A METHOD FOR PREDICTING PRESTRESS LOSSES IN A PRESTRESSED CONCRETE STRUCTURE

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A method is presented for predicting actual prestress losses in a prestressed concrete structure. It uses available information on concrete creep and shrinkage and steel stress relaxation, and includes the effect of interaction of the various factors contributing to prestress loss. The required iterative procedure and curve transfer concepts are explained.

In the past several years, there has been considerable interest within the prestressed concrete industry concerning the amount of prestressing force lost during the lifetime of a structure due to stress relaxation in the steel tendons. A substantial amount of fairly reliable stress relaxation data has been developed from testing stress-relieved wire (ASTM-A421) and strand (ASTM-A416). However, stress relaxation data are usually obtained from tests which do not allow for external load and length changes and therefore do not include the variables encountered in actual use. It is also apparent that the basis of the steel relaxation test (constant strain and stress changing with time) is inconsistent with the test basis used in concrete creep evaluation (constant stress and strain changing with time).

Therefore, any attempt to estimate prestress losses by direct prestress loss sources based on the initial conditions is subject to considerable error. Instead, an overall evaluation of the prestressed concrete structure is needed. An understanding of the effect of external load and length changes and of the interaction between prestress losses resulting from concrete creep and shrinkage and steel stress relaxation is required for such an evaluation.

This paper explains a method that

utilizes available information on concrete strain-time relationships and steel relaxation to predict the actual prestress losses in a prestressed concrete structure. Curve-transfer concepts used in developing the prestress loss estimation due to steel relaxation are presented. Also included is a description of the required iteration procedure which can be readily adapted to computer calculation.

Information required for prestress loss analysis

In developing an interaction approach for estimating prestress losses from steel relaxation and concrete creep and shrinkage, it is necessary to have parameters available describing the steel and concrete properties. Certain design and construction considerations are also required. The required parameters are:

1. Stress relaxation of steel as a function of time and stress level.
2. Average steel strength properties.
3. Creep strain of concrete as a function of time and stress level.
4. Shrinkage strain of concrete as a function of time.
5. Modulus of elasticity of the steel and of the concrete in the load range.
6. Sequence of casting, curing, stressing, and prestress transfer used in the construction procedure.
7. Stress levels expected in the steel and the concrete resulting from externally applied loads, initially and during the life of structure.

Steel stress relaxation. The prestressed concrete industry is presently using the stress relaxation test as the best indication of a steel tendon's ability to maintain the major portion of the initial prestressing force in a concrete structure. In this test, stress relaxation is defined as the time dependent loss of stress in a tendon held at constant strain. In general, stress relaxation increases with increasing stress, time, and temperature.

For prestress loss analysis, it is necessary that the stress relaxation be expressed as a function of time and stress level, assuming a relatively constant temperature. Many papers have been published reporting stress relaxation test results, but only one, that by Magura, Sozen and Siess[1], has attempted to describe relaxation in a mathematical equation including both time and stress level. The equation given is as follows:

$$\% \text{ SR} = \left[\frac{\log t}{10}(R - 0.55)\right] \times 100 \quad (1)$$

$$\text{for } R > 0.55$$

where % SR = % stress relaxation
t = test time in hrs.
R = ratio of initial stress to yield stress

This description of stress relaxation is a simple, straight line semi-log curve with 0% SR at 1.0 hr., and with the slope dependent upon the initial stress level ratio, R. An example of this curve is given in Fig. 1.

Our experience in testing for stress relaxation properties has shown that the straight line semi-log curve is not an accurate description for short periods (less than 1000 hrs.). This is understandable since the equation was developed using long-time data.

A method has been developed in our laboratory for extrapolating relatively short-time stress relaxation data to predict steel losses at much longer times. Basically, it is a quadratic equation of the form:

$$\% \text{ SR} = A + B(\ln t) + C(\ln t)^2 \quad (2)$$

where A, B and C are functions of the stress level ratio. This stress level ratio is defined as the initial steel stress divided by a measure of the steel

