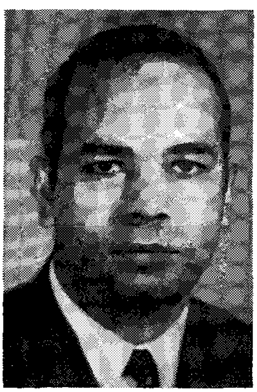
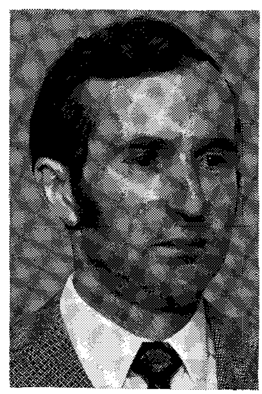
Time-dependent prestress loss and deflection in prestressed concrete members

Maher K. Tadros **Amin Ghali** **Walter H. Dilger**
Civil Engineering Department
The University of Calgary, Calgary, Alberta



Maher K. Tadros

Amin Ghali



Walter H. Dilger

The authors propose an analytical method to calculate time-dependent prestress loss, axial strain, curvature, and deflection of prestressed concrete members.
The method, which is compared with experimental data and other established methods, covers both pretensioned and post-tensioned beams and frames.

Despite the wide use of prestressing, there is no simple practical method for predicting accurately the time-dependent losses and deflections of prestressed concrete members. This is partly because of the difficulty in predicting the coefficients related to the properties of steel and concrete and the uncertain environmental conditions in which the structure will be subjected to after prestressing. Even when these factors can be predicted precisely, an accurate analysis which accounts for the interdependence of the effects of shrinkage, creep, and relaxation of steel is too complicated to be widely used in practice.

A recent paper[1] by the present authors uses a numerical procedure, which accounts accurately for the interdependence of the above effects. The current paper emphasizes practical applications without deriving the equations. Rather, the various design steps are presented and demonstrated with numerical examples. A brief explanation of the assumptions behind the proposed method is included in a Commentary in the Appendix.

The paper also attempts to answer two questions which will no doubt occur to the designer, before adopting the method:

(a) How do the basic assumptions and the results of the proposed method compare with other analytical methods?

(b) Does experimental evidence support the results of the proposed method?

In addition, the paper

(a) Shows the difference in computing prestress losses and deflections in pretensioned and post-tensioned members and

(b) Gives a procedure to find the loss (or gain) in prestress and the deflection caused by superimposed sustained load introduced some time after prestressing.

A method is presented to accurately predict the time-dependent prestress loss, axial strain, and curvature at any section in prestressed concrete beams and frames. The paper covers non-composite pretensioned and post-tensioned structures treating both simple and continuous members. Although axial strain and curvature are not directly needed in design, these values are useful in calculating deflections and, in some cases, shortening of members. When the strain and the curvature are known at various sections, deflections can be calculated by well-established methods. Equations are given to calculate midspan deflection and shortening of each span. The results of the proposed method are compared with existing experimental data showing good agreement. Primary emphasis is placed on practical applications although an explanation of the assumptions behind the method is given in the Appendix. Two fully-worked numerical examples are included and compared with approximate methods of analysis.

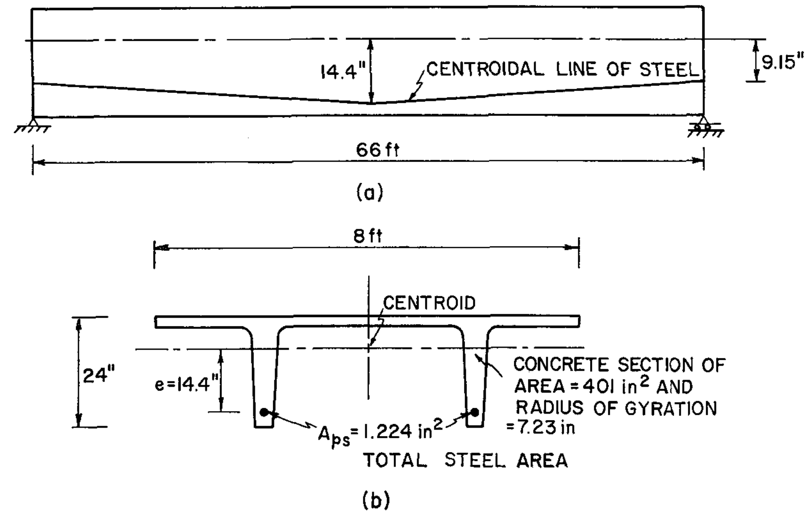


Fig. 1. Pretensioned beam considered in Examples 1 and 2. (a) tendon profile; (b) midspan cross section.

