Effect of Non-Prestressed Steel on Prestress Loss and Deflection

Maher K. Tadros
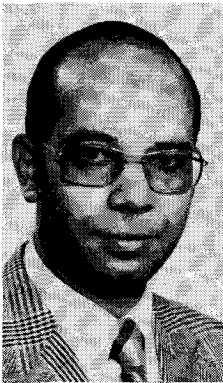Assistant Professor
Department of Civil Engineering
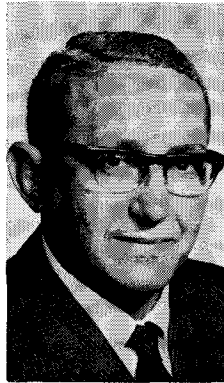West Virginia University
Morgantown, West Virginia

Amin Ghali
Professor of Civil Engineering
The University of Calgary
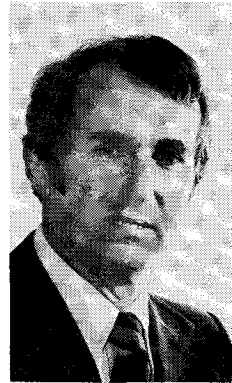Calgary, Alberta, Canada

Walter H. Dilger
Professor of Civil Engineering
The University of Calgary
Calgary, Alberta, Canada

Maher K. Tadros   Amin Ghali   Walter H. Dilger

The presence of non-prestressed reinforcement in a prestressed concrete member can have an important effect on the time-dependent stresses and deformations caused by shrinkage and creep of concrete and relaxation of steel. This is particularly true in partially prestressed members where the area of the non-prestressed reinforcement is relatively large.

In a prestressed simple beam, bottom non-prestressed steel restrains the deformations of concrete and results in reduction of prestress loss and camber (or increase in downward deflection). As concrete shrinks and creeps under compression, the non-prestressed steel continues to pick up compression and consequently increases the loss of precompression in concrete.

There are substantial differences between the various investigators[1-4] over the quantitative effects especially on

A simple accurate method is presented for calculating prestress loss, axial strain, and curvature at a cross section of a member containing prestressed as well as non-prestressed steel. Two sets of graphs are given as design aids, and their use is demonstrated by numerical examples.

The method indicates that the presence of non-prestressed steel slightly reduces the loss in tension in the prestressed steel, but can significantly reduce the prestressing force in the concrete. It has a small effect on the axial shortening but can have a much more pronounced effect on the camber or the deflection.

the loss of prestressing force transferred to concrete which is of primary importance in design.

In the present paper, a simple method is developed for the accurate evaluation of prestress loss, axial strain and curvature in a cross section of a member containing both prestressed and non-prestressed steel. Numerical examples are worked out to show the order of magnitude of the effects of non-prestressed steel in practical cases.

Problem Statement

Consider a cross section of a prestressed concrete member (Fig. 1) subjected to dead load axial force N and bending moment M introduced at time t_0, the age of concrete at transfer of prestress.

The axial force N is non-zero only in special cases, e.g., statically indeterminate post-tensioned frame. The section is assumed to have prestressed

AREA OF CONCRETE = A_c ;
ITS RADIUS OF GYRATION = r
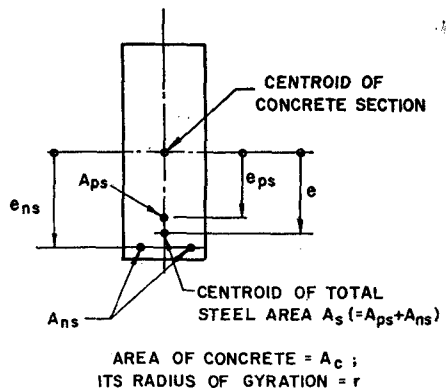
Fig. 1. Prestressed concrete section with prestressed and non-prestressed reinforcement.

and non-prestressed steel of areas A_{ps} and A_{ns}.

It is required to find at time t_k, usually the end of an extensive period, the axial strain, ϵ, and the curvature, ϕ, of the concrete section, and the forces in the concrete, the prestressed and the non-prestressed steel.

The axial strain and curvature, when evaluated for various sections along a member, can be used to determine the member shortening and deflection.

In addition to the geometric properties of the section, the following values are assumed to be known:

- s, the free shrinkage of concrete in the period $t_k - t_o$;
- v, the creep coefficient which is equal to the ratio of creep at age t_k, due to a constant sustained load applied at t_o, to the instantaneous strain;
- L_r, the intrinsic relaxation loss of steel stress in a tendon stretched between two fixed points with initial stress f_{pso} for the period $t_k - t_o$;

$\beta = \dfrac{\text{STEEL STRESS IMMEDIATELY AFTER TRANSFER}}{\text{ULTIMATE TENSILE STRENGTH}}$

$\Omega = \dfrac{\text{TOTAL PRESTRESS LOSS } - \text{ INTRINSIC RELAXATION}}{\text{STEEL STRESS IMMEDIATELY AFTER TRANSFER}}$

Fig. 2. Relaxation reduction factor ψ versus Ω.