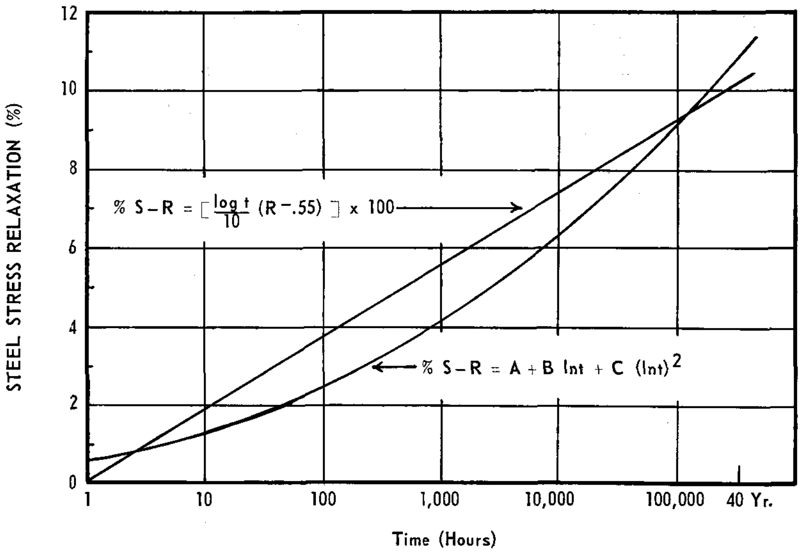


Fig. 1. Time dependent steel stress relaxation at R = 0.735

strength, which is similar to the *R*-value presented by Magura, et al., except that the yield strength can be replaced by another measure of steel strength, such as the ultimate tensile strength. As in Eq. 1, the higher the stress level ratio the greater the expected stress relaxation. The functions *A*, *B* and *C* can be determined for most steel strength measurements.

This quadratic expression for stress relaxation has the advantage of being quite accurate at short times as well as being reasonably consistent with other methods of predicting long-time stress relaxation losses. An example of a curve predicted by this equation is given in Fig. 1.

Concrete creep and shrinkage. In evaluating the concrete's contribution to prestress losses, creep and shrinkage strain have been accepted by the industry as the proper indicators of the time dependent changes in the prestress force. In the creep test, the concrete test specimen is subjected to a constant compression load and the strain change is noted as a function of time. An unloaded duplicate specimen is left in the same room in which the temperature and the humidity are controlled. The strain change of the unloaded specimen is also noted as a function of time. The strain change of the unloaded specimen is defined as the shrinkage, and the differential strain between the unloaded and loaded specimen is defined as the creep strain.

In predicting interaction prestress losses, the creep strain of concrete must be known as a function of time and stress level. One common method of expressing creep strain uses a value called the ultimate specific creep coefficient or unit creep strain[2,3]. This number specifies the ultimate strain expected in the lifetime of the concrete per unit psi of concrete stress. Along with this number, an equation which expresses the fraction or percent of the ultimate creep as a function of time must be included. Almost any mathematical form can be utilized, such as straight line semi-log, segmented line semi-log, hyperbolic function, or even a polynomial describing empirical data. Since the ultimate specific creep coefficient is the ultimate expected strain per psi of concrete stress, total creep is a function of that stress. This form of expressing creep then satisfies the requirements stated, a function of both time and stress level.

A method of describing the expected shrinkage of the concrete is also needed. Since, by definition, shrinkage is independent of applied stresses, total shrinkage strain need only be known as a function of time. One method of expressing the shrinkage is to state the ultimate shrinkage strain expected in the life of the structure, and then describe the rate of this shrinkage with a mathematical function similar to that used in the prediction of creep strain.

When describing expected creep and shrinkage, the effect of various construction and environmental factors should be considered. Construction factors that will change the expected creep and shrinkage of concrete, as determined by laboratory testing, include the type and length of curing, time from cure to prestress transfer, surface to volume ratios, the use of supplemental reinforcing steel, and the presence of biaxial stresses. The type of environment, specifically temperature and humidity, also should be evaluated as to its effect on expected concrete properties.

Elastic moduli. When the stress or strain in a prestressed concrete system changes due to non-elastic factors such as creep, shrinkage and stress relaxation, there is an elastic rebound effect. This is necessary to maintain an equilibrium of forces in the system assuming that strain differentials between the steel and concrete cannot exist. By knowing the elastic moduli of the steel and concrete, the elastic rebound or equalizing effect can be calculated using force and strain balance equations. The solution to these balancing equations is shown in Appendix I.

Construction procedure. The interaction program was developed to apply to either pretensioning or post-tensioning types of construction. Because of the differences between the two types of construction, and differences between methods used in the same type of construction, it is advantageous to know the sequence of events leading up to the initial condition for prestress loss considerations when the full steel tension is applied to the concrete. Concrete shrinkage occurs independent of the stressing situation, and stress relaxation will occur during situations where the steel tension is not applied to the concrete but is held by an external anchoring system. These factors could affect subsequent prestress losses and therefore should be considered before estimations are made.