Problem Statement

Consider a section of a concrete member (pretensioned or post-tensioned) prestressed at age t_o by a force P_o at eccentricity e (see for example the beam in Fig. 1). At the time of transfer t_o, a bending moment M and an axial force N are also introduced, representing for example the self weight effect. The axial force N is nonzero only in special applications, for example in a statically indeterminate post-tensioned frame. Superimposed load applied at a later age after prestressing produces additional internal forces. (The effect of these is discussed separately.) It is required to find at this section the prestress loss L, the axial strain ϵ, and the curvature ϕ which will occur at age t_k ($t_k > t_o$).

For the solution of this problem, three values which depend on the quality of the prestressing steel and concrete, the dimensions of the cross section, and the temperature and relative humidity of the air, are assumed to be known.

The three values are:

1. The free shrinkage s which would occur between ages t_o and t_k.
2. The creep coefficient v which is the ratio of creep at age t_k to the instantaneous strain when a load is applied at age t_o and sustained at a constant value, and
3. The "intrinsic" stress relaxation L_r. This is defined as the reduction in tension which would occur during the period $(t_k\text{-}t_o)$ if the tendon were stretched between two fixed points with initial stress $f_{so} = P_o/A_{ps}$. It is well known that the intrinsic relaxation L_r depends to a large extent on the stress level in the steel which may be expressed as the ratio β equals f_{so} divided by the ultimate strength.

The determination of the values of s and v may be guided by Reference 2 or 3.

The value of L_r is usually provided by the steel manufacturer. One of the equations to estimate L_r, based on the work of Magura, Sozen, and Siess,[4] is

$$L_r = 0.1\, f_{so}\, [f_{so}/f_y - 0.55] \log (t_n/t_r) \quad (1)$$

where $f_{so}/f_y \geqq 0.6$, and

f_y = 0.1 percent offset stress, and

t_r, t_n = time at transfer and time at which the loss is to be obtained, measured from the time of initial stressing of the tendons. In the case of post-tensioned members t_r is taken equal to 1 hour.

However, some kinds of steel are now used for prestressing with less relaxation values than that given by the above equation.[5] Furthermore, test data[6] have indicated that the relaxation loss is a function of the strand diameter.

Other material properties assumed to be known are E_s and E_c, the modulus of elasticity of steel and of concrete at age t_o, respectively.

General Description of Proposed Method

Table 1 is presented as a design aid. It was derived using a step-by-step numerical method. The procedure and derivation of the equations used for the table as well as a plot of it are given in Reference 1. The basic assumptions considered of interest to the designer are included shortly.

An equation widely used for calculating the loss due to shrinkage, creep, and relaxation is

$$L = s\, E_s + L_r + v\, n\, f_{co} \quad (2)$$

where f_{co} is the initial concrete stress at the level of the centroid of the prestressing steel, and

$$n = E_s/E_c \quad (3)$$

The loss of prestressing force, due to shrinkage and creep of concrete and steel relaxation, reduces the concrete stress and induces elastic strain and creep recoveries. The reduction in steel stress due to the shortening of the tendon results in a smaller amount of relaxation as compared to the intrinsic relaxation, L_r. For these two reasons there is an interdependence in the amount of loss caused by shrinkage, creep, and relaxation, and it is in fact not possible to separate the effect of each cause.

Eq. (2) ignores the very important aspect of the reduction in concrete strain resulting from the continuously reducing concrete stress at the level of the steel (the recovery effect). It also ignores the reduction in steel relaxation, and thus considerably overestimates the loss. One of the objectives of this paper is to provide a formula for the calculation of L in which the overestimation in Eq. (2) is avoided without appreciably complicating the formula.

The recovery effect can be accurately

Table 1. Recovery parameter μ_o.

v \ ξ	5	10	15	20	25	30	35	40	45	50
0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.5	0.101	0.049	0.037	0.029	0.024	0.020	0.017	0.015	0.014	0.012
1.0	0.239	0.122	0.090	0.070	0.058	0.049	0.042	0.037	0.033	0.030
1.5	0.410	0.217	0.159	0.124	0.102	0.087	0.075	0.066	0.059	0.054
2.0	0.609	0.332	0.243	0.190	0.156	0.133	0.115	0.102	0.091	0.083
3.0	1.084	0.620	0.454	0.357	0.294	0.250	0.217	0.192	0.172	0.156
4.0	1.642	0.976	0.719	0.568	0.469	0.400	0.348	0.308	0.276	0.251

Table 2. Relaxation reduction factor ψ.

