_B L}{3\,E\,I}$$

Set slope equal to slope due to shrinkage:

$$-\frac{M_B L}{3\,E\,I} = \frac{M_s L}{2\,E\,I}$$

$$M_B = -\frac{3}{2} M_s$$

Interior Span



$$\text{Slope at B and C} = -\frac{M_B L}{2\,E\,I}$$

Set slope equal to slope due to shrinkage:

$$-\frac{M_B L}{2\,E\,I} = \frac{M_s L}{2\,E\,I}$$

$$M_B = -M_s = M_C$$

Since different moments are required to bring the various beams to a horizontal position, the moments must be distributed to obtain the final shrinkage restraint moments.

	A–B (at B)	B–C (at B)	B–C (at C)	C–D (at C)	C–D (at D)	D–E (at D)
Distribution factor	3/7	4/7	1/2	1/2	4/7	3/7
	$-1.5\,M_s$ $+.214\,M_s$	$-M_s$ $-.286\,M_s$	$-M_s$	$-M_s$	$-M_s$ $-.286\,M_s$	$-1.5\,M_s$ $+.214\,M_s$
			$+.143\,M_s$	$+.143\,M_s$		
Final	$-1.286\,M_s$ (B)		$-.857\,M_s$ (C)		$-1.286\,M_s$ (D)	

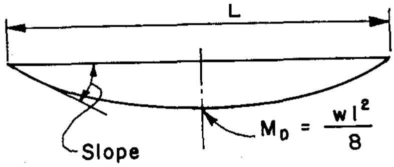
A, B, C, D, E

Final shrinkage restraint moments (not reduced by the creep effect factor) are $-1.286\,M_s$ at B and D, and $-0.857\,M_s$ at C.

Creep restraint moments. The creep restraint moment is broken into two components, one due to dead load and the other due to prestress.

Dead Load Creep Restraint Moment

$$\text{Slope} = \frac{M_D L}{3\,E\,I}$$

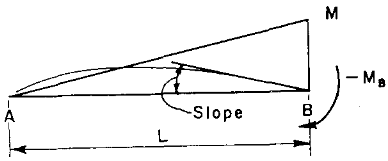


Apply moments to return beam ends to horizontal:

End Span

$$-\frac{M_B L}{3\,E\,I} = \frac{M_D L}{3\,E\,I}$$

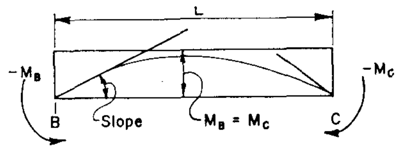
$$M_B = -M_D$$



Interior Span

$$-\frac{M_B L}{2\,E\,I} = \frac{M_D L}{3\,E\,I}$$

$$M_B = 2/3\,M_D = M_C$$



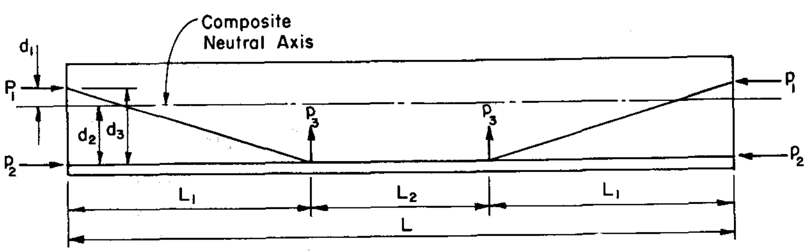
Distribute the unequal end moments due to creep under dead load:

A	D	D	C	C	D	D	E
	3/7	4/7	1/2	1/2	4/7	3/7	
	$-M_D$	$-.667M_D$	$-.667M_D$	$-.667M_D$	$-.667M_D$	$-M_D$	
	$+.14M_D$	$-.19M_D$			$-.19M_D$	$+.14M_D$	
			$+.095M_D$	$+.095M_D$			
	$-.86M_D$		$-.572M_D$		$-.86M_D$		