Expected stress levels. The applied stress levels in the prestressed concrete system have a direct effect on prestress losses. When estimating initial stress levels in the structure, it is important to consider stress changes due to factors such as elastic shortening in pretensioned systems, or friction and anchorage slip in post-tensioned systems. Even after an accurate estimate of initial stress levels is made, it may be necessary to consider changes in external loading during the life of the structure. An example of this is a beam that lies on the ground for several weeks after manufacture; then it is placed in a structure where it must support its own weight and other dead load weights. Any live or working load, if significant, should also be considered.

Interaction analysis for prestress losses

As pointed out above, the rates of steel stress relaxation and concrete creep during the life of a prestressed concrete structure are dependent upon load and time. When the steel and the concrete are combined in a prestressed structure, there is a continuing change in the stress and strain of each component. The basis of this prestress loss method of analysis is to divide the service life into small time intervals during which the stress relaxation, creep strain, and shrinkage strain can be assumed to be independent of each other. In each small time interval prestress loss due to stress relaxation and strain changes due to creep and shrinkage are calculated. At the end of each time interval, elastic expansion and/or contraction of the components is considered so that the compression load on the concrete is equal to the tension load on the steel and the total lengths of each are equal. The stress levels at the end of each time interval are used in the next successive time interval. This procedure is continued until the required service life of the structure is reached.

The basic steps used in the prestress loss estimation method for one time interval are as follows:

1. Determine the load in the steel, P_{1s}, and load in the concrete, P_{1c}, and the length of the steel, L_{1s}, and length of the concrete, L_{1c}, at the start of the time interval.

$$P_{1s} = P_{1c} \quad (3a)$$
$$L_{1s} = L_{1c} \quad (3b)$$

2. Calculate the independent creep and shrinkage strains that occur during the time increment Δt based on the load and time at the start of that increment. Assume the loads have not yet changed but the calculated length of the concrete, L_{2c} is less than the original length of the steel.

$$P_{1s} = P_{1c} \quad (4a)$$
$$L_{1s} > L_{2c} \quad (4b)$$

3. Calculate the elastic contraction of the steel and the elastic expansion of the concrete necessary to equalize lengths while maintaining equal loads. (See Appendix I for formulas.) New loads, P_{2s} and P_{2c}, and lengths, L_{2s} and L_{3c}, now describe an equilibrium situation.

$$P_{2s} = P_{2c} \quad (5a)$$
$$L_{2s} = L_{3c} \quad (5b)$$

4. Calculate the stress loss due to stress relaxation during the same time interval, Δt. The curve transfer procedure necessary for this calculation will be explained below. The lengths remain equal but the new calculated tension load on the steel, P_{3s}, is less than the compression load on the concrete, P_{2c}.

$$P_{3s} < P_{2c} \quad (6a)$$
$$L_{2s} = L_{3c} \quad (6b)$$

5. Calculate the elastic expansion of the steel and concrete necessary to increase the steel tension load and decrease the concrete compression load until they are equal. New loads, P_{4s} and P_{3c}, and new lengths L_{3s} and L_{4c}, define the load situation at the end of the time interval considered.

$$P_{4s} = P_{3c} \quad (7a)$$
$$L_{3s} = L_{4c} \quad (7b)$$

6. The prestress loss due to creep and shrinkage of concrete in this time interval is $P_{1s} - P_{2s}$.
7. The prestress loss due to stress re-

laxation of the steel during this time interval is $P_{28}-P_{48}$.

8. Load and length conditions are now defined for the next time interval as stated in Step 1 and the steps are repeated.

The length of the time intervals should be selected carefully. There must be a compromise between making it as short as possible to simulate an instantaneous situation and long enough to keep the number of computations at a manageable level. Both the concrete and the steel time dependent properties have a continuously decreasing rate with time. Therefore, it is advantageous to have the time increments at the beginning of the structure's life smaller than those at the end. This was accomplished in the computer program by defining the first increment to be from 0 to 1.1 hours, and then dividing each increasing logarithmic cycle into a given number of increments. Therefore, in the 1 to 10 hour range, the increments of time are much smaller than each succeeding log cycle—10 to 100 hrs., 100 to 1,000 hrs., etc.—until the life of the structure has been reached. The necessity of having a computer for these calculations is quite evident when using small time intervals.

For a particular post-tensioning situation studied, 50 increments per log cycle were adequate and even 20 increments per cycle gave reasonable precision. In this situation creep and stress relaxation were considered to start at the same time. If a situation is considered where this is not the case, a larger number of increments per log cycle may be necessary to keep the actual time of the increment small during the initial effects by any of the factors.