Ω \ β	0.50	0.55	0.60	0.65	0.70	0.75	0.80
0.00		1.000	1.000	1.000	1.000	1.000	1.000
0.05	0.000	0.547	0.729	0.798	0.835	0.857	0.872
0.10	0.000	0.289	0.516	0.627	0.689	0.729	0.756
0.15	0.000	0.172	0.361	0.486	0.564	0.615	0.652
0.20	0.000	0.099	0.262	0.375	0.458	0.516	0.558
0.30	0.000	0.013	0.150	0.238	0.305	0.361	0.406
0.40	0.000	0.000	0.077	0.159	0.216	0.262	0.300
0.50	0.000	0.000	0.029	0.102	0.157	0.197	0.230

accounted for by the use of the recovery parameter μ to be derived from Table 1. For the steel relaxation, a reduction factor ψ obtained from Table 2 is used.

The recovery parameter μ was derived by a step-by-step procedure. The period of prestress loss (t_k-t_o) was divided into discrete time intervals and the creep and elastic strains at the end of each interval were calculated from the updated conditions. A further explanation of the derivation of μ is included in the Appendix.

The problem of calculating loss is closely related to that of calculating time-dependent strain and curvature. Computation of prestress loss is in fact based on computing the time-dependent strain at the prestress steel level, which is related to the axial strain ϵ and the curvature ϕ. The recovery parameter μ of Table 1 is again used to obtain the values of ϵ and ϕ.

In practice, the prestressing often produces curvature of comparable magnitude and of opposite sign to the dead load curvature. Thus, the deflection (or camber) is the resultant of the difference of the two effects, and hence no accurate value of deflection can be predicted without a rational account of prestress loss.

The calculations presented here involve the following widely accepted assumptions: (a) plane cross sections remain plane after deformation and (b) the change in strain in the prestressing steel is the same as the change in strain in the adjacent concrete.

The following sections explain the computation steps for calculating L, ϵ, and ϕ, and the use of the last two values to determine the deflection and shortening of beams. Numerical examples are provided in which the results are compared with approximate methods. Finally, the method is experimentally verified and compared with existing methods.

Sign convention

P_o (and P_i) are always positive; N is positive when compressive. M is positive when it produces tension at bottom fibers of a member. Positive ϵ and ϕ correspond to positive N and M, respectively. The concrete stress is positive when compressive.

Design Procedure

The calculation of the loss L, the axial strain ϵ, and the curvature ϕ is done in three steps.

Step 1

Calculate the concrete stress at tendon level immediately after transfer

$$f_{co} = (\alpha P_o + N - Me/r^2)/A_c \quad (4)$$

where r is the radius of gyration, e is the eccentricity (positive when downward), and

$$\alpha = 1 + e^2/r^2 \tag{5}$$

The net section (less ducts) should be used for the calculation of A_c and r in post-tensioned sections unless it can be shown that the gross section does not induce too many errors. For pretensioned sections, the use of the gross concrete section involves a tolerable error.

The recovery parameter μ_o is read from Table 1 which should be entered by the creep coefficient v and the steel area parameter.

$$\xi = A_c/(\alpha n A_{ps}) \tag{6}$$

The value μ_o thus obtained corresponds to the situation when the shrinkage and relaxation are zero. For the actual situation, the recovery parameter μ is a larger value to be determined in Step 2.

Step 2

Compute the shrinkage–relaxation parameter ω defined as follows

$$\omega = (sE_s + \psi L_r)/(nf_{co}) \tag{7}$$

At this stage, the relaxation reduction factor ψ is not known, and a simple iteration is needed. At first, estimate of ψ *(I)* is used in the equation. The value of ψ can be between 0.0 and 1.0, and for most practical cases, a value of 0.7 for the first guess leads to the accurate ψ value after a single iteration.

