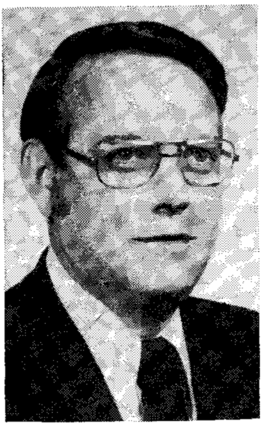
A Rational Method for Estimating Camber and Deflection of Precast Prestressed Members



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The ACI Code (ACI 318-71)[1] provides in Section 9.5.2.3 a convenient equation for estimating the additional long-time deflection of nonprestressed reinforced concrete members:

$$[2 - 1.2(A'_s/A_s)] \geq 0.6$$

where A'_s is the compressive reinforcement and A_s is the tensile reinforcement.

Unfortunately, no such convenient guide is given for prestressed concrete.

The determination of long-time cambers and deflections in precast prestressed members is somewhat more complex because of the:

1. Effect of prestress and the loss of prestress over time;
2. Strength gain of the concrete after release of prestress; and the
3. Camber or deflection is important not only at the "initial" and "final" stages, but also at erection, which occurs at some intermediate stage, usually from 30 to 60 days after casting.

Much research has been done on the effects of creep and shrinkage on prestressed concrete members, not only regarding camber/deflection behavior, but also on the related issue of prestress losses.

Some relatively precise and complex equations have been developed for predicting these long-time behaviors. However, the data on which these equations are based usually has a scatter of at least 15 to 20 percent,[2,3] using *laboratory controlled specimens.*

Synopsis

The author presents a step-by-step rational procedure for determining long-time multipliers for camber and deflection of precast prestressed concrete members.

The procedure begins with the equation in Section 9.5.2.3 of ACI 318-71 and ends with a table of numerical values (Table 3) that can be used for most members without appreciable error.

Table 2 illustrates the sensitivity of the equations to various variables and shows that except for extremely long members the variations are within the tolerances prescribed by the *PCI Manual for Quality Control.*

Table 4 illustrates how these multipliers should be used. Equations are provided for a more precise analysis.

When the various plant and field variations are considered, it seems rather futile to use these time-consuming methods for estimating long-time cambers and deflections. It is even of questionable value to program these equations for a computer, except as an exercise in computer programming.

Section 4.1 of the *PCI Design Handbook*,[4] first edition, illustrates the use of "multipliers" for determining the long-time cambers and deflections.

The *Handbook* suggests a range of 1.5 to 3.0 for these multipliers, but does not provide a guide to the designer for determining the values. In the example designs in that section, multiplier values were arbitrarily selected, based on the experience of the authors.

This paper suggests a rationale for determining long-time multipliers for precast prestressed members that is consistent with the ACI equation above for reinforced concrete members.*

It should be noted that because of the inherent variables that affect camber and deflection, such as concrete mix, storage method, time of release of prestress, time of erection and placement of superimposed loads, relative humidity, etc., and the data scatter under the most closely controlled tests,

* The multipliers used in the *PCI Design Handbook* include the initial camber or deflection, whereas the equation in ACI 318-71 is for the *additional* long-term deflection only and is added to the initial calculated (elastic) deflection. For direct comparison, the ACI equation would be $1 + [2 - 1.2\,(A'_s/A_s)]$.

calculated long-time values should never be considered any better than estimates.

Non-structural components attached to members which could be affected by camber variations, such as partitions or folding doors, should be placed with adequate allowance for error. Calculation of topping quantities should also recognize the imprecision of camber calculations.

The use of the multipliers suggested here should be limited to typical members in ordinary applications. Other, more precise methods of calculating cambers and deflections are available and can be used in special cases.

However, in such cases, at least as much care should be given to quality control, material selection, production sequence, and storage and erection procedures. Consideration should also be given to thermal gradients and to differential shrinkage in composite members.