Curve transfer procedure

For a prestress loss analysis where interaction between steel and concrete is considered, it is necessary to establish a procedure for determining steel stress relaxation losses during conditions of *changing* strain. A common procedure involves transfer from one stress level relaxation curve to another for each change in strain. The problems are to define the proper stress level curve to use after each strain change, and to define the proper starting point on that curve.

The steel stress relaxation equations previously discussed will define stress relaxation percentage vs. time curves for a given initial stress. To define the proper curve to use after a strain change, the stress equivalent to the initial stress for that situation must be determined. At least two methods are possible for selecting this initial stress.

One method uses the actual stress after the strain change. This actual stress is the algebraic sum of the initial stress and stress changes due to both strain changes and prior stress relaxation. There is no attempt to separate the contributions of each. Using the actual stress to determine a new initial stress level ratio implies that this ratio includes the effect of prior stress relaxation. However, the original equations were derived with the stress level ratio as an independent variable. Any change in this variable should be independent of prior stress relaxation. For these reasons, using the actual stress level for determining the new stress level ratio is not a satisfactory method.

A second method for selecting the proper relaxation curve uses an initial stress that is independent of prior stress relaxation. This is accomplished by introducing an *effective* initial stress which is defined as the algebraic sum of the initial applied stress and any later changes in that stress due to factors other than stress relaxation. These factors include any elastic strain changes in the steel. The effective initial stress is used in the stress level ratio, R, which is now the ratio of the effective initial stress to the yield stress. Therefore, the R-ratio is based on external load or strain changes and is not a function of the amount of stress lost due to the relaxation process. Determining the R-ratio in this manner solves the problem of defining the proper stress level curve to use after each strain change in the prestress loss analysis. The effect of prior stress relaxation is accounted for in the curve transfer procedure as explained in the following paragraphs.

To determine the proper starting point on the new stress level curve, it is necessary to define the method of transferring from the previous curve to the new stress level curve. Since the R-ratio has been defined, two variables—stress relaxation percentage and time—are left in the relaxation equation to determine a starting point on that curve. If actual time is used to define the starting point on a new stress relaxation percentage vs. time curve, the curve transfer is *vertical* from the previous curve to the new curve. However, the effect of prior stress relaxation at the higher initial stress level is neglected in this procedure. If prior stress relaxation is used to define the starting point, the curve transfer should be *horizontal* from the previous curve to the new curve. The previous stress loss due to relaxation will then define the rate of relaxation after a strain change. In other words, the R-ratio defines the curve and the previous stress relaxation defines the starting point on that curve.

To verify the proper curve transfer procedure, stress relaxation experiments were conducted using changing strain conditions on 0.250 in. dia. wire specimens. Since a strain change in the steel must, according to Hooke's Law, be accompanied by a proportional amount of stress change, the effect of a strain change is the same as changing the applied stress. In both of the experiments, two wires were tested, one at a reference stress level and another at higher inital stress levels for various periods of time.

The curves in Fig. 2 compare stress relaxation test data taken on a wire with no external stress change (Test A) vs. another wire with two external stress changes (Test B). Test A was loaded to 75% of the guaranteed ultimate tensile strength (GUTS) and had an initial R-ratio of 0.797. This test was conducted over 600 hours. Test B was loaded to a higher initial stress ratio, $R = 0.860$, then manually reduced in load after 9 min. to $R = 0.833$, and reduced again after 3 hr. 40 min. to $R = 0.797$, the same as for Test A. The problem is to define the proper transfer procedure that will determine the position on the Test A curve for predicting the stress relaxation response of Test B. It can be seen that using actual test time as the criterion for determining the proper position is the same as dropping vertically to the Test A curve. Plotting the actual data from that point shows a lower relaxation rate than would be predicted by the reference curve (Test A). The other alternative is a horizontal move to a point on the Test A curve, indicating equivalent stress loss due to stress relaxation. When the actual data are plotted in this manner, the predicted and actual relaxation rates are nearly identical. The slight initial offset is due to the necessity of resetting the strain controlling extensometer.

The curves in Fig. 3 represent a similar situation but with a lower initial stress level and a longer time in the high stress situation. Two test wires were used from the same coil, the first (Test C) originally loaded to 70% GUTS ($R = 0.728$) and continued to 400 hours. The second (Test D) was originally loaded to 76% GUTS for 24

