Presented at the Sixth Congress of the Federation Internationale de la Precontrainte, Prague, Czechoslovakia, June 1970

LOSS OF PRESTRESS, CAMBER AND DEFLECTION OF NON-COMPOSITE AND COMPOSITE PRESTRESSED CONCRETE STRUCTURES

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A systematic procedure is presented for predicting the material behavior of different weight concretes and the time-dependent structural deformation of non-composite and composite prestressed concrete structures. Continuous time functions are provided for all needed parameters, so that the general equations for predicting loss of prestress, camber and deflection readily lend themselves to computer solution.

Results computed by the material parameter equations are compared with representative data in the literature for normal weight, sand-lightweight, and all-lightweight concrete. Computed loss of prestress and camber are compared with experimental data for a sand-lightweight, composite, prestressed concrete bridge, and with data in the literature for non-composite and composite structures constructed of different weight concretes. Both laboratory specimens and actual structural members are included in the comparisons. Ranges of variation for material behavior, loss of prestress and camber are given.

Methods are also presented for predicting the effect of non-prestressed tension steel in reducing time-dependent loss of prestress and camber, and for determining short-time deflections of uncracked and cracked prestressed members (either with or without non-prestressed tension steel). Comparisons with experimental results are made for these partially prestressed methods.

The procedures in this paper for predicting time-dependent material and structural behavior represent a nominal approach for design purposes, and are neither definitive nor statistical. Probabilistic methods are needed for an accurate estimate of variability of behavior.

CONCRETE PROPERTIES

Strength and elastic properties. A study of concrete compressive strength vs. time for 88 specimens reported in References 1 to 6 indi-

Presents general equations for predicting loss of prestress and camber of both composite and non-composite prestressed concrete structures. Continuous time functions of all parameters needed to solve the equations are given, and sample results included. Computed prestress loss and camber are compared with experimental data for normal weight and lightweight concrete. Methods are also presented for predicting the effect of non-prestressed tension steel in reducing time-dependent loss of prestress and camber, and for the determination of short-time deflections of uncracked and cracked prestressed members. Comparisons with experimental results are indicated for these partially prestressed methods.

cates an appropriate general equation in the form of Eq. (1) and average value Eqs. (2) to (5) for predicting strength at any time[6,7,8].

$$(f'_c)_t = \frac{t}{a + bt}(f'_c)_{28d} \tag{1}$$

where a and b are constants; $(f'_c)_{28d}$ = 28-day compressive strength; t is age of concrete in days; and $(f'_c)_u$ refers to an ultimate (in time) value.

Moist cured concrete, Type I cement:

$$(f'_c)_t = \frac{t}{4.00 + 0.85t}(f'_c)_{28d} \tag{2}$$

or

$$(f'_c)_{7d} = 0.70\ (f'_c)_{28d}$$

$$(f'_c)_u = 1.18\ (f'_c)_{28d}$$

Moist cured concrete, Type III cement:

$$(f'_c)_t = \frac{t}{2.30 + 0.92t}(f'_c)_{28d} \tag{3}$$

or

$$(f'_c)_{7d} = 0.80\ (f'_c)_{28d}$$

$$(f'_c)_u = 1.09\ (f'_c)_{28d}$$

Steam cured concrete, Type I cement:

$$(f'_c)_t = \frac{t}{1.00 + 0.95t}(f'_c)_{28d} \tag{4}$$

or

$$(f'_c)_{2d} = 0.69\ (f'_c)_{28d}$$

$$(f'_c)_u = 1.05\ (f'_c)_{28d}$$

Steam cured concrete, Type III cement:

$$(f'_c)_t = \frac{t}{0.70 + 0.98t}(f'_c)_{28d} \tag{5}$$

or

$$(f'_c)_{2d} = 0.75\ (f'_c)_{28d}$$

$$(f'_c)_u = 1.02\ (f'_c)_{28d}$$

Eqs. (2) to (5) are compared in Fig. 1 with data from References 1 to 6, which include different weight concretes, both moist and steam curing, and Types I and III cement. The ranges of variation in the data (within about ± 20%) and the effect of type of curing and cement type (see the relative "flatness" of the strength-time curves) can be seen in Fig. 1.

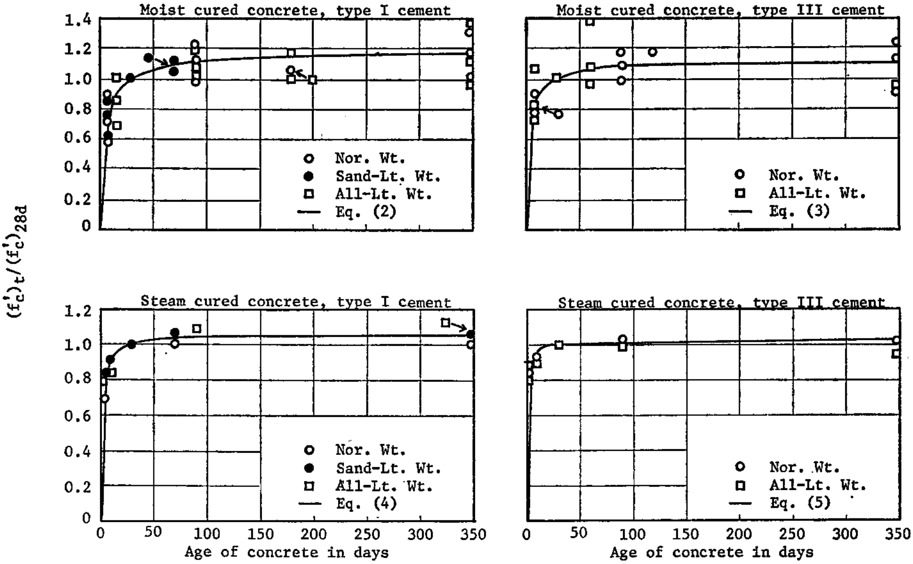

Fig. 1. Concrete strength vs. time, comparing Eqs. (2) to (5) with experimental data from References 1 to 6. Where three data points are shown for a given age, they refer to upper, lower and average values for a given set of data. Where only one data point is shown, the range is too small to indicate. Data for 88 specimens are included.

The basis for the equations is the 28-day strength. However, in the case of Eqs. (4) and (5), the type of steam curing may affect substantially the strength-time ratio in the early days following curing. Eqs. (2) to (5) were found to be equally applicable for normal weight, sand-lightweight, and all-lightweight aggregate concretes.

Eq. (6) is considered satisfactory in most cases for computing modulus of elasticity of different weight concretes[9,10].

$$E_c = 33\, w^{1.5} \sqrt{f'_c} \qquad (6)$$

where w is given in lb. per cu. ft. and f'_c and E_c in psi.

Creep and shrinkage parameters. Based largely on information from References 3 to 6 and 11 to 19, the general Eqs. (7) and (8) and the standard Eqs. (9) to (11) are recommended for predicting a creep coefficient (defined as ratio of creep strain to initial strain) and unrestrained shrinkage of concrete at any time[6,7,8].

General equations:

$$C_t = \frac{t^c}{d + t^c} C_u \qquad (7)$$

$$(\epsilon_{sh})_t = \frac{t^e}{f + t^e} (\epsilon_{sh})_u \qquad (8)$$

where c, d, e and f are constants, and t is time in days after loading for creep and, for shrinkage, time after initial shrinkage is considered.

Standard equations: Eq. (9) to (11) can be used for predicting creep and shrinkage for "standard" conditions of slump 4 in. or less, 40 percent ambient relative humidity, minimum thickness of member 6 in. or less, loading age 7 days for moist cured, and 1 to 3 days for steam cured con-

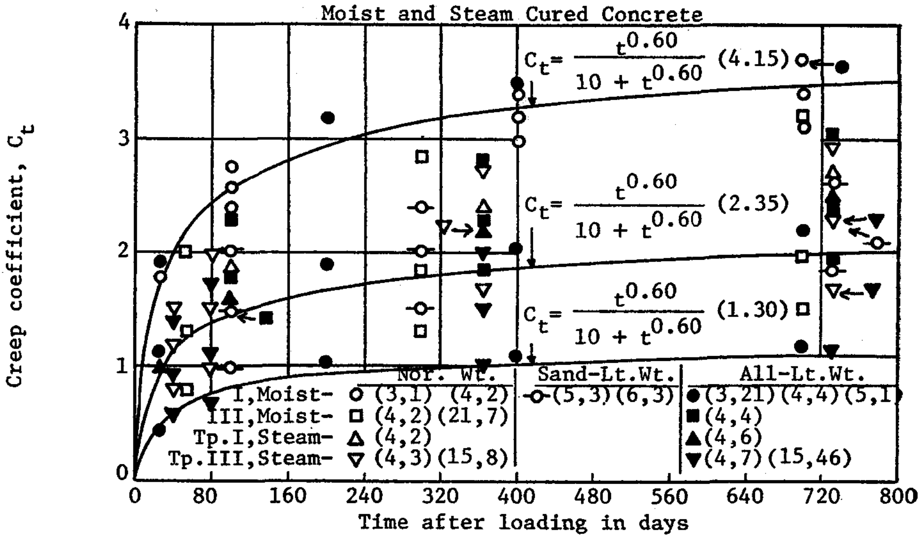

Fig. 2. Creep coefficient vs. time, comparing Eq. (9) with published data. Upper, lower and average values are plotted. All data reduced to "standard" conditions using correction factors. Legend (3, 21) indicates Reference 3 and 21 data points.

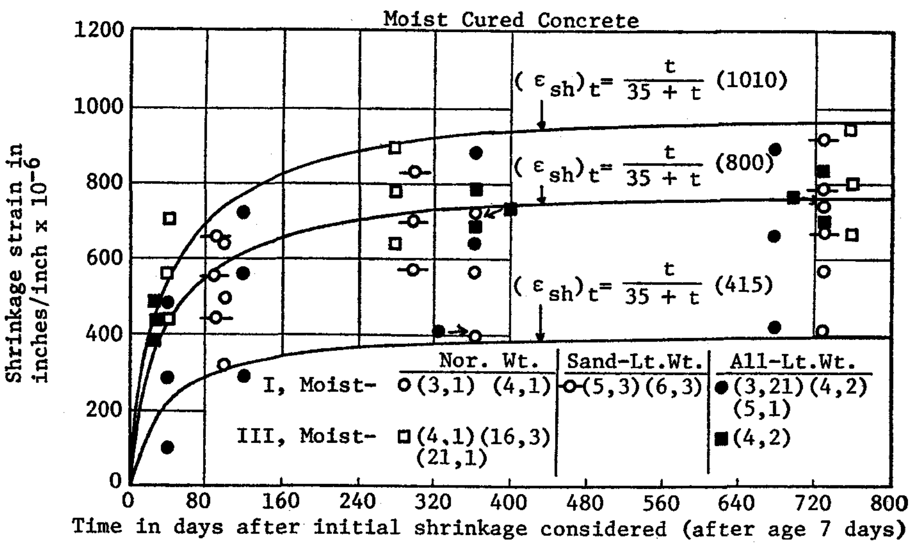

Fig 3. Shrinkage strain vs. time, comparing Eq. (10) with published data. Upper, lower and average values are plotted. Data reduced to "standard" conditions using correction factors. Legend (3, 21) indicates Reference 3 and 21 data points.

crete. For other than "standard" conditions, correction factors described in the following section must be used.