Determination of Multipliers

Because of the inherent imprecision in calculating long-time effects noted above, the multipliers calculated here are based on assumptions of typical precast prestressed members regarding concrete strength, prestress losses, etc. Also, liberal rounding of calculated values is used.

While the designer may wish to calculate his own values based on known design information, the effect on final camber and deflection is minimal, and, therefore, of questionable value, except, perhaps, on extremely long spans.

The determination of the multipliers starts with the equation in Section 9.5.2.3 (ACI 318-71). With no compressive reinforcement, the "base" factor for additional long-time deflection is:

$$\mu_b = 2 - 1.2\ (0/A_s) = 2.0 \qquad (1)$$

In non-prestressed concrete, the elastic deflection is normally based on the modulus of elasticity of full design strength concrete, while in precast prestressed concrete the multipliers are applied to the camber and deflection at the time of release.

Therefore, the long-time factor which is applied to the initial deflection caused by the dead weight of the member would be:

$$\mu_{df} = \frac{E_{ci}}{E_c}\,\mu_b \qquad (2)$$

Since the release strength of precast, prestressed members is usually about 70 percent of the 28-day strength, E_{ci} is about 85 percent of the final.

Eq. (2) would then become:

$$\mu_{df} = 0.85\mu_b = 0.85\,(2.0) = 1.7$$

The multiplier which is applied to the initial deflection would then be:

$$1 + \mu_{df} = 1 + \frac{E_{ci}}{E_c}\,\mu_b$$
$$= 1 + 1.7 = 2.7 \qquad (3)$$

To determine the upward component of the final camber or deflection, the multiplier which is applied to the initial camber must be reduced by any loss of prestress which occurs after the prestress force is released.

The long-time factor would then be:

$$\mu_{pf} = \mu_{df}\,\frac{P}{P_o} \qquad (4)$$

Total loss of prestress is usually assumed or calculated to be from 20 to 30 percent of the initial force, and the long-time part of this is usually in the range of 10 to 20 percent.

If an average value of 15 percent is used, $P = 0.85\,P_o$ and $\mu_{pf} = 1.7\,(0.85) = 1.45$.

The multiplier used to determine the upward (camber) component of the final camber or deflection is then:

$$1 + \mu_{pf} = 1 + \mu_{df}\,\frac{P}{P_o}$$
$$= 1 + 1.45 = 2.45 \qquad (5)$$

Erection Camber

The camber at the time of erection is also important. This occurs usually at 30 to 60 days following casting. Research has shown[3] that creep and shrinkage, the primary factors in long-term behavior, will have reached about 40 to 60 percent of ultimate in that time.

Therefore, it is reasonable to assume that one-half of the long-time camber, deflection, and losses will have occurred by then. The multiplier for the erection phase would then be:

For the downward component, the erection factor is:

$$\mu_{de} = 0.5\, \mu_{df} \tag{6}$$

and the multiplier applied to the initial member weight deflection is:

$$1 + \mu_{de} = 1 + 0.5\, \mu_{df} = 1 + 0.5\,(1.7) = 1.85 \tag{7}$$

For the upward component, the erection factor is:

$$\mu_{pe} = \mu_{de} \frac{P_o + P}{2P_o} \tag{8}$$

and the multiplier applied to the initial upward camber is:

$$1 + \mu_{pe} = 1 + 0.85\,(0.925) \approx 1.80 \tag{9}$$

Superimposed sustained dead loads, such as roofing or floor covering, cause immediate deflection and additional long-time deflection. Since the long-time deflection is a result of the creep caused by this additional sustained load, the long-time factor is the same as the assumed "base" factor:

$$\mu_{sd} = \mu_b \tag{10}$$

The multiplier which is applied to the elastic deflection caused by superimposed dead load is:

$$1 + \mu_{sd} = 1 + 2.0 = 3.0 \tag{11}$$

Eqs. (2), (4), (6), (8), and (10) are illustrated graphically in Fig. 1.