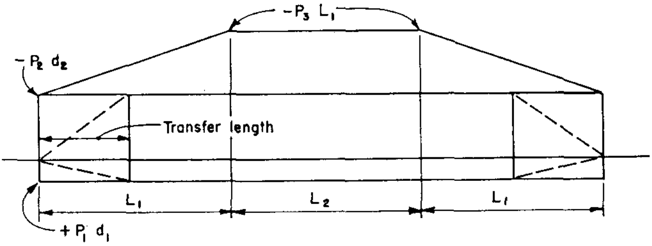
Final dead load creep restraint moments (not reduced by the creep effect factor) are $-0.86\,M_D$ at B and D, and $-0.572\,M_D$ at C.

Prestress Creep Restraint Moment

The creep under the prestress force depends on the location of the various forces with respect to the center of gravity of the composite section. The tendons will be replaced in the calculations by the equivalent forces exerted on the member.

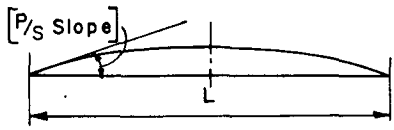


The moment diagram by parts for the above forces is as follows:



Moments might more realistically be considered to vary from zero to full value along the transfer length (dotted lines). This would reduce the creep under the prestress force. Also, any blanketed strand could be disregarded for the length of the blanketing. For convenience, the rectangular moment diagrams will be used in the following.

Slope due to prestress (abbreviated P/S)



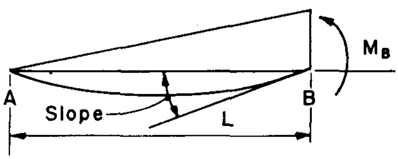
$$[\text{P/S Slope}] = \frac{P_1 d_1 L}{2EI} - \frac{P_2 d_2 L}{2EI} - \frac{P_3 L_1^2}{2EI} - \frac{P_3 L_1 L_2}{2EI}$$

Apply moments to bring beam ends at B, C and D to horizontal:

End Span

$$\frac{M_B L}{3EI} = -\ [\text{P/S Slope}]$$

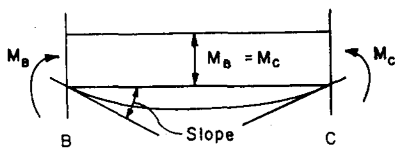
$$M_B = -\frac{3EI}{L}[\text{P/S Slope}]$$



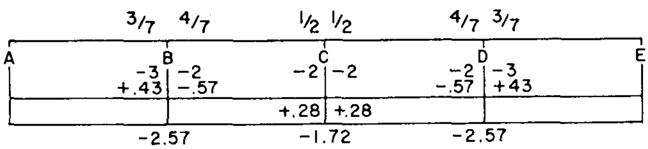
Interior Span

$$\frac{M_B L}{2EI} = -\ [\text{P/S Slope}]$$

$$M_B = M_C = -\frac{2EI}{L}[\text{P/S Slope}]$$



Distribute moments (for convenience distribute −3 and −2):



Final creep restraint moments due to prestress (not reduced by creep effect factor) are $\frac{-2.57\,EI}{L}$ [P/S Slope] at B and D, and

$\frac{-1.72\,E\,I}{L}$ [P/S Slope] at C. Recognizing that the value of [P/S Slope] will be negative, the prestress restraint moment will then have a positive value.

Summing up the various restraint moments and multiplying by the appropriate factors, the final restraint moments due to creep and shrinkage can be calculated as:

$$\text{at B and D} = \left(\frac{-2.57\,E\,I}{L}\,[\text{P/S Slope}] - 0.86\,M_D\right)(1 - e^{-\phi})$$