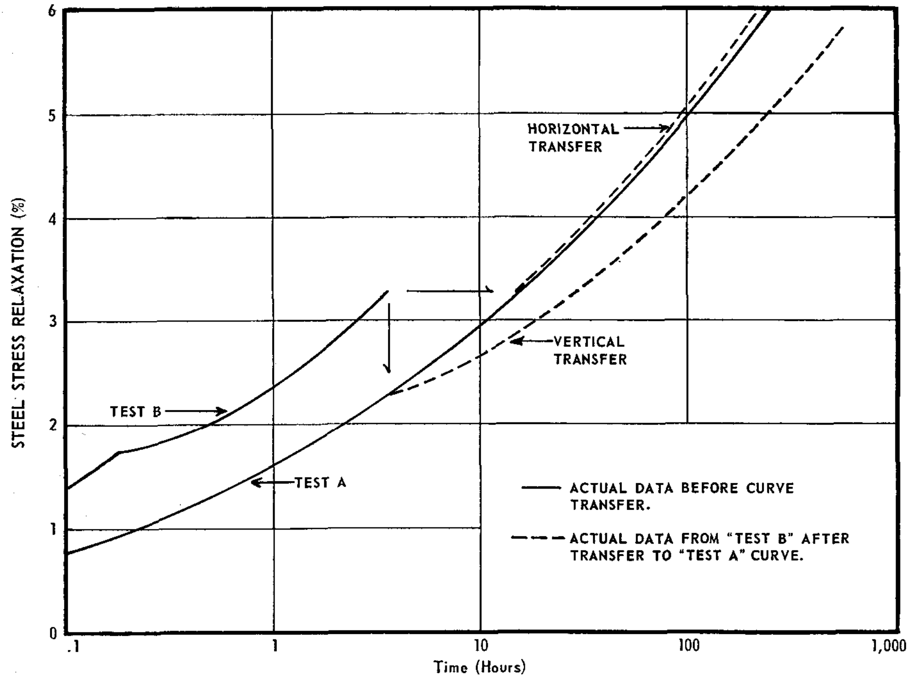


Fig. 2. Effect of curve transfer methods for Tests A and B

hr., then manually reduced by 6% of GUTS to be equivalent to the 70% GUTS test ($R = 0.728$). Again, dropping vertically at the 24 hr. point from Test D to Test C and plotting the actual data substantiates that a much lower rate of relaxation actually occurred than would be predicted from the original 70% GUTS curve. The horizontal transfer for equivalent stress relaxation was beyond the time range for Test C, but it appears to fit an extension of the Test C curve.

As the data presented in Figs. 2 and 3 illustrate, the best choice of the two alternatives of horizontal and vertical curve transfer is the horizontal method which is based on equivalent stress relaxation losses. This procedure was used in developing the full interaction approach to predicting prestressed concrete stress losses.

It is possible that the concept of the horizontal curve transfer procedure as described for steel stress relaxation is also applicable to creep strain in concrete. This possibility should be verified by test. For situations in which there is a large variation in the concrete stress during the life of the structure, an equivalent creep concept is necessary[4]. In a structure where there is little or no external stress change in the concrete, either curve transfer method could be used since concrete creep is considered to be directly proportional to the total applied stress[5,6], and therfore small changes are not as critical as they

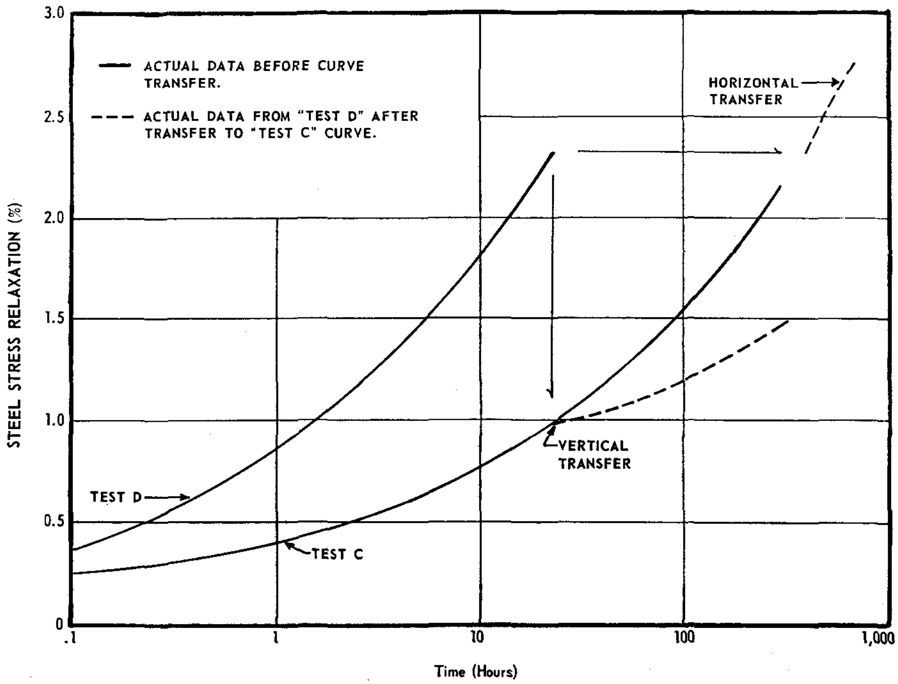


Fig. 3. Effect of curve transfer methods for Tests C and D

are in steel stress relaxation.