The standard equation for creep is

$$C_t = \frac{t^{0.60}}{10 + t^{0.60}} C_u \qquad (9)$$

The average value suggested for C_u is 2.35 when specific data for local aggregates and conditions are not available. From Eq. (14), for H = 70%, $C_u = 0.80(2.35) = 1.88$, for example. For the bridge girder sand-lightweight concrete (steam cured) herein, H was 70%, and the experimental $C_u = 1.72$.

The standard equation for shrinkage at any time after age 7 days for moist cured concrete is

$$(\epsilon_{sh})_t = \frac{t}{35 + t} (\epsilon_{sh})_u \qquad (10)$$

The average value suggested for $(\epsilon_{sh})_u$ is 800×10^{-6} in. per in., to be used when local data are not available. From Eq. (15) for H = 70%, $(\epsilon_{sh})_u = 0.70(800 \times 10^{-6}) = 560 \times 10^{-6}$ in./in., for example.

The standard equation for shrinkage at any time after age 1 to 3 days for steam cured concrete is

$$(\epsilon_{sh})_t = \frac{t}{55 + t} (\epsilon_{sh})_u \qquad (11)$$

The average value suggested for $(\epsilon_{sh})_u$ is 730×10^{-6} in. per in., to be used in the absence of local data. From Eq. (15) for H = 70%, $(\epsilon_{sh})_u = 0.70(730 \times 10^{-6}) = 510 \times 10^{-6}$ in./in., for example. For the bridge girder sand-lightweight concrete (steam cured) herein, H was 70%, and the experimental $(\epsilon_{sh})_u = 392 \times 10^{-6}$ in./in.

Eqs. (9) to (11) are compared with representative data (120 creep and 95 shrinkage specimens) in Figs. 2, 3 and 4, in which upper and lower limits and average values are shown. These equations consist of a "time-ratio" term which modifies an ultimate value (in time) for creep and shrinkage. The appropriate level of

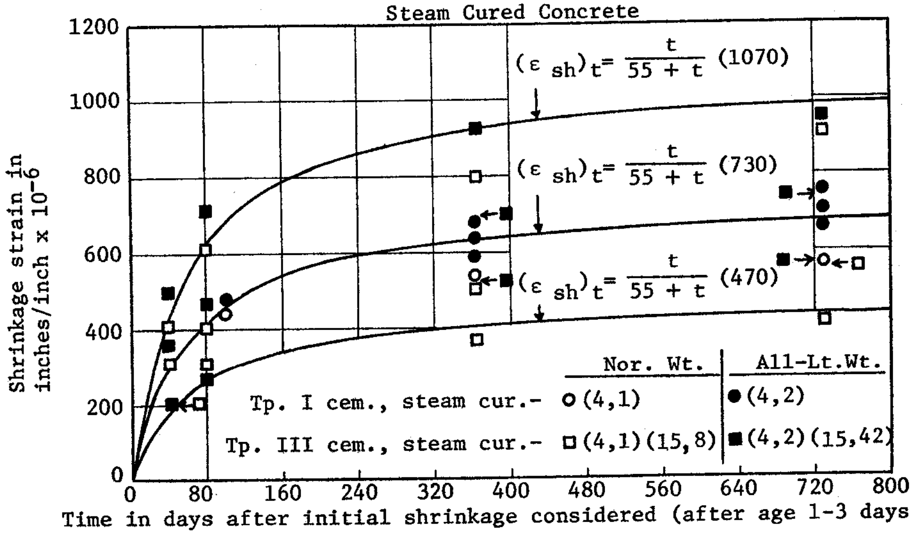

Fig. 4. Shrinkage strain vs. time, comparing Eq. (11) with published data. Upper, lower and average values are plotted. All data reduced to "standard" conditions using correction factors. Legend (15, 8) indicates Reference 15 and 8 data points.

Table 1. Time-ratio values for creep and shrinkage

	Time				
	1 month	3 months	6 months	1 year	5 years
C_t/C_u—Eq. (9)	0.44	0.60	0.69	0.78	0.90
$(\epsilon_{sh})_t/(\epsilon_{sh})_u$—Eq. (10)	0.46	0.72	0.84	0.91	0.98
$(\epsilon_{sh})_t/(\epsilon_{sh})_u$—Eq. (11)	0.35	0.62	0.77	0.87	0.97

curve for a given case is thus conveniently defined by the ultimate value, with the same time-ratio term used in general. For example, it has been shown[18] that these equations can be used to extrapolate 28-day creep and shrinkage data to complete time curves quite well for creep, and reasonably well for shrinkage, for a wide variety of data.

Normal weight, sand-lightweight, and all-lightweight concrete, using both moist and steam curing, and Types I and III cement, are included. No consistent variation was found between the different weight concretes for either creep or shrinkage. The average values of C_u and $(\epsilon_{sh})_u$ given with Eqs. (9) to (11) should be used only in the absence of specific creep and shrinkage data for local aggregates and conditions. However, the "time-ratio" terms in Eqs. (9) to (11) appear to be generally applicable (see Table 1 for values).

Correction factors. All correction factors[6,7,8] are applied to ultimate values. However, since creep and shrinkage for any period in Eqs. (9) to (11) are linear functions of the ultimate values, the correction factors in this procedure may be applied to short-term creep and shrinkage as well.

For loading ages later than 7 days for moist cured concrete and later than 1 to 3 days for steam cured concrete: Use Eqs. (12) and (13) for the creep correction factors.

$$\text{Creep } (\text{C.F.})_{LA} = 1.25\, t_{LA}^{-0.118} \quad \text{for moist cured concrete} \qquad (12)$$

$$\text{Creep } (\text{C.F.})_{LA} = 1.13\, t_{LA}^{-0.095} \quad \text{for steam cured concrete} \qquad (13)$$

where t_{LA} is the loading age in days. Representative values are shown in Table 2.

For shrinkage considered from other than 7 days for moist cured concrete and other than 1 to 3 days for steam cured concrete: Determine the differential in Eqs. (10) and (11) for any period starting after this time. For shrinkage of moist cured concrete from 1 day (can be used to estimate differential shrinkage in composite beams, for example), shrinkage C.F. = 1.20. A linear interpolation may be used between 1.20 at 1 day and 1.00 at 7 days.

For greater than 40 percent ambient relative humidity: Use Eqs. (14) to (16) for the creep and shrinkage correction factors[7,16,20].

$$\text{Creep } (\text{C.F.})_{H} = 1.27 - 0.0067\, H \quad \text{when } H \geqq 40\% \qquad (14)$$

Table 2. Creep correction factors for various non-standard loading ages, computed by Eqs. (12) and (13)

t_{LA}, days	Creep $(\text{C.F.})_{LA}$ moist cured	Creep $(\text{C.F.})_{LA}$ steam cured
10	0.95	0.90
20	0.87	0.85
30	0.83	0.82
60	0.77	0.76
90	0.74	0.74

Table 3. Creep and shrinkage correction factors for non-standard relative humidity, computed by Eqs. (14) to (16)

Relative humidity, H, percent	Creep $(C.F.)_H$	Shrinkage $(C.F.)_H$
40 or less	1.00	1.00
50	0.94	0.90
60	0.87	0.80
70	0.80	0.70
80	0.73	0.60
90	0.67	0.30
100	0.60	0.00

$$\text{Shrinkage } (C.F.)_H = 1.40 - 0.010\,H \quad \text{when } 40\% \leqq H \leqq 80\% \tag{15}$$

$$\text{Shrinkage } (C.F.)_H = 3.00 - 0.030\,H \quad \text{when } 80\% \leqq H \leqq 100\% \tag{16}$$

where H is relative humidity in percent (See Table 3).

For minimum thickness of members greater than 6 in., see Reference 7 or 8 for the creep and shrinkage correction factors as a function of loading period and length of drying period. For most design purposes, this effect can be neglected for creep of members up to about 10 to 12 in. minimum thickness, and for shrinkage of members up to about 8 to 9 in. minimum thickness(3,6,7,8,21). For large thickness members, see Reference 21 and others for relating size and shape effects for creep and shrinkage to the volume/surface ratio of the member.

For slumps greater than 4 in., see Reference 7 or 8 for the creep and shrinkage correction factors. This can normally be neglected, except for high slumps.

Other correction factors for creep and shrinkage, which are usually not excessive and tend to offset each other, are described in References 6, 7 and 8. For design purposes, in most cases, these may normally be neglected (except possibly for the effect of member size and slump as discussed above).

LOSS OF PRESTRESS, CAMBER AND DEFLECTION

Non-composite beams at any time. The loss of prestress, in percent of initial tensioning stress, is given by Eq. (17).

$$PL_t = \Bigg[\overbrace{(nf_c)}^{(1)} + \overbrace{(nf_c)C_t\left(1 - \frac{\Delta F_t}{2F_o}\right)}^{(2)} + \overbrace{(\epsilon_{sh})_t\, E_s/(1 + npk_s)}^{(3)}$$

$$+ \overbrace{\frac{f_{si}}{100}\, 1.5 \log_{10} t}^{(4)}\Bigg]\frac{100}{f_{si}} \tag{17}$$

Term (1) is the prestress loss due to elastic shortening. $f_c = \frac{F_t}{A_t} + \frac{F_t e^2}{I_t} - \frac{M_D e}{I_t}$, and n is the modular ratio at the time of prestressing. Frequently F_o, A_g, and I_g are used as an approximation instead of F_t, A_t, and I_t, where $F_o = F_t\,(1 - np)$. Only the first two terms for f_c apply at the ends of simple beams. The first term alone for f_c may yield a satisfactory average value in some cases. Term (1) must be adjusted for post-tensioned members. For continuous members, the effect of secondary moments due to prestressing should also be considered.

Term (2) is the prestress loss due to concrete creep. The expression, $C_t\left(1 - \frac{\Delta F_t}{2F_o}\right)$, was used in References 22 and 25 to approximate the creep effect resulting from the variable stress history. See Table 4 for approximate values of $\Delta F_t/F_o$ (in form of $\Delta F_s/F_o$ and $\Delta F_u/F_o$) for this secondary effect (expression in parenthesis) at 3 weeks to 1 month, 2 to 3 months, and ultimate values.

Term (3) is the prestress loss due to shrinkage. The expression, $(\epsilon_{sh})_t E_s$, somewhat overestimates (on safe side). The denominator represents the stiffening effect of the steel[31].

Term (4) is the prestress loss due to steel relaxation for stress-relieved wire or strand with a recommended maximum value $= 7.5$ percent at or above 10^5 hr. $= 11.4$ yr.[6,29,30]. In this term, t is time after initial stressing in hours. This expression applies when f_{si}/f_y is between 0.60 and 0.90, in which f_y is the 0.1 percent offset yield strength. For low-relaxation steel, use $f_{si} \times 0.60 \log_{10} t \div 100$ with a recommended maximum value $= 3.0$ percent.