$$- (1.286\,M_s)\left(\frac{1 - e^{-\phi}}{\phi}\right)$$

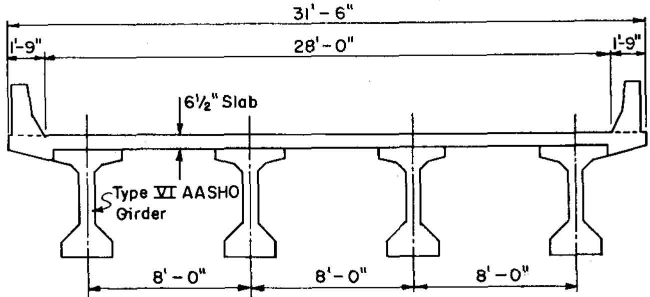
$$\text{at C} = \left(\frac{-1.72\,E\,I}{L}\,[\text{P/S Slope}] - 0.572\,M_D\right)(1 - e^{-\phi})$$

$$- (0.857\,M_s)\left(\frac{1 - e^{-\phi}}{\phi}\right)$$

For a given value of ϕ, the values of $(1 - e^{-\phi})$ and $\left(\frac{1 - e^{-\phi}}{\phi}\right)$ may be taken directly from Fig. 9.

SAMPLE CALCULATIONS FOR A BRIDGE WITH FOUR EQUAL 130-FT. SPANS

This design example for the interior girder of a bridge with four equal 130-ft. spans was chosen largely to illustrate that this type of construction is applicable to the entire range of bridge spans for which precast, prestressed girders would usually be considered. More numerous applications will undoubtedly occur for shorter spans. In states which have laws prohibiting highway transportation of the longer and heavier precast girders, the girders may be cast in segments and connected at the bridge site. A comprehensive study of methods of connecting precast segments to achieve long spans has been published by the Prestressed Concrete Institute[20].



SECTION

Design criteria:

	Concrete Strength	E
Prestressed beam at release:	4000 psi	3.66×10^6 psi
Prestressed beam at 28 days:	5000 psi	4.05×10^6 psi
Deck slab and diaphragm:	4500 psi	3.88×10^6 psi

Loading: AASHO HS20-44

Prestressing strand: 1/2″ ϕ, 270 k

Area = 0.153 in.^{2}

Final force (after losses) = 23.6 k/strand

Section properties:

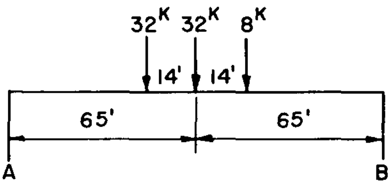
Prestressed beam		Composite girder (interior)	
Area =	1085 in.^{2}	Area =	1676 in.^{2}
$\bar{y}$ =	36.38 in.	STC =	59,977 in.^{3}
I =	733,320 in.^{4}	SBC =	26,266 in.^{3}
ST =	20,587 in.^{3}	STSC =	46,252 in.^{3}
SB =	20,157 in.^{3}	I =	1,313,811 in.^{4}

Design of prestressing reinforcement

Impact = 50/255 = 19.6%

$$LL = \frac{8.0}{5.5 \times 2} = 0.727 \text{ lanes}$$

Moment at 0.5 Span AB:



Using influence lines for 4 equal continuous spans:

$0.1517 \times 32 \times 130 = 631$ ft. kips
$0.1998 \times 32 \times 130 = 830$
$0.1449 \times 8 \times 130 = 150$
Total for 1 lane $= 1611$ ft. kips

LL + I moment at
0.5 Span AB $= 1.196 \times 0.727 \times 1611 = 1403$ ft. kips

Dead load moments at 0.5 Span AB:

Beam $= 1085/144 \times 0.150 \times 130^2/8 = 2390$ ft. kips
Slab $= 96 \times 6.5/144 \times 0.150 \times 130^2/8 = 1372$
Diaphragm $= 3.30 \times 130/3 = 143$
$M_D = 3905$ ft. kips

Stresses at 0.5 Span AB

	Top	Bottom
Slab	802 psi	1425 psi
Beam	1395	817
Diaphragm	83	86
LL + I	282	643
	2562 psi	2971 psi
Allowable tension	212	212
	2774 psi	2759 psi