Application of prestress loss analysis

A computer program was developed based on the procedures described for interaction analysis and curve transfer. The program was written so that all available pertinent information could be included in the prestress loss calculations. Use of the computer allows a large number of situations to be considered using small time interval interactions.

Results. Table 1 and 2 present a summary of estimated prestress losses assuming a particular post-tensioning situation. The assumed initial conditions are as follows:

Creep and shrinkage as proposed by PCI Committee on Prestress Losses.

Yield strength of steel = 220,000 psi

Modulus of elasticity of steel = 29×10^6 psi

Original stress on steel = 168,000 psi

Modulus of elasticity of concrete = 4.2×10^6 psi

Original stress on concrete = 1,500 psi

Total shrinkage strain = 500×10^{-6} in./in.

Time before prestress transfer = 30 days

Average temperature = 68 F

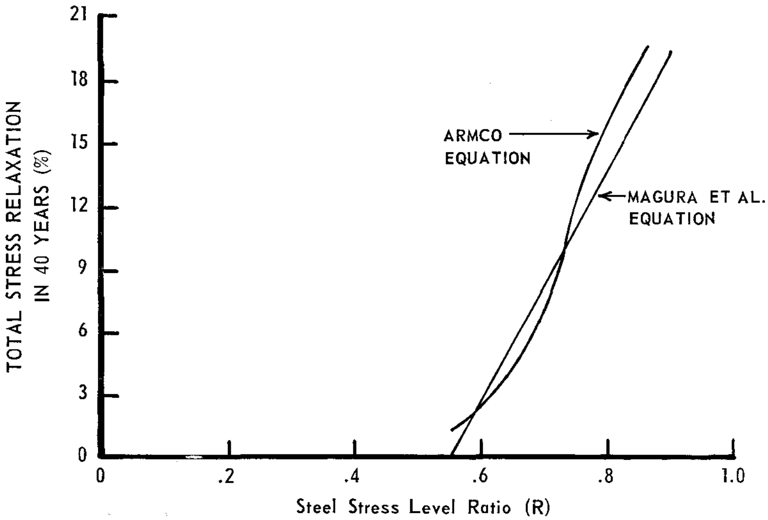
Table 1 includes estimations using a

Table 1. Estimated load losses in a prestressed concrete structure using various steel and concrete grades and based on Eq. 2

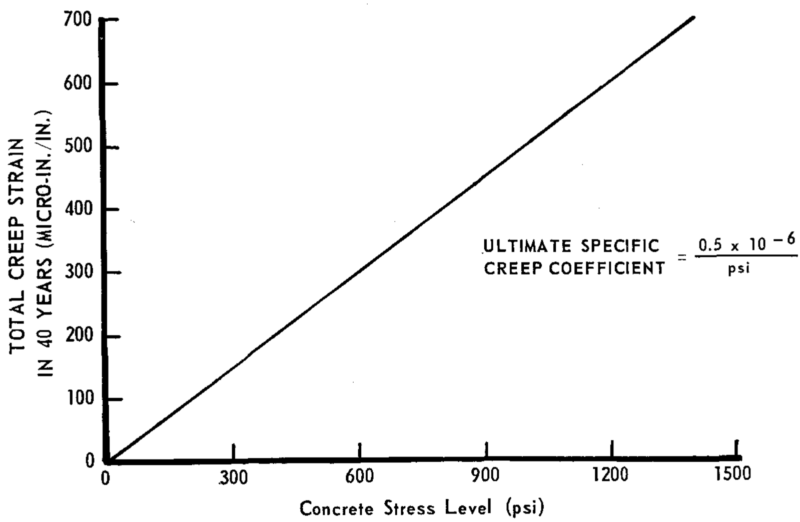
Steel type	Ultimate specific creep coef. $\times 10^{-6}$/psi	Estimated percent load loss in 40 years					
		Due to steel relaxation		Due to concrete creep and shrinkage		Total load loss	
		No interaction	Interaction with concrete	No interaction	Interaction with steel	No interaction	Interaction
1	0.2	13.7	6.9	10.1	9.2	23.8	16.1
	0.4	13.7	5.2	15.3	13.4	29.0	18.6
	0.8	13.7	3.4	25.6	20.9	39.3	24.3
2	0.2	8.5	4.3	10.1	9.3	18.6	13.6
	0.4	8.5	3.0	15.3	13.6	23.8	16.6
	0.8	8.5	2.0	25.6	21.2	34.1	23.2
3	0.2	3.9	2.1	10.1	9.4	14.0	11.5
	0.8	3.9	2.0	15.3	13.7	19.2	15.7
	0.4	3.9	1.8	25.6	21.2	29.5	23.0