Now, the recovery parameter is calculated from

$$\mu = \mu_o + \frac{(1 + 0.6\, v)\omega}{1 + 0.6\, v + \xi} \tag{8}$$

The time-dependent loss (excluding the instantaneous loss at transfer) is given by

$$L = sE_s + \psi L_r + (v - \mu)n\, f_{co} \tag{9}$$

The accuracy of the assumed value of $\psi = \psi$ *(I)* is now examined by using Table 2, which is entered by β and the following parameter

$$\Omega = (L - L_r)/f_{so} \tag{10}$$

If the value of ψ obtained from the table is different from the assumed value, Step 2 is repeated using $\psi = \psi$ *(II)*, the last value obtained from the table. This repetition if needed, will in most cases, give accurate values of the recovery parameter μ and prestress loss L, and no further iteration will be necessary.

Step 3

Calculate the values of the axial strain ϵ and the curvature ϕ at age t_k (including the instantaneous deformations occurring at t_o).

$$\epsilon = s + \frac{P_o + N}{A_c E_c}(1 + v) - \frac{f_{co}}{\alpha E_c}\mu \tag{11}$$

$$\phi = \frac{M - P_o e}{r^2 A_c E_c}(1 + v) + \frac{e\, f_{co}}{\alpha r^2 E_c}\mu \tag{12}$$

Instantaneous Loss in Pretensioned Members

In the case of pretensioned beams, the value of the initial tension immediately before transfer P_i may be more readily available than P_o used above. The difference between the two values is caused by the instantaneous stress loss.

$$L_{es} = (P_i - P_o)/A_{ps} \tag{13}$$

which may be calculated as follows

$$L_{es} = n\, f_{ci}/(1 + 1/\xi) \tag{14}$$

where f_{ci} is the concrete stress at the steel level which would occur had there been no instantaneous loss, and is given by

$$f_{ci} = (\alpha\, P_i - M\, e/r^2)/A_c \tag{15}$$

Eq. (14) was derived by equating the change in strain of steel and concrete at the tendon level occuring at transfer.

Comments and Applications

Comparison of Eq. (2) with the more accurate equation for the time-dependent loss [Eq. (9)], shows clearly how ignoring the reduction factor ψ and the recovery parameter μ results in overestimation of the loss.

It should be noted that ψ and μ are interdependent. A subsequent numerical example will show the difference in the results of the two equations.

Similarly, the last terms in each of Eqs. (11) and (12) represent the deformations resulting from prestress loss.

For computing the deflection it may be convenient to separate ϕ into two parts, such that $\phi = \phi_{mp} + \phi_{pl}$. The values ϕ_{mp} *and* ϕ_{pl} are the curvature caused by M and P_o, and the curvature resulting from prestress loss.

$$\phi_{mp} = \frac{M - P_o e}{r^2 A_c E_c}(1 + v) \qquad (16)$$

$$\phi_{pl} = \frac{e f_{co} \mu}{\alpha\, r^2 E_c} \qquad (17)$$

For simple beams, the variation of ϕ_{mp} along the span depends on the variation of the dead load moment M and the tendon profile. If P_o is assumed constant along the span, then the instantaneous deflection δ_o can be calculated using the equations listed on p. 11-10 of Reference 7, and at time t_k the deflection excluding the prestress loss effect is

$$\delta_{mp} = \delta_o (1 + v) \qquad (18)$$

Thus, it appears that the calculation of ϕ_{mp} is not needed in this case.

The deflection caused by the loss of prestress depends on the form of variation of ϕ_{pl} along the span. If three values are calculated: ϕ_1, ϕ_2, ϕ_3 at the left end, at the center and at the right end, the deflection at the center, assuming parabolic interpolation is given by

$$\delta_{pl} = (l^2/96)(\phi_1 + 10\,\phi_2 + \phi_3)_{pl} \qquad (19)$$

where positive deflection is measured downwards.

The total deflection is

$$\delta = \delta_{mp} + \delta_{pl} \qquad (20)$$

The axial shortening Δl of a member of length l may be obtained by Simpson's rule.

$$\Delta l = (l/6)(\epsilon_1 + 4\epsilon_2 + \epsilon_3) \qquad (21)$$

where ϵ_1, ϵ_2, and ϵ_3 are the axial strain at the left end, at the center and at the right end of the beam, respectively.

Effect of Superimposed Sustained Load

When an added load is applied some time after prestressing and sustained on the structure up to the time t_k, its contribution to the prestress loss, axial strain, and curvature can be calculated in a similar manner. Appropriate E_c and v values which correspond to the new age of loading should be used; the age at loading t_o in this case is the time of application of the added load. First, the instantaneous loss (or gain) of prestress is calculated by Eq. (14), with $P_i = 0$.