The camber for non-composite beams is given by Eq. (18). It is suggested that an average of the end and midspan loss of prestress be used for straight tendons and 1-point harping, and the midspan loss of prestress for 2-point harping (bridge girders herein)[6].

$$\Delta_t = \overbrace{(\Delta_i)_{F_o}}^{(1)} - \overbrace{(\Delta_i)_D}^{(2)} + \overbrace{\left[-\frac{\Delta F_t}{F_o} + \left(1 - \frac{\Delta F_t}{2F_o}\right) C_t\right](\Delta_i)_{F_o}}^{(3)}$$

$$- \overbrace{C_t (\Delta_i)_D}^{(4)} - \overbrace{\Delta_L}^{(5)} \qquad (18)$$

Term (1) is the initial camber due to the initial prestress force after elastic loss, F_o. See Appendix B for common cases of prestress moment diagrams, with formulas for computing camber $(\Delta_i)_{F_o}$. Here $F_o = F_i\,(1 - n f_c/f_{si})$, where f_c is determined as in Term (1) of Eq. (17). For continuous members, the effect of secondary moments due to prestressing, which normally results in a reduction in camber, should also be included.

Term (2) is the initial dead load deflection of the beam. $(\Delta_i)_D = K M L^2/(E_{ci} I_g)$. I_g is suggested instead of I_t for practical reasons. See Notation in Appendix A for K and M.

Term (3) is the creep (time-dependent) camber of the beam due to the prestress force. This expression includes the effects of creep and loss of prestress, that is, the creep effect under variable stress. ΔF_t refers to the total loss at any time minus the elastic loss. The term, $\Delta F_t/F_o$, refers to the steel stress or force after elastic loss; the prestress loss in percent, PL (as used herein), refers to the initial tensioning stress or force. The two are related as:

$$\frac{\Delta F_t}{F_o} = \frac{1}{100}(PL_t - PL_{el})\frac{f_{si}}{f_o}$$

and can be closely approximated by:

$$\frac{\Delta F_t}{F_o} = \frac{1}{100}(PL_t - PL_{el})\frac{1}{1 - np}$$

Term (4) is the dead load creep deflection of the beam.
Term (5) is the live load deflection of the beam.

Unshored and shored composite beams at any time. Subscripts 1 and 2 are used to refer to the slab (or effect of the slab such as slab dead load) and precast beam, respectively. The loss of prestress, in percent of initial tensioning stress for unshored and shored composite beams, is given by Eq. (19).

$$PL_t = \Bigg[\overbrace{(nf_c)}^{(1)} + \overbrace{(nf_c)C_{s2}\left(1 - \frac{\Delta F_s}{2F_o}\right)}^{(2)} + \overbrace{(nf_c)(C_{t2} - C_{s2})\left(1 - \frac{\Delta F_s + \Delta F_t}{2F_o}\right)\frac{I_2}{I_c}}^{(3)}$$

$$+ \overbrace{(\epsilon_{sh})_t E_s/(1 + npk_s)}^{(4)} + \overbrace{\frac{f_{si}}{100} 1.5 \log_{10} t}^{(5)} - \overbrace{(mf_{cs})}^{(6)} - \overbrace{(mf_{cs}) C_{t1}\frac{I_2}{I_c}}^{(7)} - \overbrace{PG_{DS}}^{(8)}\Bigg]\frac{100}{f_{si}} \quad (19)$$

Term (1) is the prestress loss due to elastic shortening. See explanation of Term (1) of Eq. (17) for the calculation of f_c.

Term (2) is the prestress loss due to concrete creep up to the time of slab casting. C_{s2} is the creep coefficient of the precast beam concrete at the time of slab casting. See Term (2) of Eq. (17) for comments concerning the reduction factor, $\left(1 - \frac{\Delta F_s}{2F_o}\right)$.

Term (3) is the prestress loss due to concrete creep for any period following slab casting. C_{t2} is the creep coefficient of the precast beam concrete at any time after slab casting. The reduction factor, $\left(1 - \frac{\Delta F_s + \Delta F_t}{2F_o}\right)$, with the incremental creep coefficient, $(C_{t2} - C_{s2})$, estimates the effect of creep under the variable prestress force that occurs after slab casting. The reduction factor term was modified from previous references. The expression, I_2/I_c, modifies the initial value and accounts for the effect of the composite section in restraining additional creep curvature (strain) after slab casting.

Term (4) is the prestress loss due to shrinkage. See Term (3) of Eq. (17) for comment.

Term (5) is the prestress loss due to steel relaxation for stress-relieved wire or strand. In this term t is time after initial stressing in hours. See comments for Term (4) of Eq. (17) for the maximum value and limitating conditions, and corresponding information for low-relaxation steel.

Term (6) is the elastic prestress gain due to slab dead load, and m is the modular ratio at the time of slab casting. $f_{cs} = (M_{s,Dt})e/I_g$. $M_{s,Dt}$ refers to slab or slab plus diaphragm dead load. e and I_g refer to the precast beam section properties for unshored construction and the composite beam section properties for shored construction.

Term (7) is the prestress gain due to creep under slab dead load. C_{t1} is the creep coefficient for the slab loading, where the age of the precast beam concrete at the time of slab casting is considered. For shored construction, drop the term, I_2/I_c.

Term (8) is the prestress gain due to differential shrinkage. $PG_{DS} = mf_{cd}$, where $f_{cd} = Q\, y_{cs}\, e_c/I_c$, and f_{cd} is the concrete stress at the steel c.g.s. Since this effect results in a prestress gain, not loss, and is normally small (see Table 8), it may usually be neglected.

The camber of unshored and shored composite beams is given by Eqs. (20) and (21), respectively.

Unshored construction:

$$\Delta_t = \overbrace{(\Delta_i)_{F_o}}^{(1)} - \overbrace{(\Delta_i)_2}^{(2)} + \overbrace{\left[-\frac{\Delta F_s}{F_o} + \left(1 - \frac{\Delta F_s}{2F_o}\right)C_{s2}\right](\Delta_i)_{F_o}}^{(3)}$$

$$+ \overbrace{\left[-\frac{\Delta F_t - \Delta F_s}{F_o} + \left(1 - \frac{\Delta F_s + \Delta F_t}{2F_o}\right)(C_{t2} - C_{s2})\right](\Delta_i)_{F_o}\frac{I_2}{I_c}}^{(4)} - \overbrace{C_{s2}(\Delta_i)_2}^{(5)}$$

$$- \overbrace{(C_{t2} - C_{s2})(\Delta_i)_2\frac{I_2}{I_c}}^{(6)} - \overbrace{(\Delta_i)_1}^{(7)} - \overbrace{C_{t1}(\Delta_i)_1\frac{I_2}{I_c}}^{(8)} - \overbrace{\Delta_{DS}}^{(9)} - \overbrace{\Delta_L}^{(10)} \quad (20)$$

Term (1) is the initial camber due to the initial prestress force after elastic loss, F_o. See Term (1) of Eq. (18) for additional comments.

Term (2) is the initial dead load deflection of the precast beam. $(\Delta_i)_2 = K\,M_2\,L^2/(E_{ci}\,I_g)$. See Term (2) of Eq. (18) for additional comments.

Term (3) is the creep (time-dependent) camber of the beam, due to the prestress force, up to the time of slab casting. See Term (3) of Eq. (18) and Terms (2) and (3) of Eq. (19) for additional comments.

Term (4) is the creep camber of the composite beam, due to the prestress force, for any period following slab casting. See Term (3) of Eq. (18) and Terms (2) and (3) of Eq. (19) for additional comments.

Term (5) is the creep deflection of the precast beam up to the time of slab casting due to the precast beam dead load. C_{s2} is the creep coefficient of the precast beam concrete at the time of slab casting.

Term (6) is the creep deflection of the composite beam for any period following slab casting due to the precast beam dead load. See Term (3) of Eq. (19) for additional comments.

Term (7) is the initial deflection of the precast beam under slab dead load. $(\Delta_i)_1 = K\,M_1\,L^2/(E_{cs}\,I_g)$. See Notation for K and M. When diaphragms are used, add to $(\Delta_i)_1$:

$$(\Delta_i)_{1D} = \frac{M_{1D}}{E_{cs}\,I_g}\left(\frac{L^2}{8} - \frac{a^2}{6}\right)$$

where M_{1D} is the moment between diaphragms, and a is $L/4$, $L/3$, etc., for two symmetrical diaphragms at the quarter points, third points, etc., respectively.

Term (8) is the creep deflection of the composite beam due to slab dead load. C_{t1} is the creep coefficient for the slab loading, where the age of the precast beam concrete at the time of slab casting is considered. See Term (3) of Eq. (19) for comment concerning I_g/I_c.

Term (9) is the deflection due to differential shrinkage. For simple spans, $\Delta_{DS} = Q\,y_{cs}L^2/8E_{cs}I_c$, where $Q = DA_1E_1/3$. The factor 3 provides for the gradual increase in the shrinkage force from day 1, and also approximates the creep and varying stiffness effects[27]. This factor 3 is also consistent with the data herein and elsewhere. In the case of continuous members, differential shrinkage produces secondary moments (similar to the effect of prestressing but normally opposite in sign) that should be considered[32].

Term (10) is the live load deflection of the composite beam, in which the gross-section flexural rigidity, E_cI_c, is normally used.

Shored construction:

$$\Delta_t = \text{Eq. (20), with Terms (7) and (8) modified as follows:} \qquad (21)$$

Term (7) is the initial deflection of the composite beam under slab dead load. $(\Delta_i)_1 = K\ M_1\ L^2/E_{cs}\ I_c$. Term (8) is the creep deflection of the composite beam under slab dead load $= C_{t1}\,(\Delta_i)_1$. The composite section effect is already included in Term (7).

It is suggested that the 28-day moduli of elasticity for both slab and precast beam concretes, and the gross I (neglecting the steel), be used in computing the composite moment of inertia, I_c, in Eqs. (19) to (21).

Special cases of ultimate values. For computing ultimate values of loss of prestress, camber and deflection, Eqs. (22) to (26) correspond term by term to Eqs. (17) to (21), respectively.

Loss of prestress for non-composite beams, as per Eq. (17):

$$PL_u = \Big[\overbrace{(nf_c)}^{(1)} + \overbrace{(nf_c)C_u\left(1 - \frac{\Delta F_u}{2\,F_o}\right)}^{(2)} + \overbrace{(\epsilon_{sh})_u\,E_s/(1 + npk_s)}^{(3)} + \overbrace{0.075\,f_{si}}^{(4)}\Big]\frac{100}{f_{si}} \qquad (22)$$

Camber of non-composite beams, as per Eq. (18):

$$\Delta_u = \underbrace{(\Delta_i)_{F_o}}_{(1)} - \underbrace{(\Delta_i)_D}_{(2)} + \underbrace{\left[-\frac{\Delta F_u}{F_o} + \left(1 - \frac{\Delta F_u}{2\,F_o}\right) C_u\right](\Delta_i)_{F_o}}_{(3)} - \underbrace{C_u(\Delta_i)_D}_{(4)} - \underbrace{\Delta_L}_{(5)} \tag{23}$$

Loss of prestress for unshored and shored composite beams, as per Eq. (19):

$$PL_u = \left[\underbrace{(nf_c)}_{(1)} + \underbrace{(nf_c)\,(\alpha_s\,C_u)\left(1 - \frac{\Delta\,F_s}{2\,F_o}\right)}_{(2)} + \underbrace{(nf_c)\,(1-\alpha_s)C_u\left(1 - \frac{\Delta\,F_s + \Delta\,F_u}{2\,F_o}\right)\frac{I_2}{I_c}}_{(3)}\right.$$

$$\left. + \underbrace{(\epsilon_{sh})_u\,E_s/(1 + npk_s)}_{(4)} + \underbrace{0.075 f_{si}}_{(5)} - \underbrace{(mf_{cs})}_{(6)} - \underbrace{(mf_{cs})\,(\beta_s C_u)\frac{I_2}{I_c}}_{(7)} - \underbrace{PG_{DS}}_{(8)}\right]\frac{100}{f_{si}} \tag{24}$$