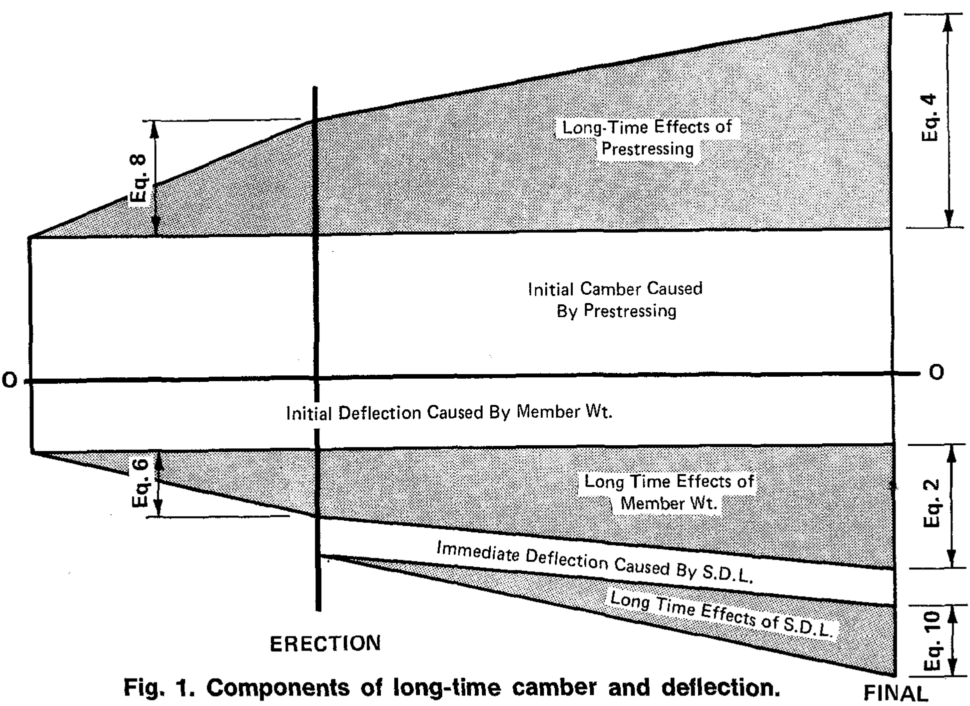

Fig. 1. Components of long-time camber and deflection.

Composite Members

For members with composite topping, the final multipliers should be modified by the effect of the increased moment of inertia after the topping is added.

If the section becomes composite at about the time of erection, the difference between non-composite long-time factors at erection and final should be multiplied by the ratio of non-composite to composite moments of inertia, I_o/I_c.

The long-time factors would then be as follows:

$$\mu_{dfc} = \mu_{de} + (\mu_{df} - \mu_{de})(I_o/I_c) \quad (12)$$

and the multiplier applied to the initial downward component is:

$$1 + \mu_{dfc} = 1 + 0.85 + (0.85)(I_o/I_c)$$
$$= 1.85 + 0.85\, I_o/I_c \quad (13)$$

For commonly used members with a 2-in. composite topping, this ratio will vary from about 0.5 for 8-in. hollow-core slabs to about 0.8 for deep-stemmed members.

Using an average value of 0.65, Eq. (13) becomes:

$$1.85 + 0.85\,(0.65) = 2.40 \quad (13a)$$

Similarly, the long-time factor for the upward component is:

$$\mu_{pfc} = \mu_{pe} + (\mu_{pf} - \mu_{pe})\, I_o/I_c \quad (14)$$

and the multiplier applied to the initial upward (camber) component is:

$$1 + \mu_{de}\left(\frac{P_o + P}{2}\right)\left(1 - \frac{I_o}{I_c}\right) + \mu_{df}\left(\frac{P}{P_o}\right)\left(\frac{I_o}{I_c}\right) \quad (15)$$

$$= 1 + 0.85\,(0.925)\,(035) + 1.7\,(0.85)\,(0.65)$$
$$\approx 2.20 \quad (15a)$$

The immediate deflection caused by the placement of the topping is calculated using the non-composite section properties, so the long-time effects are also modified by the ratio of I_o/I_c:

$$\mu_t = \mu_{sd}\,(I_o/I_c) \quad (16)$$

and the multiplier is then:

$$1 + \mu_{sd}\,(I_o/I_c) = 1 + 2.0\,(0.65)$$
$$= 2.30 \quad (17)$$

The deflection caused by other superimposed loads is calculated using the composite section properties so the multiplier is the same as in Eq. (11).