Assume eccentricity of strand center of gravity from bottom of beam = 4.70 in. Eccentricity from center of gravity:

$$36.38 - 4.70 = 31.68 \text{ in.}$$

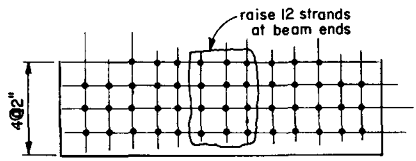
Calculate force required to give a bottom flange stress of 2759 psi:

$$2759 = F/1085 + F \times 31.68/20.157$$
$$F = 1{,}110{,}000 \text{ lb.}$$

Strands required = 1,110,000/23,600 = 47.1 strands
Use 48 – 1/2″ ϕ 270 k strands

Check c.g. of strands compared to assumed value:

13 at 2 =	26
13 at 4 =	52
13 at 6 =	78
9 at 8 =	72
48	228



228/48 = 4.75 in.
(close enough to assumed value)
48 strands at 23.6 = 1132 kips
36 strands at 23.6 = 850 kips
12 strands at 23.6 = 283 kips

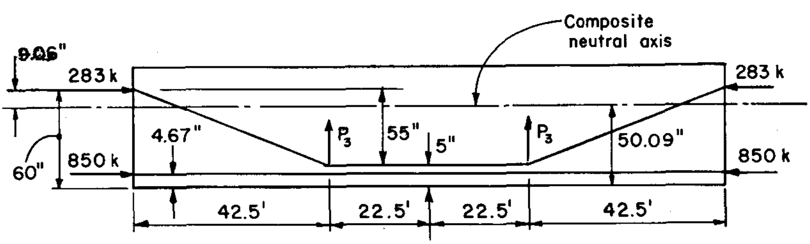
Check eccentricity at girder ends for no tension under working loads:

$$0 = 1132/1085 - 1132e/20{,}587$$
$$e = 19 \text{ in. (17.38 in. from bottom)}$$

Center of gravity of 12 harped strands must be about 55 in. from the bottom to provide this eccentricity. Raise center of gravity of 12 strands 60 in. at ends to reduce compressive stress in bottom fibers.

Design for positive moments at piers

Prestress Creep Restraint Moments (not adjusted by creep effect factor)



$$P_3 = 55/42.5 \times 12 \times 283 = 30.5 \text{ kips}$$

$$[\text{P/S Slope}] = \frac{P_1 d_1 L}{2\,E\,I} - \frac{P_2 d_2 L}{2\,E\,I} - \frac{P_3 L_1^2}{2\,E\,I} - \frac{P_3 L_1 L_2}{2\,E\,I}$$

$$\frac{P_1 d_1 L}{2\,E\,I} = 283 \times 9.91 \times 130 \times 12/2 \quad = + \quad 2{,}190{,}000 \text{ in. kips}$$

$$-\frac{P_2 d_2 L}{2\,E\,I} = -850 \times 45.42 \times 130 \times 12/2 = -\ 30{,}100{,}000$$

$$-\frac{P_3 L_1^2}{2\,E\,I} = -30.5 \times 42.5^2 \times 144/2 \quad = - \quad 3{,}960{,}000$$

$$-\frac{P_3 L_1 L_2}{2\,E\,I} = -30.5 \times 42.5 \times 45 \times 144/2 = - \quad \underline{4{,}200{,}000}$$

$$[\text{P/S Slope}] \quad = -\frac{36{,}070{,}000}{E\,I} \text{ in. kips}$$

$$\text{at B} = \frac{-2.57\,EI}{130 \times 12}\left[-\frac{36{,}070{,}000}{E\,I}\right] = 59.4 \times 10^6 \text{ in. lb.} = 4960 \text{ ft. kips}$$

$$\text{at C} = \frac{-1.72\,EI}{130 \times 12}\left[-\frac{36{,}070{,}000}{E\,I}\right] = 39.8 \times 10^6 \text{ in. lb.} = 3320 \text{ ft. kips}$$