Camber of unshored composite beams, as per Eq. (20):

$$\Delta_u = \underbrace{(\Delta_i)_{F_o}}_{(1)} - \underbrace{(\Delta_i)_2}_{(2)} + \underbrace{\left[-\frac{\Delta\,F_s}{F_o} + \left(1 - \frac{\Delta\,F_s}{2\,F_o}\right)\alpha_s\,C_u\right](\Delta_i)_{F_o}}_{(3)}$$

$$+ \underbrace{\left[-\frac{\Delta\,F_u - \Delta\,F_s}{F_o} + \left(1 - \frac{\Delta\,F_s + \Delta\,F_u}{2\,F_o}\right)(1-\alpha_s)C_u\right](\Delta_i)_{F_o}\frac{I_2}{I_c}}_{(4)} - \underbrace{\alpha_s C_u(\Delta_i)_2}_{(5)}$$

$$- \underbrace{(1-\alpha_s)C_u(\Delta_i)_2\frac{I_2}{I_c}}_{(6)} - \underbrace{(\Delta_i)_1}_{(7)} - \underbrace{\beta_s C_u(\Delta_i)_1\frac{I_2}{I_c}}_{(8)} - \underbrace{\Delta_{DS}}_{(9)} - \underbrace{\Delta_L}_{(10)} \tag{25}$$

Camber of shored composite beams, as per Eq. (21):

Δ_u = Eq. (25), except that the composite moment of inertia is used in Term (7) to compute $(\Delta_i)_1$, and the ratio, I_2/I_c, is eliminated in Term (8). (26)

Eqs. (17) to (26) could be greatly shortened by combining terms and substituting the approximate parameters given in the next section, but are presented in the form of separate terms in order to show the separate effects or contributions to the behavior, such as effects due to prestress force, dead load, creep, shrinkage, etc., that occur both before and after slab casting.

Grossly approximate equations.

Non-composite beams

$$\Delta_u = \Delta_i + \Delta_i C_u\left(1 - \frac{\Delta\,F_u}{2\,F_o}\right) \tag{27}$$

with $\Delta_i = (\Delta_i)_{F_o} - (\Delta_i)_D$

Composite beams

$$PL_u = \left[nf_c \left(1 + \frac{C_u}{2}\right) - nf_{cs} + (\epsilon_{sh})_u E_s + 0.075 f_{si} \right] \frac{100}{f_{si}} \tag{28}$$

$$\Delta_u = \Delta_i + \Delta_i \, C_u \, (I_2/I_c) \tag{29}$$

$$\text{with } \Delta_i = (\Delta_i)_{F_o} - (\Delta_i)_2 - (\Delta_i)_1$$

Summary of general or average parameters. Continuous time functions are provided for all needed material parameters (and for different weight concretes, moist and steam cured), so that the equations readily lend themselves to computer solution. Certain other read-in data (such as for the effect of behavior before and after slab casting—α_s, β_s, m, and $\Delta F_s/F_o$) are also included.

The loss of prestress ratios at time of slab casting and at ultimate, given in Table 4, are suggested for most calculations. These are defined as the total loss (at slab casting and ultimate) minus the initial elastic loss divided by the prestress force after elastic loss. The different values for the different weight concretes are due primarily to different initial strains (because of different E) for normal stress levels.

Table 5 gives average modular ratios based on $f'_{ci} = 4000$ to 4500 psi for both moist cured (M.C.) and steam cured (S.C.) concrete and Type I cement; up to 3 months $f'_c = 6360$ to 7150 psi (using Eq. 2) for moist cured and $f'_c = 6050$ to 6800 psi (using Eq. 4) for steam cured; and for both 250K and 270K prestressing strands. $E_s = 27 \times 10^6$ psi for 250K strands, $E_s = 28 \times 10^6$ psi for 270K strands.

α_s and β_s given in Table 6 may be used for all concrete weights, both Type I and Type III cement, moist or steam cured, and for the "standard" conditions of Eq. (9). α_s refers to the part of the total creep that takes place before slab casting

$$\alpha_s = \left(\frac{t^{0.60}}{10 + t^{0.60}} \right),$$

as per Eq. (9) and β_s—average creep $(\text{C.F.})_{LA}$ from Eqs. (12) and (13)—is the creep correction factor for the precast beam concrete age when the

Table 4. Loss of prestress ratios for different concretes

Ratio	Normal weight concrete (w = 145 pcf)	Sand-light-weight concrete (w = 120 pcf)	All-light-weight concrete (w = 100 pcf)
$\Delta F_s/F_o$—for 3 weeks to 1 month between prestressing and slab casting	0.10	0.12	0.14
$\Delta F_s/F_o$—for 2 to 3 months between prestressing and slab casting	0.14	0.16	0.18
$\Delta F_u/F_o$	0.18	0.21	0.23

Table 5. Average modular ratios

Time condition		Normal weight (w = 145 pcf)		Sand-lightweight (w = 120 pcf)		All-lightweight (w = 100 pcf)	
		M.C.	S.C.	M.C.	S.C.	M.C.	S.C.
n at release of prestress		7.3	7.3	9.8	9.8	12.9	12.9
m for indicated time between prestressing and slab casting	3 weeks	6.1	6.2	8.1	8.3	10.7	10.9
	1 month	6.0	6.2	8.0	8.2	10.5	10.7
	2 months	5.9	6.1	7.9	8.2	10.2	10.6
	3 months	5.8	6.0	7.7	8.0	10.2	10.5

slab is cast (under slab dead load). See Eqs. (9) to (11), and the correction factors herein, for suggested values of C_u and $(\epsilon_{sh})_u$.

Numerical calculations. Computed ultimate loss of prestress at end and mid-span, using Eqs. (19), (24) and (28), and ultimate mid-span camber, using Eqs. (20), (25) and (29), are shown in Table 7 for the sand-lightweight, steam cured, composite bridge girders[6,33] with moist cured slab shown in Fig. 5. Both experimental parameters and general or average parameters are used.

Although the agreement is good (note the camber is near zero due to the slab effect) by these methods, the approximate method may be suitable in many cases for rough calculations only. Also, the calculations needed by the approximate methods are not significantly fewer than by the other methods. The more reliable equations should be preferable for computer use.

Table 6. Average values of α_s and β_s

Time between prestressing and casting	α_s	β_s
3 weeks	0.38	0.85
1 month	0.44	0.83
2 months	0.54	0.78
3 months	0.60	0.75

COMPARISON OF MEASURED AND COMPUTED LOSSES AND CAMBER

Sand-lightweight unshored composite bridge. The measured and computed mid-span camber vs. time curves for 5 bridge girders (Fig. 5) are shown in Fig. 6. Computations were based on Eq. (20) with experimental parameters. The results are reasonably good, but not precise, and probably indicate the nature of the correlation that might be expected, at best, for this type of behavior.

The computed ultimate values of loss of prestress and camber (using general Eqs. 19 and 20 with experimental parameters) are shown term by term in Tables 8 and 9 as an illustration of the separate contributions to the total effect.

The ultimate loss of prestress for

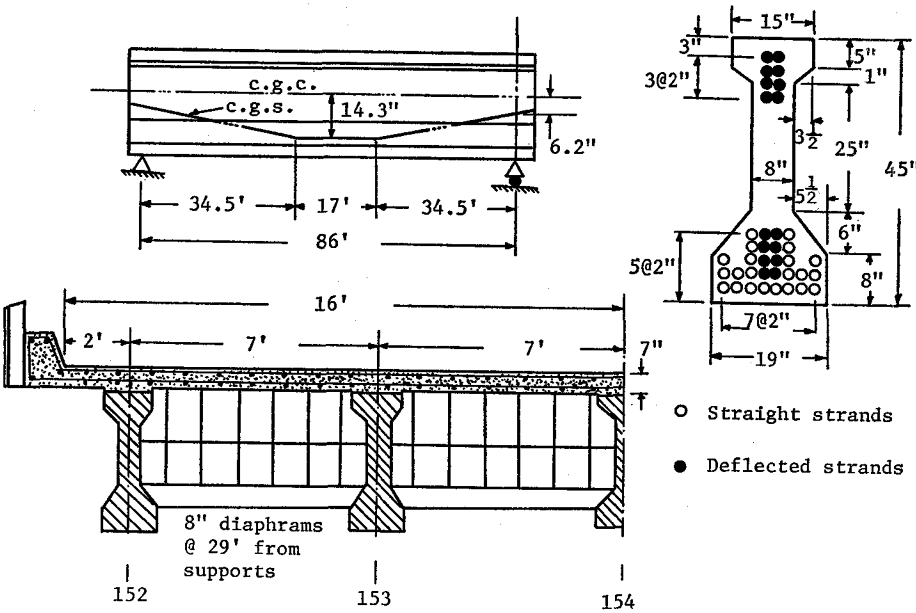

Fig. 5. Sand-lightweight concrete prestressed bridge girders composite with normal weight concrete deck slab

the sand-lightweight concrete bridge girders was 29 to 31 percent (See Fig. 5 and Table 7). It was determined that loss percentages for bridges under similar conditions using normal weight concrete will normally be of the order of 25 percent, and using all-lightweight concrete will normally be of the order of 35 percent or higher. Higher losses for the lighter concretes are due primarily to the lower modulus of elasticity (higher elastic strains for a given stress level),

Table 7. Computed ultimate loss of prestress at end and mid-span and mid-span camber for sand-lightweight bridge girders

Girder No.	Prestress loss, percent						Mid-span camber, in.		
	Eq. (19) with exp. parameters		Eq. (24) with general parameters		Eq. (28) with general parameters		Eq. (20) with exp. parameters	Eq. (25) with general parameters	Eq. (29) with general parameters
	End	Mid	End	Mid	End	Mid			
152	29.4	29.6	30.7	34.3	30.5	35.0	0.43	0.51	0.53
153	30.2	30.0	30.6	33.6	30.5	34.4	0.16	0.14	0.14
154	30.2	30.0	30.6	33.6	30.5	34.4	0.16	0.14	0.14
155	29.3	28.7	30.6	33.6	30.5	34.4	0.01	0.14	0.14
156	30.5	31.0	30.7	34.3	30.5	35.0	0.50	0.51	0.53