Table 2. Estimated load losses in a prestressed concrete structure using various steel and concrete grades and based on Eq. 1

Steel type	Ultimate specific creep coef. $\times 10^{-6}$/psi	Estimated percent load loss in 40 years					
		Due to steel relaxation		Due to concrete creep and shrinkage		Total load loss	
		No interaction	Interaction with concrete	No interaction	Interaction with steel	No interaction	Interaction
1	0.2	13.7	7.7	10.1	9.1	23.8	16.8
	0.4	13.7	6.2	15.3	13.3	29.0	19.5
	0.8	13.7	4.8	25.6	20.7	39.3	25.5
2	0.2	7.4	5.1	10.1	9.3	17.5	14.4
	0.4	7.4	4.1	15.3	13.5	22.7	17.6
	0.8	7.4	3.1	25.6	21.0	33.0	24.1
3	0.2	2.5	1.7	10.1	9.4	12.6	11.1
	0.4	2.5	1.3	15.3	13.8	17.8	15.1
	0.8	2.5	0.9	25.6	21.4	28.1	22.3



(b) – Total Stress Relaxation at 40 Yrs. vs. Steel Stress Level Ratio



(a) – Total Creep Strain at 40 Yr. vs. Concrete Stress

Fig. 4. Response of concrete creep and steel relaxation to stress levels

stress relaxation equation of the type given as Eq. 2, and Table 2 gives estimations using the straight line semi-log Eq. 1. In each of these tables, three steel types are considered based on their basic "no interaction" stress relaxation properties. Also, three concrete situations are considered by using different unit creep strains. The lower "interaction" load loss due to stress relaxation using Eq. 2 (Table 1) is due to the lower predicted stress relaxation rate during the important initial time periods (see Fig. 1).

Discussion. As would be expected, in every situation that was investigated, the interaction consideration yielded lower total prestress losses than if the loss causes were treated independently. The decrease in expected losses due to the steel relaxation was generally more significant than the decrease in losses due to concrete creep. The reason for this is the variation in the response to stress levels as shown in Fig. 4. The applied stresses in the concrete are usually less than one half the ultimate compressive strength and, in this range, the creep is considered to be directly proportional to the applied stress (Fig. 4a). Significant steel stress relaxation occurs only at relatively high stresses starting above $R = 0.60$ (Fig. 4b). A typical initial stress ratio in the steel would be 0.75. Therefore, the expected steel relaxation can change quite drastically compared to the expected creep strain for an equivalent amount of load change.

Most prestressed concrete structures are designed using a minimum prestress force requirement during the expected life of the structure. Therefore, the expected prestress losses are compensated for by using additional prestressing force in the initial construction. This additional prestressing force is provided by increasing the number of steel tendons since the stress on each tendon is limited by construction codes and cannot be increased. For this reason, an accurate estimate of prestress losses is directly related to the economics of the construction because an over-estimation of losses will result in unnecessary costs.

With recent emphasis in the industry on steel tendons specially processed to produce low relaxation properties, it is interesting to note the effect of these properties on total load loss when the interaction approach is considered. As shown in Table 1, on the basis of total percent load loss, the interaction effect for each steel type in a relatively poor concrete (creep coefficient = 0.8) indicates that there is very little economic advantage to a low relaxation steel (Type 3). The total decrease in expected percent load losses based on the same original load, replacing normal stress-relieved material (Type 1) with low relaxation material (Type 3), is only 1.3%. This improvement is significantly less than the 9.8% decrease expected when not considering the interaction effects.

The effect of improved concrete properties (changing the creep coefficient from 0.8 to 0.2) on total percent load loss is not as significant when considering interaction. For example, a load loss decrease of 15.5% is predicted using no interaction with stress relieved steel compared to 8.2% when considering the interaction effect. Improving the concrete properties is more effective when using a steel tendon with lower relaxation properties as shown by the 11.5% decrease in interaction losses using Type 3 steel.

Evaluation of the data in Tables 1 and 2 points out that if the most economical method of construction is desired, it is necessary to have a good balance of steel and concrete properties. The use of low relaxation steel has more of an advantage when higher quality concrete is used, and vice-versa. The decision to use higher priced materials to decrease prestress losses should be based on the best estimate of the total effect of these materials and the resulting economic advantage.