Dead Load Creep Restraint Moments (not adjusted by creep effect factor)

at B $= -0.86 \times 3905 = -3360$ ft. kips

at C $= -0.572 \times 3905 = -2230$ ft. kips

Shrinkage Retraint Moments (not adjusted by creep effect factor)

Assuming continuity connection made 28 days after prestress release, exposure at 50 percent relative humidity, and ultimate shrinkage = 0.0006 in./in.:

$\epsilon_s = 0.0006 \times 0.4 \times 1.0 = 0.00024$ in./in.
(factors from Figs. 8 and 10)

$$M_s = \epsilon_s \times E_b \times A_b \left(e_2' + \frac{t}{2}\right)$$

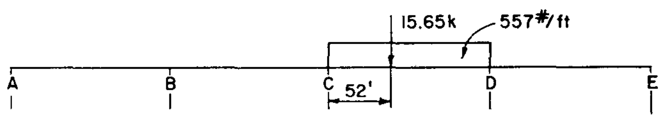
$$M_s = 0.00024 \times 3.88 \times 10^6 \times 96 \times 6.5\,(21.91 + 3.25)$$
$$M_s = 14{,}600 \text{ in. kips} = 1215 \text{ ft. kips}$$

The elastic shrinkage restraint moments are then:

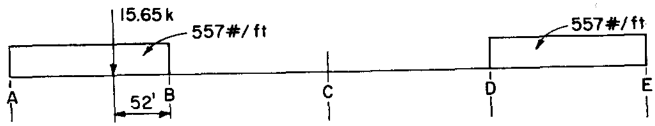
at B $= -1.286 \times 1215 = -1565$ ft. kips

at C $= -0.857 \times 1215 = -1040$ ft. kips

Live Load Positive Moments



Using influence lines for 4 equal continuous spans, the positive LL + I moment at B is calculated to be = 169.5 ft. kips



The LL + I moment for this loading at C = 392 ft. kips

Evaluation of Creep Factors

From Fig. 5, for an elastic modulus at strand release $= 3.66 \times 10^6$ psi, specific creep at 20 years $= 0.41 \times 10^{-6}$ in./in./psi.

For release of the prestress force at 2 days, the value of specific creep is adjusted by Fig. 6 to:

$1.66 \times 0.41 \times 10^{-6} = 0.68 \times 10^{-6}$ in./in./psi

From Fig. 7 and Table 1, the volume/surface ratio correction revises the specific creep to:

$1.18 \times 0.68 \times 10^{-6} = 0.802 \times 10^{-6}$ in./in./psi

From Fig. 8, at 28 days 0.4 of the creep has occurred, leaving 0.6 to occur or:

$0.6 \times 0.802 \times 10^{-6} = 0.481 \times 10^{-6}$ in./in./psi

The value of $\phi = \epsilon_c/\epsilon_e$ for the structure with a connection made at 28 days would then be:

$$\phi = \epsilon_c/\epsilon_e = \epsilon_c \times E = 0.481 \times 10^{-6} \times 4.05 \times 10^6 = 1.95$$

From Fig. 9, for $\phi = 1.95$:

$$(1 - e^{-\phi}) = 0.88$$

$$\left(\frac{1 - e^{-\phi}}{\phi}\right) = 0.45$$

Summation of Positive Restraint Moments at Piers

Shrinkage plus creep:

at B $= (4960 - 3360) \times 0.88 - 1565 \times 0.45 = 705$ ft. kips
at C $= (3320 - 2230) \times 0.88 - 1040 \times 0.45 = 491$ ft. kips