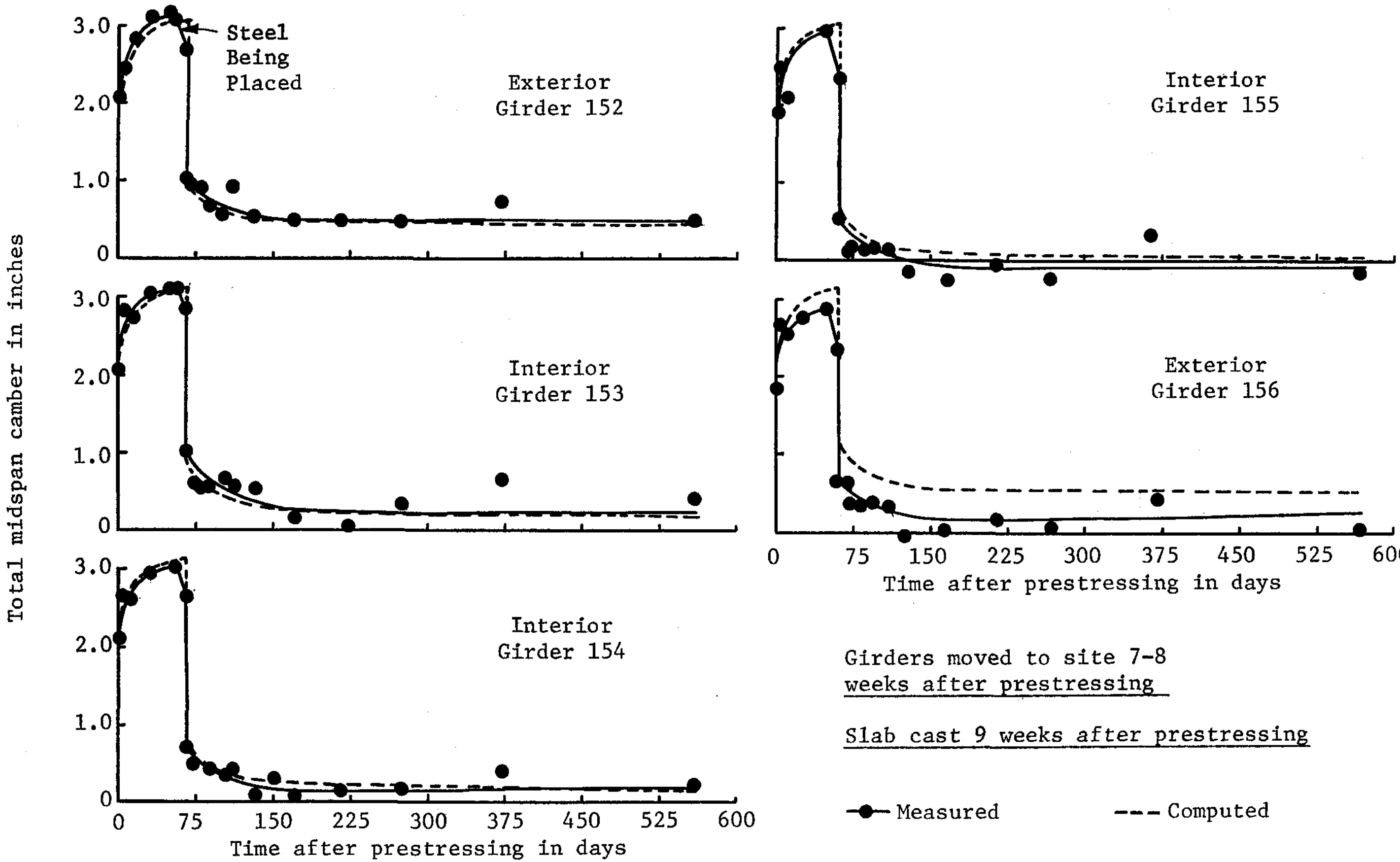

Fig. 6. Measured and computed camber for the sand-lightweight concrete prestressed bridge girders

Table 8. Computed loss of prestress at mid-span using Eq. (19) with experimental parameters

Girder No.	Elastic loss	Creep loss before slab is cast	Creep loss after slab is cast	Shrinkage loss	Relaxation loss	Elastic gain due to slab	Creep gain due to slab	Gain due to differential shrinkage	Total loss
152	11.5	9.8	2.1	4.5	7.5	−3.7	−1.5	−0.6	29.6
153	12.0	10.3	2.2	4.5	7.5	−4.2	−1.7	−0.6	30.0
154	12.0	10.3	2.2	4.5	7.5	−4.2	−1.7	−0.6	30.0
155	11.5	9.6	2.2	4.5	7.5	−4.3	−1.7	−0.6	28.7
156	12.3	10.3	2.3	4.5	7.5	−3.8	−1.5	−0.6	31.0

and not, necessarily, to greater creep and shrinkage.

Additional comparisons with data from four other studies. For each of four studies[6,22,23,26], the mid-span loss of prestress and camber predicted by Eqs. (17) to (20) at various times, using both exerimental material parameters reported in the respective studies and general parameters given herein, were compared with the observed prestress loss and camber. These tests and comparisons are described in Figs. 7, 8, Table 11, and below. Some 27 laboratory specimens and 10 actual structural members are included.

The University of Florida tests[22] involved 10 post-tensioned normal weight concrete laboratory beams of 19 ft. 6 in. spans. The cross-sections were 8 x 12 in. with 5 composite slabs, 2 ft. 2 in. x 3 in., cast on half of the ten beams. The test period was 5 months. The experimental creep and shrinkage parameters were slightly larger than the general creep and shrinkage parameters.

Table 9. Computed mid-span camber using Eq. (20) with experimental parameters

Girder No.	Initial camber due to prestress	Initial defl. due to beam DL	Creep camber up to slab cast	Creep camber after slab cast	Creep defl. up to slab cast	Creep defl. after slab cast	Elastic defl. due to slab	Creep defl. due to slab	Defl. due to diff. shrinkage	Total camber
152	3.71	−1.56	2.33	0.65	−1.42	−0.36	−1.96	−0.78	−0.18	0.43
153	3.87	−1.64	2.39	0.68	−1.49	−0.38	−2.21	−0.87	−0.19	0.16
154	3.87	−1.64	2.39	0.68	−1.49	−0.38	−2.21	−0.87	−0.19	0.16
155	3.72	−1.57	2.28	0.71	−1.40	−0.37	−2.26	−0.91	−0.19	0.01
156	3.96	−1.68	2.38	0.73	−1.50	−0.39	−2.01	−0.81	−0.18	0.50

Notes for Tables 7, 8 and 9: All losses are expressed in percent of initial stress. The girders were prestressed at age 2 to 3 days. The experimental material parameters are given in Reference 6. The experimental creep and shrinkage factors (after correction factors for $H = 70\%$ and 8-in. web thickness) were:

Precast beam creep, $C_u = 1.62$

Precast beam shrinkage, $(\epsilon_{sh})_u = 352 \times 10^{-6}$ in./in.

Slab shrinkage, from day 1, $(\epsilon_{sh})_u = 330 \times 10^{-6}$ in./in.

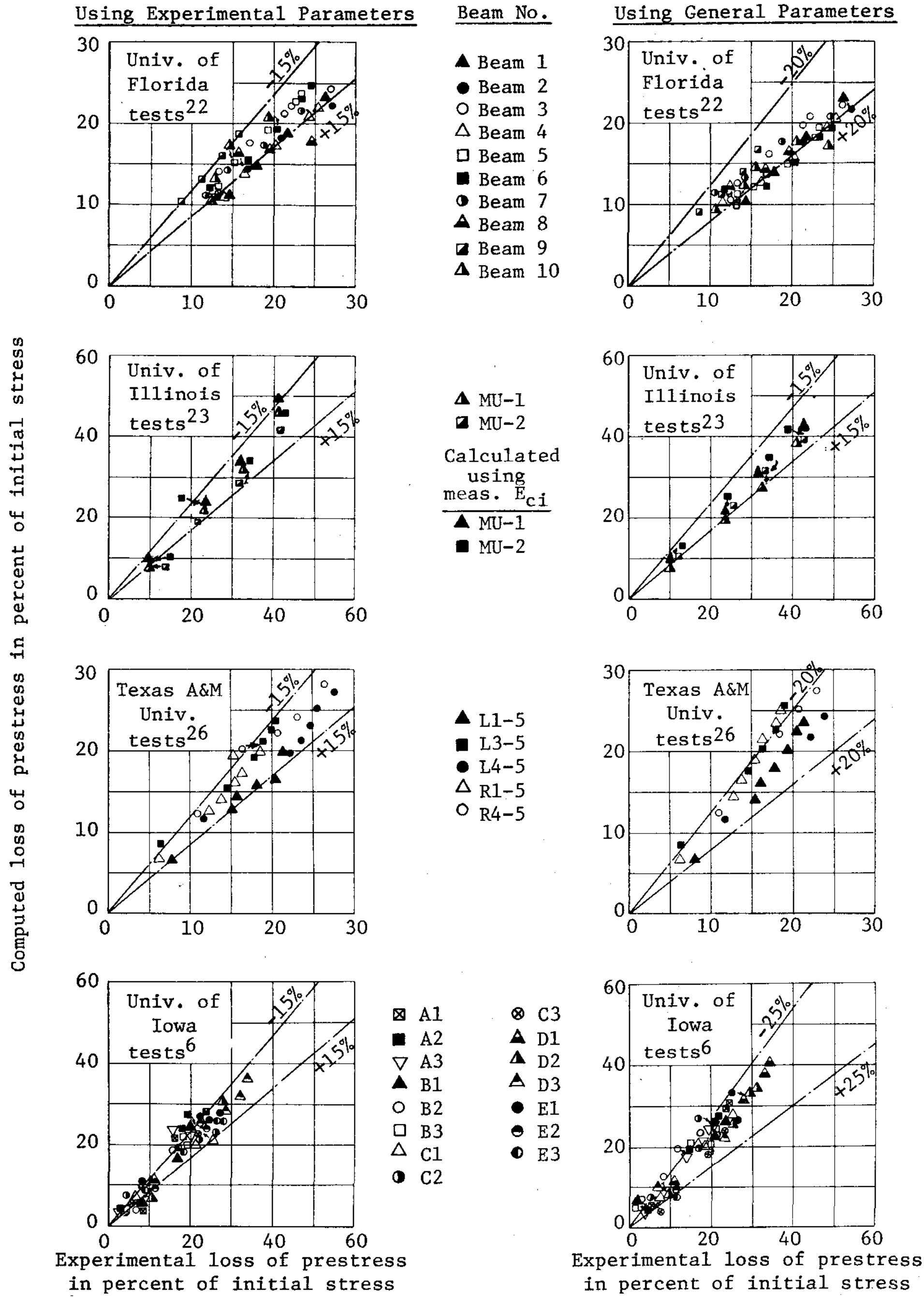

Fig. 7. Comparison of experimental and computed mid-span loss of prestress at various ages.