Sensitivity of Cambers and Deflections to the Variables

In order to determine the sensitivity of the foregoing equations to the variables encountered, cambers of five different precast prestressed members were computed using the stated assumptions, and then were recalculated by changing the variables one at a time. The five members used in this calculation are described below:

Member No. 1

Double-tee, 8 ft wide, 24 in. deep (*PCI Design Handbook,* Section 8DT24), normal weight concrete, prestressed with 14–½-in. diameter, 270-ksi strands, depressed at midpoint. $f'_c = 5000$ psi, $f'_{ci} = 3500$ psi. A superimposed dead load of 10 psf was assumed. Span = 70 ft.

Member No. 2

Double-tee with composite topping, 8 ft wide, 24 in. deep with 2-in. topping (*PCI Design Handbook,* Section 8DT24+2), normal weight concrete, 12–½-in. diameter, 270-ksi strands, depressed at midpoint. $f'_c =$ 5000 psi, $f'_{ci} = 3500$ psi. f'_c (topping) = 3000 psi (composite moment of inertia is calculated by the normal method of assuming that the width of the topping is reduced by the ratio of E_{ct}/E_c). Superimposed dead load = 15 psf. Span = 64 ft.

Member No. 3

Hollow-core slab, 4 ft wide, 8 in. deep, prestressed with 7–½-in. diameter, 270-ksi parallel strands. Lightweight concrete (115 pcf). Superimposed dead load = 10 psf. Span = 33 ft.

Member No. 4

Hollow-core slab, same as Member No. 3 except normal weight concrete and 2-in. composite concrete. Superimposed dead load = 15 psf. Span = 35 ft.

Member No. 5

Single tee, 10 ft wide, 48 in. deep (*PCI Design Handbook*, Section 10DT48), lightweight concrete, 24–½-in. diameter, 270-ksi strands, depressed at midpoint. $f'_c = 5000$ psi, $f'_{ci} = 3500$ psi. Superimposed dead load = 10 psf. Span = 120 ft.

The initial assumptions made for each member (Members 1 through 5) were as described in the foregoing and are summarized in Table 1.

The changes in the variables and the results of the camber calculations are shown in Table 2. Note that only with Member No. 5, which is an extremely long span for a precast product, does the difference exceed ¾ in., which is the maximum tolerance allowed (from the calculated value) by the *PCI Manual for Quality Control*.[6]

It is suggested, therefore, that the numerical values for camber multipliers shown in Table 3 will be adequate for most cases. The precision of these numerical values should be as good as those obtained by the use of the factor in Section 9.5.2.3 of ACI 318-71 for non-prestressed members.

Table 4 illustrates, in tabular form, how these multipliers are used for Member No. 2. The table is similar to that shown in the *PCI Design Handbook*.[4]

Table 1. Initial assumptions used in camber calculations.

Basic time-dependent factor	2.0
Initial loss of prestress (percent)	8.0
Time-dependent loss of prestress (percent)	15.0
Percent of total camber/deflection change at erection	50.0
Ratio of I_o/I_c	0.65

Reduction of Cambers and Deflections with Mild Steel Reinforcing

Shaikh and Branson[5] have shown that long-time cambers and deflections can be reduced by adding mild steel reinforcing near the level of the prestressing steel. They suggest that the long-time effects can be reduced by a factor:

$$\alpha = \frac{1}{1 + A_s/A_{ps}}$$

where A_s is the mild steel and A_{ps} is the area of prestressing steel.