The same procedure outlined above is then employed with a fictitious $P_o = -A_{ps}L_{es}$ [see Eq. (13)] and $\omega = 0$. The total loss, the axial strain, and the curvature between transfer and age t_k are then obtained by superposition. The value of the total loss (excluding instantaneous loss at transfer), should be used to evaluate the relaxation reduction factor to check if further iteration is needed.

An alternative solution suggested by Huang[8] is to assume that the superimposed load acts at the same time as the first loading, i.e., at transfer. This is a reasonable assumption, particularly for loss calculation, as long as v and E_c do not differ appreciably for the two loading ages and the superimposed load is small as compared to load at transfer.

Numerical Examples

Example 1

Find the values of the instantaneous loss and the time dependent loss at midspan of the pretensioned beam in Fig. 1, at $t_k = \infty$. Also find ϵ and ϕ at this age. The prestressing is applied by strands of ultimate strength 270 ksi. Other data* for the section are:

$A_c = 401$ sq. in., $r = 7.23$ in., $e = 14.40$ in., $A_{ps} = 1.224$ sq. in., $P_i = 231$ kips, $E_s = 28{,}000$ ksi, self weight of beam = 0.418 kips per ft.

The prestressing force is applied at age $t_o = 1$ day. The following values correspond to the two ages t_o and t_k and are used in the analysis: $E_c = 3587$ ksi, $v = 2.0$, $s = 300 \times 10^{-6}$, $L_r = 13$ ksi.

The self weight of the beam gives at midspan:

$M = 0.418\,[(66)^2/8]\,12 = 2731$ in.-kips and $N = 0$.

Using Eqs. (5) and (6), $\alpha = 4.97$ and $\xi = 8.44$. Eqs. (14) and (15) give $f_{ci} = 0.987$ ksi, $n = 7.81$, and the instantaneous loss $L_{cs} = 6.89$ ksi. Thus, immediately after transfer, $P_o = 222.6$ kips and $f_{so} = 181.86$ ksi.

The three steps can now be followed.

(1) $f_{co} = 0.883$ ksi. For $v = 2.0$ and $\xi = 8.44$, Table 1 gives $\mu_o = 0.418$.

(2) For a first estimate, take $\psi = \psi\,(I) = 0.7$, which when used in Eqs. (7)-(9), gives $\omega\,(I) = 2.54$, $\mu\,(I) = 0.944$, and $L\,(I) = 24.78$ ksi. Using this value of the loss to evaluate Ω by Eq. (10), $\Omega\,(I) = 0.06$. Corresponding to this value and $\beta = 181.86/270 = 0.67$, Table 2 gives, $\psi = \psi\,(II) = 0.78$. Repetition of Step 2 with the new value of ψ gives $\omega\,(II) = 2.69$, $\mu\,(II) = 0.975$, and $L\,(II) = 25.61$ ksi. No further iteration is necessary. Thus, the loss occurring after transfer at time $t_k = \infty$ is 25.61 ksi.

(3) The corresponding axial strain and curvature calculated by Eqs. (11) and (12) are: $\epsilon = 716 \times 10^{-6}$ and $\phi = -5.63 \times 10^{-6}$ in.$^{-1}$.

For use in the example to follow, separate the value of ϕ into its two components using Eqs. (16) and (17): $\phi_{mp} = -18.93 \times 10^{-6}$ in.$^{-1}$ and $\phi_{pl} = 13.30 \times 10^{-6}$ in.$^{-1}$.

Comparison with approximate method: The use of Eq. (2) for calculating the loss gives $L = 35.19$ (37 percent overestimation).

* The data used in this example are chosen to conform to a cross section commonly used in current practice (see Reference 7).

Example 2

Calculate the midspan deflection immediately after transfer as well as the midspan deflection and the axial shortening at time $t_k = \infty$ for the beam of Example 1.

Computations similar to those in Example 1 for the end sections give:

$P_o = 217.5$ kips, $L = 31.80$ ksi, $\epsilon = 696 \times 10^{-6}$, $\phi_{mp} = -79.41 \times 10^{-6}$ in.$^{-1}$ and $\phi_{pl} = 10.14 \times 10^{-6}$ in.$^{-1}$.

The approximate Eq. (2) gives a value of loss, $L = 40.42$ ksi.