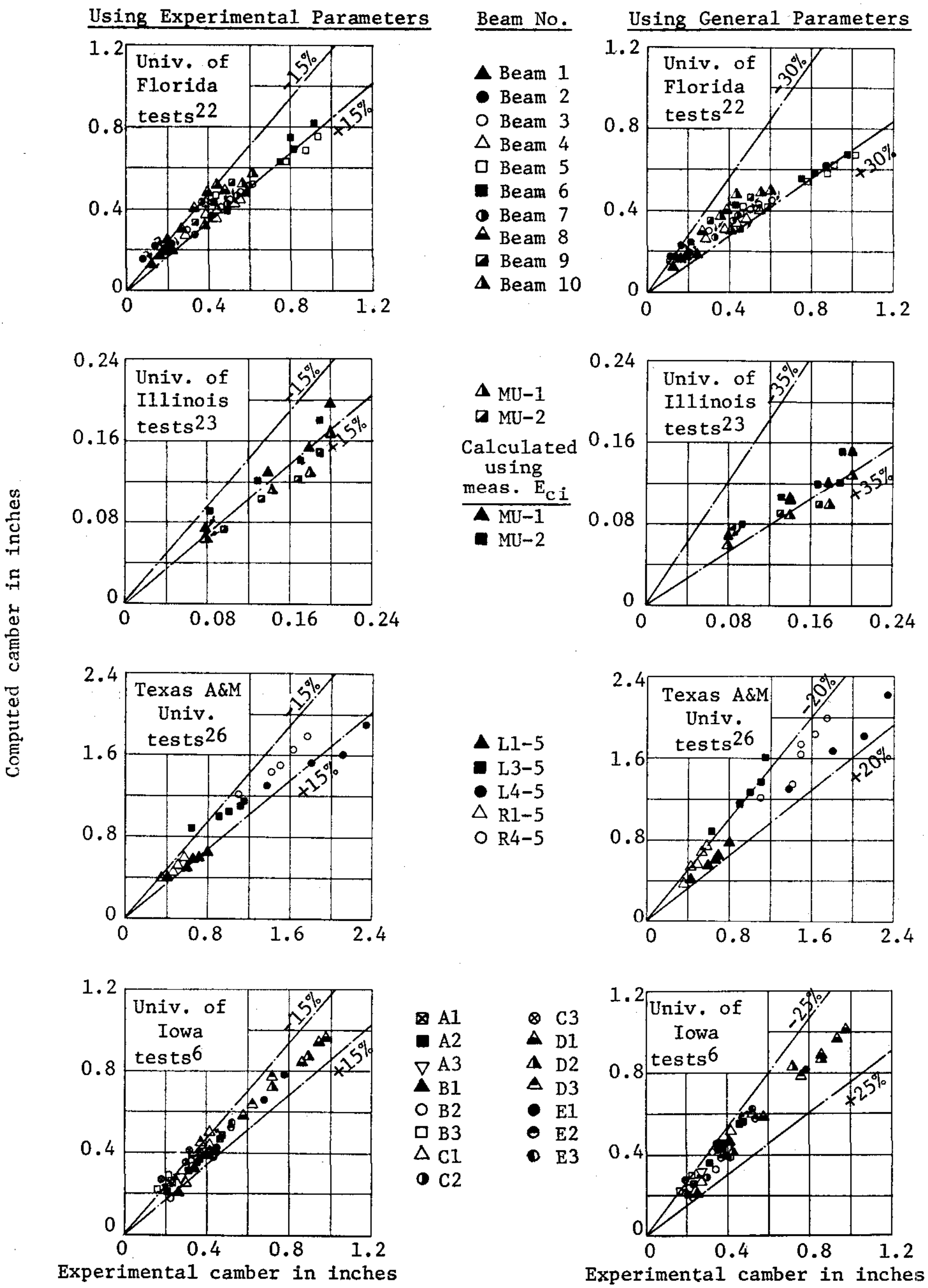

Fig. 8. Comparison of experimental and computed mid-span camber at various ages.

Table 10. Variation between experimental and computed mid-span loss of prestress and camber at various ages

Reference	Using experimental material parameters from the papers		Using general material parameters	
	Mid-span loss of prestress	Mid-span camber	Mid-span loss of prestress	Mid-span camber
U. of Florida[22]	±15%	±15%	−10% to +25%	±30%
U. of Illinois[23]	±15%	−10% to +20%	0% to +15%	+5% to +40%
Texas A & M Univ.[26]	±15%	±15%	±20%	±20%
U. of Iowa[6]	±15%	±15%	±25%	±25%

The University of Illinois specimens[23] consisted of 2 pretensioned non-composite rectangular beams (4 x 6 in.) of normal weight concrete and 6-ft. spans. The beams were observed for two years under laboratory conditions. The experimental creep and shrinkage parameters were somewhat larger than the corresponding general parameters. The measured modulus of elasticity was also greater than the computed value (based on compressive strength), and this tended to compensate for the smaller general creep and shrinkage parameters when used to obtain computed results.

The Texas A & M University tests[26] involved 5 non-composite pretensioned Type B Texas Highway Department bridge girders (4 lightweight and 1 normal weight) of 38 to 45-ft. spans. The girders were observed in the field for a period of one year. The experimental creep and shrinkage parameters were slightly smaller than the general creep and shrinkage parameters.

The University of Iowa specimens[6] consisted of 15 pretensioned laboratory beams (6 x 8 in.) of 15-ft. spans. Twelve were sand-lightweight concrete and three were all-lightweight concrete. Nine of the beams were non-composite and six were composite (slabs 20 x 2 in. and 20 x 3 in.). The test period was 6 months for 12 of the beams and 1 year for 3 beams. The experimental creep and shrinkage parameters were slightly smaller than the corresponding general parameters.

The steel relaxation expression in the equations was modified in the prestress loss calculations for post-tensioned members. In the calculations using experimental parameters, E_c was computed using the measured f'_c and Eq. (6), except as noted.

The comparison of experimental loss of prestress and camber observed in these four studies with values predicted by the equations of this paper is summarized in Table 10. The two left-hand columns show the percentage by which observed values varied from those predicted using experimental parameters; columns on the right make a similar comparison with predictions based on average parameters.

It appears that the procedures presented for predicting loss of prestress and camber will normally agree with actual results within ±15 percent when using experimentally determined material parameters. The use of the general or average material parameters gave predicted results that agree with actual results in the range of ±30 percent. With some knowledge of the time-dependent behavior of concrete using local aggregates and under local conditions, it is concluded that one would normally be able to predict loss of prestress and camber within about ±20 percent. In each case, it is noted that most of the results are considerably better than these limits.

PARTIALLY PRESTRESSED MEMBERS

Effect of non-prestressed tension steel in reducing time-dependent loss of prestress and camber. Based on the following energy condition, Eqs. (30) and (31) were developed[34]:

(Work done by forces in steel)
− (Work done against the beam dead load)
= (Change in internal strain energy) (30)

Due to creep:

$$(k_r)_{cp} = \cfrac{1}{1 + \cfrac{E'_s A'_s \int_0^L (\epsilon'_{ci})^2\, dx}{E_s A_s \int_0^L (\epsilon_{ci})^2\, dx}} \tag{30a}$$

Due to shrinkage:

$$(k_r)_{sh} = \cfrac{1}{1 + \cfrac{E'_s A'_s}{E_s A_s}} \tag{30b}$$

where k_r is the reduction factor for the effect of non-prestressed tension steel in reducing time-dependent loss of prestress and camber. When $e' = e$ and $E'_s = E_s$ (an approximate design condition in most cases), both Eqs. (30a) and (30b) reduce to Eq. (31).

Table 11. Experimental and theoretical values of the reduction factor, k_r

Series No.	I			II			III			IV		
Beam No.	1	2	3	1	2	3	1	2	3	1	2	3
A'_s/A_s	1.15	2.30	3.46	0.50	1.73	3.46	0	0	1.30	0.50	1.94	3.76
Experimental range of k_r during period, %	100 to 100	70 to 77	60 to 65	100 to 100	62 to 66	32 to 40	100 to 100	70 to 80	43 to 50	100 to 100	78 to 85	39 to 45
Average exp. value of k_r, %	100	74	63	100	65	37	100	77	48	100	83	42
Theoretical value of k_r, %	100	74	56	100	56	42	100	73	48	100	54	44

The experimental value of k_r is simply the ratio of the time-dependent camber for two beams. In each series, the beam with the greatest time-dependent camber (smallest amount of non-prestressed tension steel, for example) was the reference beam.

The theoretical values of k_r were determined using Eq. (30a) for the Series III beams in which $A'_s/A_s = 0$ for the reference beam; a more general equation from Reference 34 was used for the beams of Series I, II and IV, which contained varying amounts of non-prestressed tension steel including the reference beams in each series.

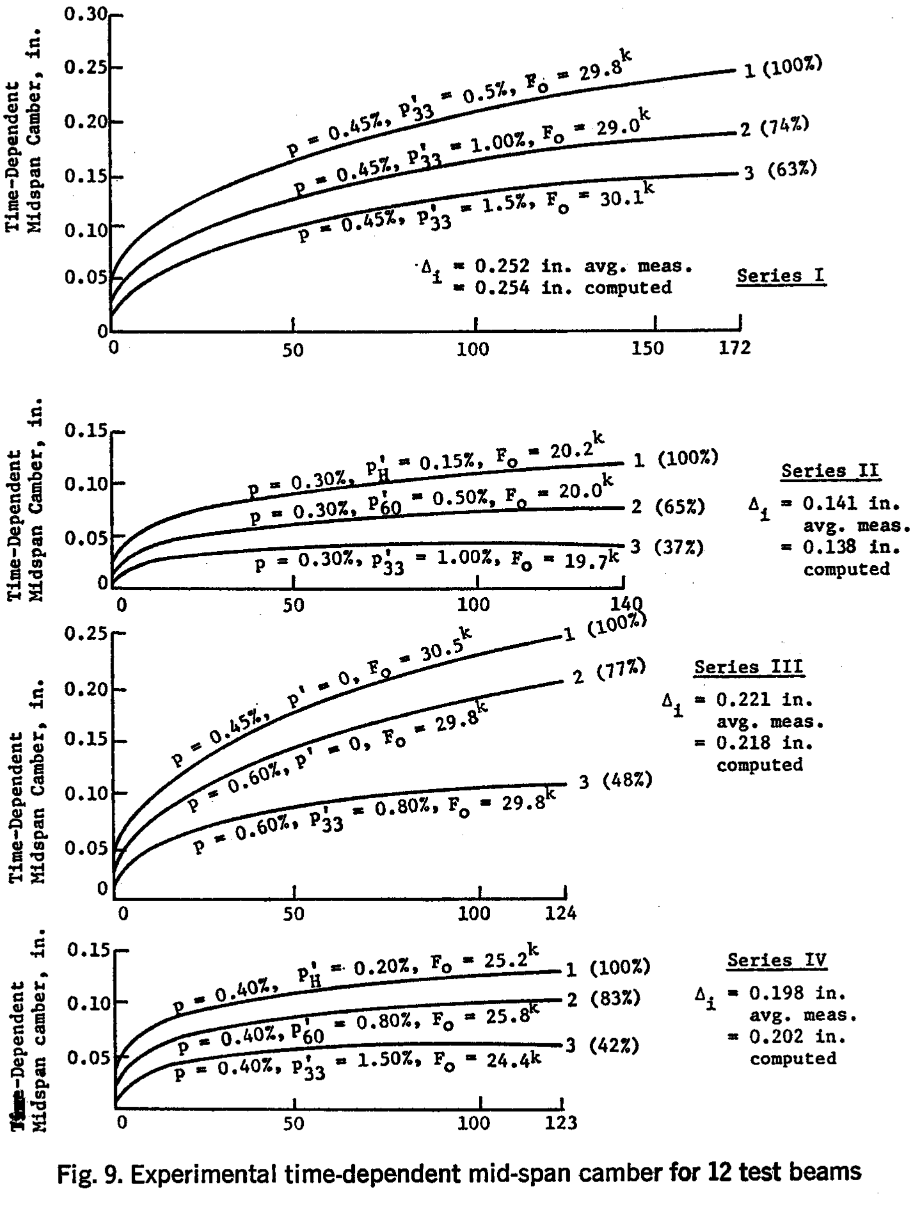

Fig. 9. Experimental time-dependent mid-span camber for 12 test beams

$$k_r = 1/[1 + (A'_s/A_s)] \qquad (31)$$

The application of k_r in the prestress loss and camber equations (Eqs. 17 to 29) is accomplished by replacing all values of the creep coefficient and shrinkage strain with modified values—$k_r\, C_t$ and $k_r\, (\epsilon_{sh})_t$—in all terms, including those due to composite slab effects[28]. k_r can also be applied as a single reduction factor for time-dependent loss of prestress and camber in approximate calculations (see Table 11).