For example, if one #7 bar is added in the stem of a member which has 7–½-in. diameter, 270 ksi strands, $A_s = 0.60$, $A_{ps} = 1.07$ and $\alpha = 0.64$. The "base" factor would then be:

$$\mu_b = 2\,(0.64) = 1.28$$

All other multipliers would be similarly modified.

Allowable Deflection

ACI 318-71 sets forth requirements for allowable deflection in Table 9.5(b). This table, for members which have non-structural elements attached to them, requires that the deflection to be considered is "that part of the total deflection which occurs after attachment of the non-structural elements . . ."

In precast prestressed concrete construction, this can be considered to be

Table 2. Sensitivity of camber to variables.

		Member #1		Member #2		Member #3		Member #4		Member #5	
		Camber	Difference from 1	Camber	Difference from 1	Camber	Difference from 1	Camber	Difference from 1	Camber	Difference from 1
(1) *Using assumptions in Table I*	Erection	2.51	0	2.26	0	2.35	0	1.00	0	3.88	0
	Final	2.00	0	-0.35	0	2.56	0	-0.61	0	2.60	0
(2) Change basic factor from 2.0 to 2.5	Erection	2.76	+.21	2.49	+.23	2.60	+.25	1.10	+.10	4.24	+.36
	Final	2.20	+.20	-0.47	+.12	2.90	+.34	-0.75	-.14	2.77	+.17
(3) Change initial loss from 8% to 6%	Erection	2.69	+.18	2.39	+.13	2.45	+.10	1.07	+.07	4.25	+.37
	Final	2.24	+.24	-0.18	+.17	2.70	+.14	-0.52	+.09	3.11	+.51
(4) Change initial loss from 8% to 10%	Erection	2.34	-.17	2.12	-.14	2.25	-.10	0.92	-.08	3.50	-.38
	Final	1.76	-.24	-0.51	-.16	2.43	-.13	-0.71	-.10	2.09	-.51
(5) Change time-dependent loss from 15% to 10%	Erection	2.61	+.10	2.33	+.07	2.40	+.05	1.04	+.04	4.08	+.20
	Final	2.38	+.38	-0.13	+.22	2.78	+.22	-0.49	+.12	3.42	+.82
(6) Change time-dependent loss from 15% to 20%	Erection	2.41	-.10	2.18	-.08	2.29	-.06	0.96	-.04	3.67	-.21
	Final	1.61	-.39	-0.56	-.21	2.35	-.21	-0.73	-.12	1.79	-.81
(7) Change % of camber/deflection change at erection from 50% to 40%	Erection	2.36	-.15	2.11	-.15	2.18	-.17	0.94	-.06	3.68	-.20
	Final	2.00	0	-0.39	-.04	2.56	0	-0.63	-.02	2.60	0
(8) Change % of camber/deflection change from 50% to 70%	Erection	2.75	+.24	2.50	+.24	2.65	+.30	1.09	+.09	4.11	+.23
	Final	2.00	0	-0.26	+.09	2.56	0	-0.58	+.03	2.60	0
(9) Change I_o/I from 0.65 to 0.50	Erection	--	--	2.26	0	--	--	1.00	0	--	--
	Final	--	--	+0.06	+.41	--	--	-0.35	+.26	--	--
(10) Change I_o/I from 0.65 to 0.80	Erection	--	--	2.26	0	--	--	1.00	0	--	--
	Final	--	--	-0.75	-.40	--	--	-0.87	-.26	--	--

Table 3. Suggested multipliers to be used as a guide in estimating long-time cambers and deflections for typical members.

	Without Composite Topping	With Composite Topping
At erection:		
(1) Deflection (downward) component - apply to the elastic deflection due to the member weight at release of prestress.	1.85	1.85
(2) Camber (upward) component - apply to the elastic camber due to prestress at the time of release of prestress.	1.80	1.80
Final:		
(3) Deflection (downward) component - apply to deflection calculated in (1) above.	2.7	2.4
(4) Camber (upward) component - apply to camber calculated in (2) above.	2.45	2.2
(5) Deflection (downward) - apply to elastic deflection due to superimposed dead load only.	3.0	3.0
(6) Deflection (downward) - apply to elastic deflection caused by the composite topping.	--	2.30

Table 4. Tabular method of calculating long-time cambers and deflections.