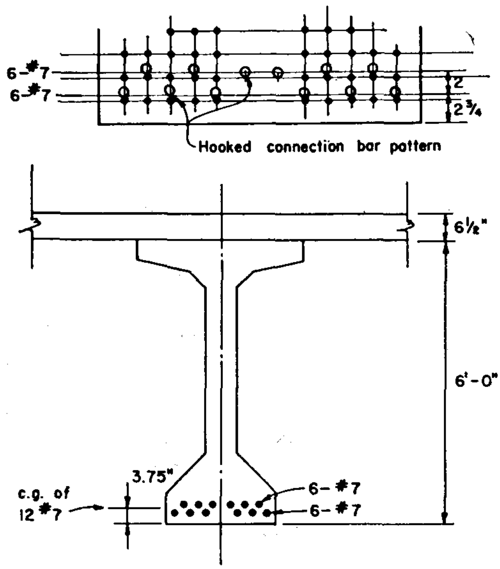
Add LL + I:

at B $= 705 + 169.5 = 874.5$ ft. kips
at C $= 491 + 392 = 883$ ft. kips

Design of the Positive Moment Connection

To design for the 883 ft. kips moment at C, use intermediate grade reinforcement with an allowable stress $= 0.6 \times f_y = 0.6 \times 40 = 24$ ksi. Obtain approximate area of steel required using an estimate of $jd = 68$ in.:

$A_s = M/f_s jd$
$A_s = 883{,}000 \times 12/24{,}000 \times 68 = 6.50$ sq. in.
Try 12 No. 7 bars, $A_s = 7.20$ sq. in.



Assume $kd = 8.76$ in. (from trial computation)

Section	A	y	Ay	Ay²	I₀
Slab, 96 × 6.5 × 0.96	600	− 3.25	−1950	6,330	2530
Top flange, 2.26 × 42	95	− 7.63	− 725	5,530	—
Steel, 7.20 × 7.16	51.6	−74.75	−3860	289,000	—
Totals	746.6		6535	300,860	2530

$$\frac{6535}{746.6} = 8.75 \text{ in. (ok)}$$

$$\begin{aligned} 300{,}860 + 2530 &= \quad 303{,}390 \\ -746.6 \times 8.75^2 &= \quad -57{,}200 \\ I &= \quad 246{,}190 \text{ in.}^4 \end{aligned}$$

Distance from neutral axis to steel $= 78.5 - 8.75 - 3.75 = 66.0$ in.

$$\text{Steel stress} = \frac{883{,}000 \times 12 \times 66.0}{246{,}190} \times 7.16 = 20{,}300 \text{ psi}$$

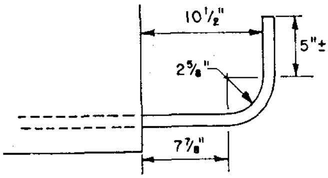
Since the stress is less than 24,000 psi the design is satisfactory. However, in view of the low stress, consideration might be given to reducing the size or number of bars in the connection.

Embedment Length and Bend Details

Allowable bond stress $= 0.10 \times 4500 = 450$ psi, use 350 psi maximum. Perimeter of a No. 7 bar $= 2.749$ in.

Assume distance from the end face of the precast girder to the inside face of the hook $= 12 \times$ bar diameter $= 12 \times 7/8 = 10.5$ in.

Minimum hook radius $= 3 \times 7/8 = 2\frac{5}{8}$ in.



Required embedment length

$$= \frac{20{,}300 \times 0.60}{350 \times 2.749} = 12.2 \text{ in.}$$

1/4 bend = 4.80 in.

7.875 in. + 4.80 in. = 12.675 in.
12.2 − 12.7 = no extension required
Use 5 in. ± extension—gives 18 in. embedment

To ensure that bars extend into the part of the beam where the prestress force is effective, and to avoid terminating all connection bars at one point, extend 6 bars 3 ft. 0 in. into the beam, and extend the other 6 bars 4 ft. 6 in. into the beam.

Design negative moment reinforcement over piers. The live load plus impact negative moments over piers are calculated as 1499 ft. kips at B, and 1359 ft. kips at C. Design the negative moment reinforcement at B.