Experimental camber for the 12 beams of Reference 34 (6 x 8 in., span = 15 ft.) are shown in Fig. 9 and experimental vs. computed reduction factors are shown in Table 11.

Deflection of uncracked and cracked prestressed concrete members (either with or without non-prestressed tension steel). The method for computing deflections of reinforced beams[8,28,35,36], ACI Building Code (ACI 318-71), is shown in Reference 34 to apply equally well to bonded prestressed beams, either with or without non-prestressed tension steel, loaded into the cracking range. Eq. (32) from the 1971 ACI Code applies when $M_{max} \geqq M_{cr}$, otherwise $I_{eff} = I_g$.

$$I_{eff} = (M_{cr}/M_{max})^3\, I_g + [1 - (M_{cr}/M_{max})^3]\, I_{cr} \quad (32)$$

Experimental vs. computed results are shown in Figs. 10 and 11 for the 12 beams of Reference 34. For loads up to 80 percent of ultimate, with cracking loads of about 30 percent of ultimate, the maximum deviation between measured and computed deflections was 19 percent. The measured modulus of rupture, f_r, equal to about $11\sqrt{f'_c}$, and a calculated value for E_c were used in obtaining the computed results.

The 1971 ACI Code uses $f_r = 7.5\sqrt{f'_c}$ for computing deflections of normal weight concrete beams. In Reference 36, f_r ranges from $7.5\sqrt{f'_c}$ to $12\sqrt{f'_c}$, and in Reference 8, ave. $f_r = 7.8\sqrt{f'_c}$ to $8.4\sqrt{f'_c}$ for normal weight concrete. Based on the constant 7.8, the following expression for modulus of rupture of concretes of different weight is recommended:

$$f_r = 0.65\sqrt{w\, f'_c} \quad (33)$$

For computing deflections into the cracking range under a superimposed load, or moment $(M_L)_{max}$, use Eqs. (34), (35) or (36) for $(M_L)_{cr}$ in Eq. (32).

For non-composite prestressed beams:

$$(M_L)_{cr} = F\,e + \frac{F\, I_g}{A_g\, y_t} - M_D + \frac{f_r\, I_g}{y_t} \quad (34)$$

For unshored composite prestressed beams:

$$(M_L)_{cr} = F\, e_2 \frac{(y_t)_2\,(I_g)_c}{(y_t)_c\,(I_g)_2} + \frac{F\,(I_g)_c}{(A_g)_2\,(y_t)_c} - (M_1 + M_2)\frac{(y_t)_2\,(I_g)_c}{(y_t)_c\,(I_g)_2} + \frac{f_r\,(I_g)_c}{(y_t)_c} \quad (35)$$

Subscripts 2 and c refer to precast and composite sections, respectively, and $(M_1 + M_2)$ is the slab plus precast beam dead load moment.

For shored composite prestressed beams:

$(M_L)_{cr}$ = same as Eq. (35), (except delete M_1 in the third term), $- M_1$ **(36)**

An average I_{eff} for simple spans is given by Eq. (32). All terms in Eqs. (32) to (36) refer to the maximum moment section, as at midspan for symmetrical simple spans. For continuous beams, see References 38 and 39 for suggested procedures for obtaining appropriate average values of I_{eff}, relative to the positive and negative moment regions of a given span. The use of the uncracked transformed section properties, instead of the gross section properties, in Eqs. (32) to (36) will yield only small refinements in most cases.

Deflections under superimposed loads are computed by Eq. (37), as shown in Appendix C, where $E_c\, I_{eff}$ is seen to be a "secant rigidity."

$$\Delta_L = K\, M_L\, L^2 / E_c\, I_{eff} \quad (37)$$

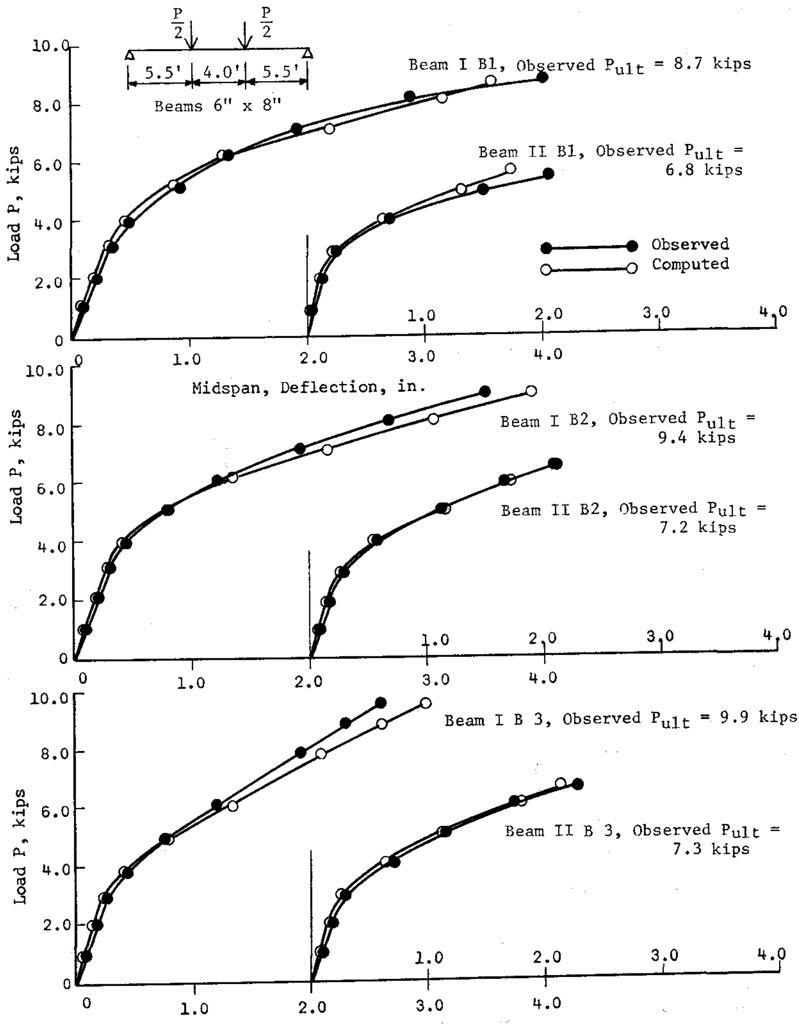

Fig. 10. Experimental and computed loss vs. mid-span deflection for the Series I and II beams of Reference 34

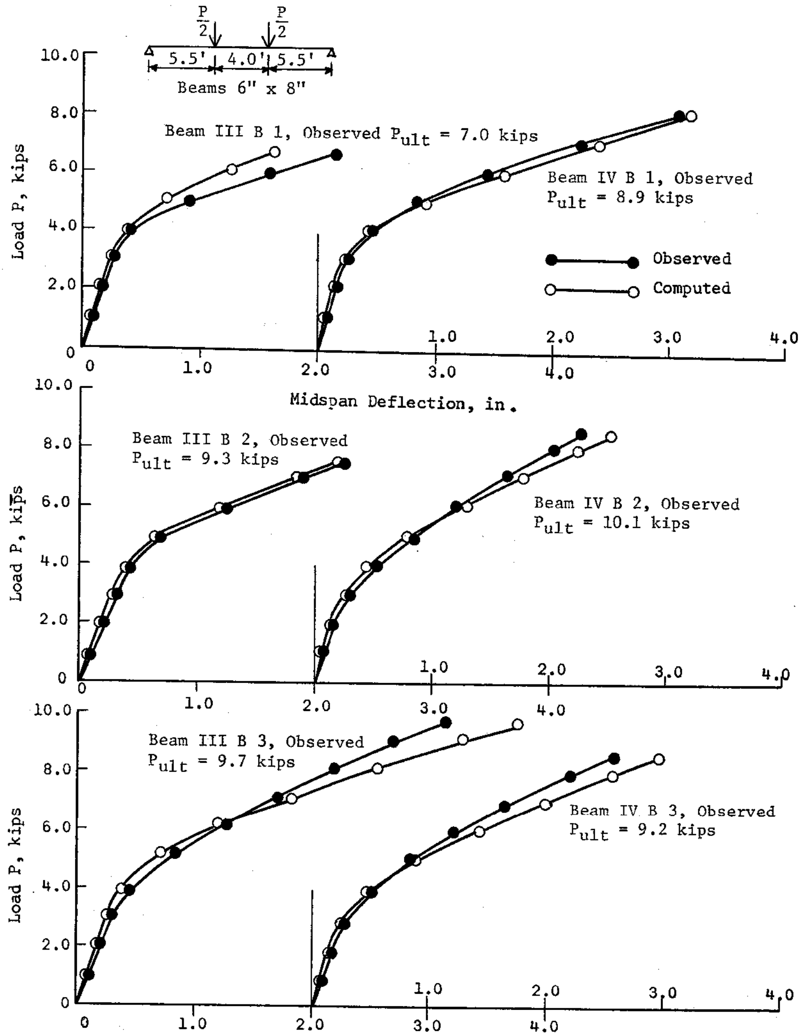

Fig. 11. Experimental and computed loss vs. mid-span deflection for the Series III and IV beams of Reference 34

ACKNOWLEDGMENTS

This is a report of research conducted under Iowa State Highway Commission Research Project No. HR-137, initiated in February 1968, with the final report submitted August 1970. Acknowledgment is made of the assistance of Dr. B. L. Meyers, University of Iowa; Messrs. S. E. Roberts, Research Engineer, C. Pestotnik, Bridge Engineer, Y. H. Gee, Assistant Bridge Engineer, and J. A. Young, Research Technician, of the Iowa Highway Commission; and Mr. J. H. Boehmler, Jr., President, Prestressed Concrete of Iowa, Inc. The contribution of Dr. A. F. Shaikh, University of Wisconsin—Milwaukee, in connection with the material on partial prestressing is also acknowledged.