	(1) Release	Multiplier	(2) Erection	Multiplier	(3) Final
Prestress	3.43 ↑	1.8 x (1)	6.17 ↑	2.2 x (1)	7.55 ↑
Member weight	-2.10 ↓	1.85 x (1)	-3.89 ↓	2.4 x (2)	-5.04 ↓
Net	1.33 ↑		2.28 ↑		2.51 ↑
Topping DL			-0.84 ↓	2.30 x (2)	-1.93 ↓
Net			1.44 ↑		0.58 ↑
Superimposed DL			-0.33 ↓	3.0 x (2)	-0.99 ↓
Net			1.11 ↑		0.41 ↑

Note: The values in this table differ slightly from those shown for the same member in Table 2 because of rounding of the multipliers.

the live load deflection plus the difference between the final camber and the camber after placement of superimposed dead loads.

For example, the member shown in Table 4, the deflection to be used in meeting these requirements would be $1.11 + 0.41 = 1.52$ plus the instantaneous live load deflection.

The span in this example is 64 ft, so the allowable deflection to meet the requirements of Table 9.5(b) would range between $(64 \times 12)/480 = 1.60$ in. and $(64 \times 12)/240 = 3.20$ in. (for floor members).

References

1. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-71)," American Concrete Institute, Detroit, 1971, p. 27.
2. Subcommittee 5, ACI Committee 435, "Deflections of Prestressed Concrete Members," *ACI Journal*, Proceedings V. 60, No. 12, Dec. 1963, pp. 1697-1728.
3. Subcommittee 2, ACI Committee 209, "Prediction of Creep, Shrinkage, and Temperature Effects in Concrete Structures," *Designing for Effects of Creep, Shrinkage, and Temperature in Concrete Structures*, SP-27, American Concrete Institute, Detroit, 1971, pp. 51-93.
4. *PCI Design Handbook* (first edition, second printing), Prestressed Concrete Institute, Chicago, 1972, pp. 4-3 to 4.7.
5. Shaikh, A. F., and Branson, D. E., "Non-Tensioned Steel in Prestressed Concrete Beams," PCI JOURNAL, V. 15, No. 1, Feb. 1970, pp. 14-36.
6. *Manual for Quality Control for Plants and Production of Precast Prestressed Concrete Products*, MNL-116-70, Prestressed Concrete Institute, Chicago, 1970.

Notation

A_s = non-prestressed tensile reinforcement, sq in.

A'_s = non-prestressed compressive reinforcement, sq in.

E_c = 28-day modulus of elasticity of concrete, psi

E_{ci} = modulus of elasticity at time of transfer of prestress force, psi

f'_c = 28-day compressive strength of concrete, psi

f'_{ci} = compressive strength at time of transfer of prestress force, psi

I_c = moment of inertia of composite section, in.^{4}

I_o = moment of inertia of precast section, in.^{4}

P = total prestress force after all losses, kips

P_o = prestress force at transfer, kips

μ_b = "base" factor for additional long-time deflection

μ_{de} = factor for time-dependent deflection at erection applied to initial deflection caused by member weight

μ_{df} = factor for final long-time deflection applied to initial deflection caused by member weight

μ_{dfc} = μ_{df} for a composite section

μ_{pe} = factor for time-dependent camber at erection applied to initial camber caused by prestressing force

μ_{pf} = factor for final long-time camber applied to initial camber caused by prestressing force

μ_{pfc} = μ_{pf} for a composite section

μ_{sd} = factor for final long-time deflection caused by superimposed dead load

μ_t = factor for final long-time deflection applied to deflection caused by topping in a composite section