In the computation of the above values for the end sections, the intrinsic relaxation loss L_r was taken equal to 10 ksi. (Note that because of the elastic loss effect, the initial steel stress at the end sections is less than that at midspan.)

Using an average value for $P_o = 220.1$ kips along the span, the central deflection at time t_o computed by well-known equations (see p. 11-10 of Reference 7) is $\delta_o = -0.531$ in. This deflection is the sum of a downward deflection of 2.373 in. due to self weight and an upward deflection of -2.904 in. due to P_o.

The deflection at time t_k excluding the effect of prestress loss [Eq. (18)] is $\delta_{mp} = -1.593$ in.

Similarly, the prestress loss produces a deflection [Eq. (19)] of $\delta_{pl} = 1.002$ in.

Table 3. Data of test beams.

Reference No.	Beam designation	ℓ ft.	Midspan e in.	A_c in^2	r in.	A_{ps} in.^{2}	E_{co} ksi	E_s ksi	P_o kip	Midspan moment kip-in.	t_o days	t_k days	υ	$s \times 10^6$	L_r/f_{so}	β
7*	A1	15.00	2.00	48.00	2.31	0.218	3680	27000	37.0	13.9	7	187	1.21	546	0.065	0.69
7*	A2	15.00	2.00	48.00	2.31	0.173	3680	27000	29.6	13.9	7	187	1.21	546	0.065	0.69
7*	A3	15.00	2.00	48.00	2.31	0.138	3680	27000	23.4	13.9	7	187	1.21	546	0.065	0.69
8*	MU1	6.00	1.03	23.34	1.72	0.181	2540	30000	22.5	1.3	5	698	2.53	650	0.037	0.57
8*	MU2	6.00	1.03	23.34	1.72	0.181	2560	30000	22.6	1.3	5	698	2.10	650	0.037	0.57
9**	-	90.55	41.10	1517.00	25.50	10.91	5689	30000††	1523.0	19876.0	60-68	206	0.66	89	0.008†	0.70
9**	-	90.55	41.10	1517.00	25.50	10.91	5689	30000††	1523.0	19876.0	60-68	416	0.81	16	0.010†	0.70

* Pretensioned beams, straight tendon.

** Post-tensioned beam, parabolic tendon, eccentricity at ends = 10.8 in.

† Assumed values based on the formula given in Ref. 4.

†† Assumed values of E_s.

Table 4. Measured and computed prestress loss and deflection.

Reference No.	Beam designation	Prestress loss L (ksi)				Midspan deflection (in.)	
		End section		Midspan			
		Measured	Computed	Measured	Computed	Measured	Computed
7	A1	32.40	30.52	30.33	29.77	-0.44	-0.50
7	A2	29.58	29.19	27.57	28.39	-0.35	-0.37
7	A3	27.38	27.92	27.16	27.05	-0.27	-0.28
8	MU1	-	59.67	55.80	59.67	-0.20	-0.21
8	MU2	-	55.52	55.80	54.70	-0.20	-0.19
9	-	-	7.42	9.98	10.97	-1.46	-1.50

The total deflection at time t_k is $\delta = -0.591$ in. (upward).

The axial shortening occurring between time t_o and time t_k [Eq. (21)] is $\Delta l = 0.562$ in.

Comparison with approximate methods: Approximate methods presently used for long-term deflections apply the creep coefficient to the initial deflection due to self weight and due to one of the following criteria (a) (P_o-loss), or (b) P_o.

When the loss L is taken as the average of the two values obtained by Eq. (2) for midspan and end sections, the deflection by these two criteria will be

$$\delta = 2.373\,(1 + v) - 2.904\,(1 - LA_{ps}/P_o)\,(1 + v) = 0.238 \text{ in. (downwards)}$$

or

$$\delta = 2.373\,(1 + v) - 2.904\,(1 + v) = -1.593 \text{ in. (upwards)}$$

This above example shows clearly how the two methods result in wide variations.

Experimental Verification

The numerical results of the proposed method are compared with published experimental values on simple beams. The data for these beams and the comparison are presented in Tables 3 and 4. These experiments represent a wide variation in material properties and conditions; they include pretensioned and post-tensioned beams, made of both lightweight and of normal weight concretes, and tested in laboratory and field conditions. Additional experimental results are available in the literature, but in many cases the full data required for the computation are not reported.