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APPENDIX A

NOTATION

- $_1$ = subscript denoting cast-in-place slab or the effect of the slab (such as under slab dead load)
- $_2$ = subscript denoting precast beam
- A_g = area of gross section, neglecting the steel
- A_s = area of prestressing steel
- A'_s = area of non-prestressed tension steel
- A_t = area of uncracked transformed concrete section
- b = width of compression face
- $C.F.$ = correction factor
- C_s = creep coefficient at time of slab casting
- C_t = creep coefficient, defined as ratio of creep strain to initial strain, at any time
- C_{t1} = creep coefficient of the composite beam under slab dead load
- C_{t2} = creep coefficient due to precast beam dead load
- C_u = ultimate creep coefficient
- $_c$ = subscript denoting composite section; also used to denote concrete
- $_{cr}$ = subscript denoting cracking
- D = differential shrinkage strain; also used to denote dead load
- $_{DS}$ = subscript denoting differential shrinkage
- d = effective depth of section
- E_c = modulus of elasticity of concrete, such as at 28 days
- E_{ct} = modulus of elasticity of concrete at the time of transfer of prestress
- E_{cs} = modulus of elasticity of concrete at the time of slab casting
- E_s = modulus of elasticity of prestressing steel
- E'_s = modulus of elasticity of non-prestressed tension steel
- e, e' = eccentricity of prestressing steel and non-prestressed tension steel, respectively
- e_c = eccentricity of steel at center of beam (see Appendix B); also used to denote eccentricity of steel in composite section
- e_o = eccentricity of prestressing steel at end of beam (see Appendix B)
- F = prestress force after losses
- F_i = initial tensioning force
- F_o = prestress force at transfer (after elastic loss)
- ΔF = loss of prestress due to time-dependent effects only (such as creep, shrinkage, etc.); the elastic loss is deducted from F_i to obtain F_o
- ΔF_s = total loss of prestress at slab casting minus the initial elastic loss
- ΔF_t = total loss of prestress at any time minus the initial elastic loss
- ΔF_u = total ultimate loss of prestress minus the initial elastic loss
- f_c = concrete stress such as at steel c.g.s. due to prestress and precast beam dead load in the prestress loss equations
- f_{cd} = concrete stress at steel c.g.s. due to differential shrinkage
- f_{ct} = concrete stress at the time of transfer of prestress

f_{cs} = concrete stress at steel c.g.s. due to slab dead load (plus diaphragm or other dead load when applicable)

f_r = modulus of rupture of concrete

f_{si} = initial or tensioning stress in prestressing steel

$(f'_c)_t$ = compressive strength of concrete at any time

$(f'_c)_u$ = ultimate (in time) compressive strength of concrete

f_y = yield strength of steel

H = relative humidity in percent; also subscript denoting relative humidity

I_1 = moment of inertia of slab

I_2 = moment of inertia of precast beam

I_c = moment of inertia of composite section with transformed slab; the slab width is divided by E_{c2}/E_{c1}

I_{cr} = moment of inertia of cracked transformed concrete section

I_{eff} = effective moment of inertia

I_g = moment of inertia of gross section, neglecting the steel

I_t = moment of inertia of uncracked transformed concrete section

i = subscript denoting initial value

K = deflection coefficient; for example, for beams of uniform section and uniformly loaded:
cantilever beam, $K = 1/4$
simple beam, $K = 5/48$
one end continuous, $K = 8/185$
both ends continuous, $K = 1/32$

k_r = reduction factor for the effect of non-prestressed tension steel in reducing time-dependent loss of prestress and camber

k_s = $1 + e^2/r^2$, where e is the steel eccentricity and $r^2 = I_g/A_g$

L = span length; also used as a subscript to denote live load

LA = subscript denoting loading age

M = bending moment; when used as the numerical maximum moment for beams of uniform section and uniformly loaded:
cantilever beam, $-M = q\,L^2/2$
simple beam, $+M = q\,L^2/8$
one end continuous, $-M = q\,L^2/8$
both ends continuous, $-M = q\,L^2/12$

M_1 = maximum bending moment under slab dead load

M_2 = maximum bending moment under precast beam dead load

M_{cr} = cracking moment

M_{max} = maximum moment

m = modular ratio, E_s/E_{cs}, at the time of slab casting

n = modular ratio, E_s/E_{ci}, at release of prestress

P = transverse load

PG = prestress gain in percent of initial tensioning stress or force

PL = total prestress loss in percent of initial tensioning stress or force

PL_{el} = prestress loss due to elastic shortening

PL_t = total prestress loss in percent at any time

PL_u = ultimate prestress loss in percent

p = steel percentage, A_s/bd; also $p = A_s/A_g$ in approximate equation for F_o

p' = steel percentage for non-prestressed tension steel, A'_s/bd (p'_{33} for 33 ksi yield steel, p'_{60} for 60 ksi yield steel, and p'_H for high strength steel)

Q = differential shrinkage force $= D\,A_1\,E_1/3$. The factor 3 provides for the gradual increase in the shrinkage force from day 1, and also approximates the creep and varying stiffness effects

q = uniformly distributed load

s = subscript denoting time of slab casting; also used to denote steel

t = time in general; time in hours in the steel relaxation equations, and time in days in other equations

u = subscript denoting ultimate (in time) value

w = unit weight of concrete in pcf

y_{cs} = distance from centroid of composite section to centroid of slab

y_t = distance from centroid of section to extreme fiber in tension

α_s = ratio of creep coefficient at the time of slab casting to the ultimate creep coefficient, or C_s/C_u

β_s = creep correction factor for the precast beam concrete age when slab cast

Δ = maximum camber or deflection

Δ_i = initial camber or deflection

$(\Delta_i)_1$ = initial deflection under slab dead load

$(\Delta_i)_2$ = initial deflection under precast beam dead load

$(\Delta_i)_{F_o}$ = initial camber due to the initial prestress force, F_o

Δ_D = dead load deflection

Δ_L = live load deflection

Δ_t = total camber or deflection at any time

Δ_u = ultimate camber or deflection

ϵ_{ci} = initial concrete strain at level of prestressing steel

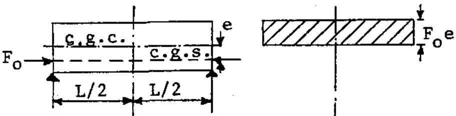
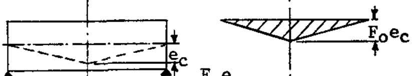
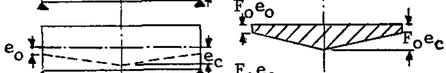
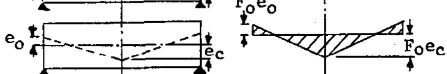
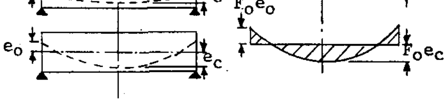
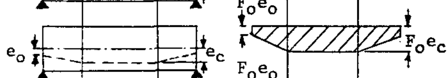
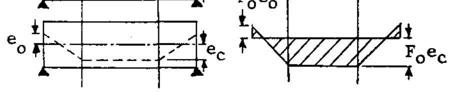
ϵ'_{ci} = initial concrete strain at level of non-prestressed tension steel

$(\epsilon_{sh})_t$ = shrinkage strain at any time

$(\epsilon_{sh})_u$ = ultimate (in time) shrinkage strain

APPENDIX B

COMMON CASES OF PRESTRESS DIAGRAMS WITH FORMULAS FOR COMPUTING CAMBER

Prestressed Beam	F_o e Moment Diagram	Midspan Camber Due to F_o e Moments
		$(\Delta_i)_{F_o} = F_o e\, L^2/8\, E_{ci} I_g$
		$(\Delta_i)_{F_o} = F_o e_c L^2/12\, E_{ci} I_g$
		$(\Delta_i)_{F_o} = \dfrac{F_o(e_c - e_o)L^2}{12\, E_{ci}\, I_g} + \dfrac{F_o e_o L^2}{8E_{ci} I_g}$
		$(\Delta_i)_{F_o} = \dfrac{F_o(e_c + e_o)L^2}{12\, E_{ci}\, I_g} - \dfrac{F_o e_o L^2}{8E_{ci} I_g}$
		$(\Delta_i)_{F_o} = 5\, F_o e_c\, L^2/48\, E_{ci}\, I_g$
		$(\Delta_i)_{F_o} = \dfrac{5F_o(e_c - e_o)L^2}{48\, E_{ci}\, I_g} + \dfrac{F_o e_o L^2}{8E_{ci} I_g}$
		$(\Delta_i)_{F_o} = \dfrac{5F_o(e_c + e_o)L^2}{48\, E_{ci}\, I_g} - \dfrac{F_o e_o L^2}{8E_{ci} I_g}$
		$(\Delta_i)_{F_o} = \dfrac{F_o\, e_c}{E_{ci} I_g}\left(\dfrac{L^2}{8} - \dfrac{a^2}{6}\right)$
		$(\Delta_i)_{F_o} = \dfrac{F_o(e_c - e_o)}{E_{ci}\, I_g}\left(\dfrac{L^2}{8} - \dfrac{a^2}{6}\right) + \dfrac{F_o e_o L^2}{8E_{ci} I_g}$
		$(\Delta_i)_{F_o} = \dfrac{F_o(e_c + e_o)}{E_{ci}\, I_g}\left(\dfrac{L^2}{8} - \dfrac{a^2}{6}\right) - \dfrac{F_o e_o L^2}{8E_{ci} I_g}$

APPENDIX C

For computing I_{eff} and Δ_L as below

$$(M_L)_{cr}/(M_L)_{max} = (f_{pe} - f_{DL} + f_r)/(f_{pe} - f_{DL} + f_t)$$

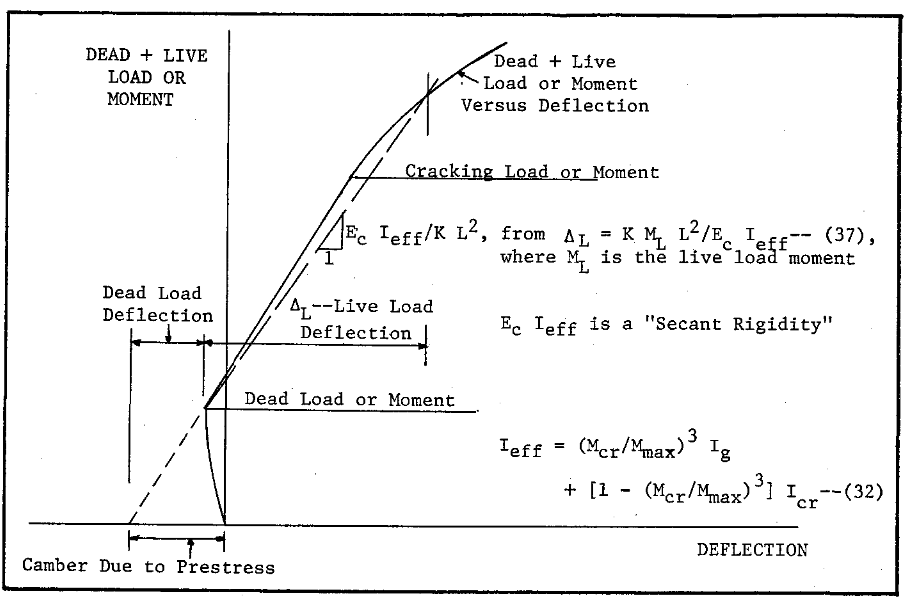
f_{pe} = compressive stress in concrete due to prestress only after all losses, at the extreme fiber of a section at which tensile stresses are caused by applied loads

f_{DL} = stress due to service dead load at the extreme fiber in tension

f_r = modulus of rupture of concrete

f_t = total computed tensile stress (above the cracking stress), that is, between f_r and the maximum allowable stress which, by the 1971 ACI Code, is $12\sqrt{f'_c}$

Note that, for the 1971 ACI Code, f_r is $7.5\sqrt{f'_c}$, $6.4\sqrt{f'_c}$ and $5.6\sqrt{f'_c}$ for normal weight, sand-lightweight and all-lightweight concrete, respectively, and the upper limit value of $f_t = 12\sqrt{f'_c}$. Also for $f_{pe} - f_{DL}$ = something less than $0.45 f'_c$ (say 1500 psi), $f'_c = 5000$ psi, and I_{cr}/I_g = a lower value of 0.20, then $I_{eff}/I_g = 0.71$, 0.66, 0.62 for normal weight, sand-lightweight and all-lightweight concrete, respectively. Thus it is seen that a *maximum* reduction in the effective I is about 29, 34 and 38 percent, respectively, for the live load deflection increment for normal weight, sand-lightweight and all-lightweight concrete, according to the 1971 ACI Code maximum allowable stress for partially prestressed members.



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