Ultimate negative moment = 2.5 × 1499 = 3740 ft. kips

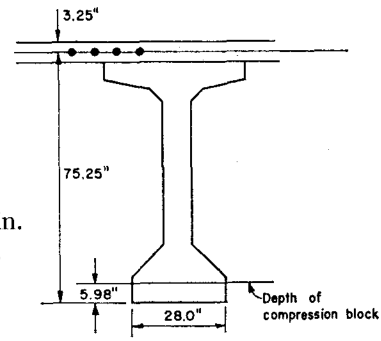
$$\frac{M_u}{bd^2} = \frac{3{,}740{,}000 \times 12}{28 \times 75.25^2} = 283$$

From Fig. 13 (the arrows in Fig. 13 apply to this design example):

$p = 0.0074$

$A_s = 0.0074 \times 28 \times 75.25 = 15.6$ sq. in.
Try 16 No. 9 bars, $A_s = 16.00$ sq. in.

Check ultimate moment provided:



$$a = \frac{A_s f_y}{0.85 f'_c b}$$

$$a = \frac{16 \times 40{,}000}{0.85 \times 4500 \times 28}$$

$a = 5.98$ in.

Note depth of compression block < bottom flange thickness.

$$M_u = A_s f_y \left(d - \frac{a}{2}\right) = 16.00 \times 40{,}000\,(75.25 - 2.99)$$

$M_u = 46{,}300{,}000$ in. lb. = 3850 ft. kips

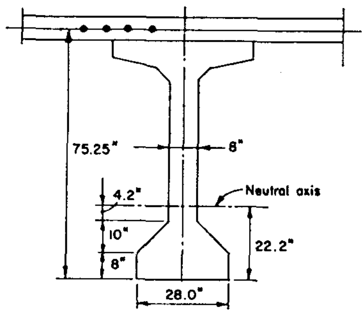
M_u provided > required 3740 ft. kips.

Compressive stress in girders at piers. From a trial calculation, assume $kd = 22.2$ in.

$n = E_s/E_c$

$= \frac{29 \times 10^6}{4.05 \times 10^6}$

$= 7.16$



Section	A	y	Ay	Ay²	I_o
16 × 7.16 (steel)	114.2	−75.25	−8600	647,000	—
8 × 28	224	− 4	− 896	3,580	1195
10 × 18	180	−11.63	−2095	24,400	1221
8 × 4.2	33.6	−20.1	− 640	12,850	—
Totals	551.8		−12,231	687,830	2416

$$\text{N.A} = \frac{12{,}231}{551.8} = 22.2 \text{ in.}$$

$$\begin{aligned} 687{,}830 + 2416 &= 690{,}246 \\ -551.8 \times 22.2^2 &= -272{,}000 \\ I &= 418{,}246 \text{ in.}^4 \end{aligned}$$

$$f_c\,(\text{LL} + \text{I}) = \frac{1{,}499{,}000 \times 12}{418{,}246} = 952 \text{ psi}$$

$$f_c\,(\text{prestress}) = \frac{1{,}132{,}000}{1085} + \frac{1{,}132{,}000 \times 18.88 \times 36.38}{733{,}320} = 2105 \text{ psi}$$

$$= 3057 \text{ psi}$$

This stress will not actually occur because of the tendon transfer length. The stress is also reduced by the positive moment induced by creep. The theoretical stress can be reduced to 2000 psi by blanketing 14 bottom strands for a short distance.

Tensile stresses in girders at midspan. In span AB, the positive creep restraint moment at midspan would be $0.5 \times 829 = 414.5$ ft. kips. This moment causes a tension of:

$$f_c = \frac{414{,}500 \times 12}{26{,}266} = 189 \text{ psi}$$

Adding the tension due to gravity loads of about 94 psi, the total tensile stress becomes 283 psi. The permissible value to avoid cracking would be $0.8 \times 7.5\sqrt{5000} = 426$ psi. Since the tensile stress is much less than the allowable, no additional prestressing reinforcement is required.

In span BC, the positive creep restraint moment at midspan is

$\frac{570 + 829}{2} = 699.5$ ft. kips. This causes a tensile stress of 319 psi. Since there is no tensile stress in this span under gravity loads, this section is also satisfactory for tensile stresses due to creep.

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