The particular values stated in the previous paragraphs are only examples based on one particular set of conditions and described properties. The effect of all material properties and construction techniques must be considered for a proper evaluation.

A specific example of construction techniques is the case of the pretensioned I-beam. Normally the steel is stressed to 70% of the guaranteed ultimate tensile strength and anchored to abutments. The concrete is then cast around the steel tendons. A normal procedure is to steam cure this concrete at temperatures around 150 F for at least one night. After the designated concrete strength is reached, the steel tendons are cut from the anchorages and the stress is transferred to the concrete. Because of the expected high amount of relaxation at 150 F, and the subsequent loss of stress due to elastic shortening of the beam, it is possible that the relaxation of the steel after the prestress transfer is insignificant. An accurate description of the stress and temperature levels during the curing process and relaxation test data under these conditions is necessary to substantiate this possibility.

Program limitations. The main limitation to the interaction program for estimating prestress losses is the quality of the available input data. The accuracy of the estimate based on interaction effects can be no greater than the accuracy of the data determined from "no interaction" tests. It is important to the acceptance of this program that the best information be made available to those who wish to use it. This means that materials testing should be continued and intensified.

The major deficiency in most of the data now available is that the effect of environmental conditions has not been determined. Concrete creep and shrinkage strain are known to be affected by temperature and humidity, and steel relaxation is also known to be substantially affected by temperature. Yet these parameters are not well described for most situations actually encountered, such as the pretensioning situation described above.

Conclusions

1. The basis of the interaction method of predicting prestress losses described in this paper is a stress and strain balancing technique which is not new to the industry, although it has not been used extensively.
2. The method of horizontal curve transfer for predicting stress relaxation rates after strain changes occur is a new approach and has not been used before. It is an important part of the interaction method.
3. The interaction of concrete creep and shrinkage strains and steel stress relaxation reduces the total expected prestress losses.
4. Both the stress losses due to steel stress relaxation and those due to concrete creep and shrinkage are decreased when the interaction approach is used. The reduction is generally more significant for steel stress relaxation.
5. Improving the steel stress relaxation properties alone by a given amount will not reduce the total expected losses by an equivalent amount.
6. The accuracy of the interaction method of predicting prestress losses is dependent upon the accuracy of the material property

determinations and on the proper description of actual stress levels and the environment of the structure during its lifetime.

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APPENDIX I

Known conditions at load equilibrium:

Load in concrete = Load in steel

$$P_c = P_s$$

If a length differential ΔL_T exists, then to equalize that differential

$$\Delta L_T = \Delta L_s + \Delta L_c$$

or $$\Delta \epsilon_T = \Delta \epsilon_s + \Delta \epsilon_c$$

Therefore:

$$P_c(\text{original}) = P_s(\text{original})$$

$$A_c \sigma_{co} = A_s \sigma_{so}$$

where A = cross-sectional area

$$\frac{A_c}{A_s} = \frac{\sigma_{so}}{\sigma_{co}}$$

But any change in loads must be equal

$$\Delta P_c = \Delta P_s$$

$$A_c\, \Delta\sigma_c = A_s\, \Delta\sigma_s$$

$$A_c\, E_c\, (\Delta\epsilon_c) = A_s\, E_s\, (\Delta\epsilon_s)$$

$$\Delta\epsilon_c = \frac{A_s\, E_s}{A_c\, E_c}\, \Delta\epsilon_s$$

$$\Delta\epsilon_c = \frac{\sigma_{co}\, E_s}{\sigma_{so}\, E_c}\, \Delta\epsilon_s$$

$$\Delta\epsilon_c = \frac{\sigma_{co}\, E_s}{\sigma_{so}\, E_c}\, (\Delta\epsilon_T - \Delta\epsilon_c)$$

$$\Delta\epsilon_c \left(1 + \frac{\sigma_{co}\, E_s}{\sigma_{so}\, E_c}\right) = \frac{\sigma_{co}\, E_s}{\sigma_{so}\, E_c}\, \Delta\epsilon_T$$

Therefore:

$$\Delta\epsilon_c = \frac{\sigma_{co}\, E_s}{\sigma_{so}\, E_c + \sigma_{co}\, E_s}\, \Delta\epsilon_T$$

$$\Delta\epsilon_s = \Delta\epsilon_T - \Delta\epsilon_c$$

Discussion of this paper is invited.
Please forward your discussion to PCI Headquarters by July 1 to permit publication in the July-August 1972 issue of the PCI JOURNAL.