The comparison in Table 4 shows good agreement of the measured and computed loss and deflection values.

Other Methods of Calculation

It has been shown above that the prediction of prestress loss by Eq. (2) results in a large overestimation. Several authors have suggested more accurate methods; the most significant of these are reported in References 5 and 12, and the following discussion will be limited to these.

PCI Committee Method[5]

This is a step-by-step method, in which the loss period is divided into m time intervals, and the steel and concrete stresses are adjusted at the end of each interval using the following equation:

$$L = \sum_{j=1}^{m} (CR + SH + RET)_j \qquad (22)$$

The symbols $(CR)_j$, $(SH)_j$, and $(RET)_j$ represent the loss of steel stress due to creep, shrinkage, and relaxation occurring in the jth interval. For each interval, the values of these coefficients depend on the length of the period and the updated stress in prestressing steel and in the adjacent concrete.

When appropriate values of coefficients are chosen, the method gives accurate results, and its only inconvenience are the lengthy calculations.

ACI Committee 209 Method[12]

ACI Committee 209 adopted a method developed by Branson *et al.*[9] Essentially, the method gives a simple equation which can be put in the form:

$$L = \frac{s\,E_s}{1 + 1/\xi} + 0.75\,L_r + v\left(1 - \frac{L}{2f_{so}}\right) n\,f_{co} \tag{23}$$

This equation is now compared with Eq. (9) above. The shrinkage effect in Branson's equation is accounted for by the term $sE_s/(1 + 1/\xi)$, while in Eq. (9) the shrinkage effect is represented by the first term and the recovery resulting from shrinkage loss is included in the last term of the equation. If a hypothetical case is assumed in which the creep and relaxation are zero ($v = L_r = 0$), Eqs. (9) and (23) give identical results.

The empirical coefficient 0.75 in Branson's equation, is replaced in Eq. (9) by the relaxation reduction factor ψ (which may vary between zero and 1.0) and a recovery effect included in the last term.

The inclusion of the term $(-n\,f_{co}\,v\,L/2\,f_{so})$ appears to be a recognition of the effect of the continuous reduction of the concrete stress as the loss develops. The recovery effect given by Eq. (23) may be regarded as that of a negative prestress force, equal to one-half the loss, applied at transfer.

In Eq. (9) the above approximations are avoided by the use of the coefficients ψ and μ.

ACI Committee 209 also proposes[12] an equation for the central deflection based on the work of Branson *et al.*[9] This method involves similar approximations, which are again avoided in the method presented here.

References

1. Ghali, A., Tadros, M. K., and Dilger, W. H., "Accurate Evaluation of Time-Dependent Deflection of Prestressed Concrete Frames," *Deflections of Concrete Structures*, SP-43, American Concrete Institute, Detroit, 1974, pp. 357-375.

2. Branson, D. E., and Christiason, M. L., "Time-Dependent Concrete Properties Related to Design-Strength and Elastic Properties, Creep and Shrinkage," SP-27, American Concrete Institute, Detroit, 1971, pp. 257-277.

3. CEB-FIP Report, *International Recommendations for the Calculation and Execution of Concrete Structures*, Prague, Cement and Concrete Association, London, June 1970, 80 pp.

4. Magura, D. D., Sozen, M. A., and Seiss, C. P., "A Study of Stress Relaxation in Prestressing Reinforcement," PCI JOURNAL, V. 9, No. 2, April 1964, pp. 13-26.

5. PCI Committee on Prestress Losses, "Tentative Recommendations for Estimating Prestress Losses," Final Draft, 1974. (To be published in July-August 1975 PCI JOURNAL.)

6. Schultchen, E., Ying, H-T., and Huang, T., "Relaxation Behavior of Prestressing Strands," Fritz Engineering Laboratory Report No. 339.6, Lehigh University, Bethlehem, Pennsylvania, March 1972, 55 pp.

7. *PCI Design Handbook*, Prestressed Concrete Institute, Chicago, Illinois, 1972.

8. Huang, Ti, "Prestress Losses in Pre-

tensioned Concrete Structural Members," Fritz Engineering Laboratory Report No. 339.9, Lehigh University, August 1973, 100 pp.

9. Branson, D. E., Meyers, B. L., and Kripananayanan, "Loss of Prestress, Camber, and Deflection of Noncomposite and Composite Structures Using Different Weight Concretes," Final Report No. 70-6, Iowa State Highway Commission Research Project HR-137, August 1970, 229 pp.

10. Corley, W. G., Sozen, M. A., and Siess, C. P., "Time-Dependent Deflections of Prestressed Concrete Beams," Bulletin No. 307, Highway Research Board, 1961, pp. 1-25.

11. Delarue, J., "Fluage et Béton Précontraint (Creep of Prestressed Concrete)," International Association of Testing and Research Laboratories for Materials and Structures, RILEM Bulleton No. 4, October 1959, pp. 32-52.

12. ACI Committee 209, "Prediction of Creep, Shrinkage, and Temperature Effects in Concrete Structures," SP-27, American Concrete Institute, Detroit, 1971, pp. 51-93.

Appendix—Commentary

The basic assumptions needed for deriving the equations used in the paper are here briefly described. The time-dependent strain of the concrete is obtained by superposition. The effect of the reduction in stress due to the loss is accounted for by considering the loss as a negative prestress force applied in steps at the middle of arbitrarily chosen time intervals. At the middle of each interval a force (tensile on concrete) is introduced. These forces result in instantaneous and creep recoveries, and their total effect is obtained by summation. The magnitude of each of these prestress decrements is obtained from the condition that the strain in the concrete at the tendon level is compatible with the steel strain at the end of each time interval.

The step-by-step computation described above requires that a time-variation function of creep, shrinkage, relaxation, and the modulus of elasticity of concrete be assumed. Studies[1] have shown that the final value of strain (or loss) is sensitive only to the final values of s, v, and L_r but not to their time variation, and a negligible error is involved if E_c is assumed constant equal to the value of the elasticity modulus at transfer. This makes it possible to produce a table for the recovery parameter μ (Table 1) which represents the effect of the continuous reduction in the compressive stress in concrete. The expressions for the time variation of creep and shrinkage and stress relaxation are given in Reference 1. These expressions are used in producing Table 1 and in deriving the equations given in this paper.

Appendix—Notation

A_c = net cross-sectional area of concrete

A_{ps} = cross-sectional area of prestressing steel

E_c = modulus of elasticity of concrete at age of loading t_o

E_s = modulus of elasticity of prestressing steel

e = eccentricity of prestressing tendon taken positive downward from centroid of concrete section

f_{ct}, f_{co} = concrete compressive stress at level of centroid of tendons, defined by Eqs. (15) and (4)

f_{si}, f_{so} = initial steel tensile stress immediately before and after transfer

L = total loss of stress in prestressing steel in period $(t_k - t_o)$ excluding instantaneous loss at transfer

L_{es} = instantaneous loss of steel stress at transfer

L_r = intrinsic relaxation loss of steel stress of tendon stretched between two fixed points

l = span length

Δl = axial shortening of member at time t_k

M, N = bending moment and normal force in section due to applied loads; positive N is compressive and positive M produces tension at bottom fibers of beam

n = modular ratio at time $t_o = E_s/E_c$

P_i, P_o = initial prestressing force immediately before and after transfer

r = radius of gyration of concrete section

s = free shrinkage of concrete in period $(t_k - t_o)$

t = age of concrete in days, subcripts o and k refer to the age at loading and age at which loss and displacements are required

α = dimensionless coefficient $= 1 + e^2/r^2$

β = ratio of initial steel stress, f_{so}, to its ultimate strength

δ_o, δ = instantaneous midspan deflection at time t_o and total midspan deflection at time t_k

δ_{mp}, δ_{pl} = components of δ referring to combined effect of applied loads and prestress, and to effect of prestress loss

ϵ = axial strain at time t_k; subscripts 1, 2, and 3 refer to left end, center and right end of member

μ = recovery parameter defined by Eq. (8). Symbol μ_o represents value of μ when shrinkage and relaxation are zero

ξ = steel area parameter defined by Eq. (6)

v = creep coefficient equals ratio of creep at age t_k to instantaneous strain when age at loading is t_o

ψ = relaxation reduction factor

Ω = dimensionless quantity [Eq. (10)]

ω = shrinkage-relaxation parameter defined by Eq. (7)

ϕ = curvature at time t_k

ϕ_{mp}, ϕ_{pl} = components of ϕ due to combined effect of applied load and prestress, and to effect of prestress loss. Subscripts 1, 2, and 3 refer to values of ϕ_{pl} at left end, center and right end of member.

Discussion of this paper is invited. Please forward your discussion to PCI Headquarters by October 1, 1975.