E_c, the modulus of elasticity of concrete at t_o; and

E_s, the modulus of elasticity of steel, assumed to be the same for both prestressed and non-prestressed reinforcement.

Sign convention

The prestress forces P_{ps} in steel and P_c in concrete are always positive. N is positive when compressive. M is positive when it produces tension at the bottom fiber of a member. Positive ϵ and ϕ correspond to positive N and M, respectively. The concrete stress is positive when compressive.

The above sign convention is chosen to conform to the *PCI Design Handbook*.[5]

General Description of Method

The same problem was discussed in an earlier paper[6] with the only difference that additional non-prestressed steel is here considered. The procedure to be followed in design is fully given in the present paper, but some of the development details included in Reference 6 are not repeated.

An approximate equation to calculate the loss of stress in prestressed steel due to shrinkage, creep and relaxation is:

$$L_{ps} = s E_s + L_r + v n f_{co} \qquad (1)$$

where f_{co} is the initial concrete stress at the level of the prestressed steel, and:

$$n = E_s/E_c \qquad (2)$$

Eq. (1) overestimates the loss because it ignores the continuous reduction in the concrete stress accompanying the development of the prestress loss. It also ignores that the reduction in steel stress due to the shortening of the tendon results in a smaller amount of re-
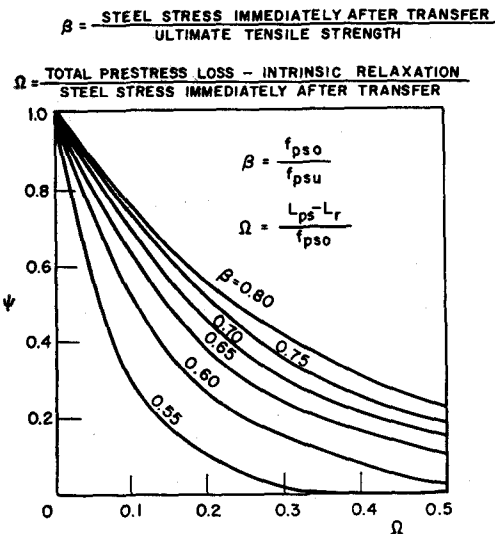
In the figure:

$$\mu = \frac{\text{STRAIN CAUSED BY PRESTRESS LOSS}}{\text{INSTANTANEOUS STRAIN AT TRANSFER}}$$

μ_0 = THE VALUE OF μ IN A HYPOTHETICAL CASE WHEN SHRINKAGE AND RELAXATION ARE ZERO

$$\xi = \frac{\lambda_c}{\alpha n \lambda_s}$$
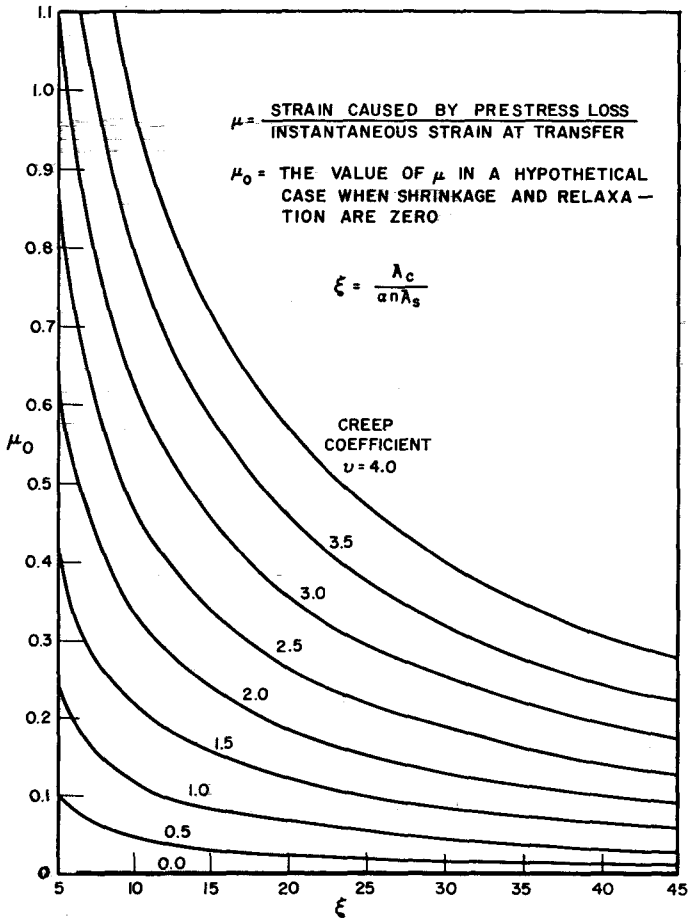
CREEP COEFFICIENT
$\nu = 4.0$

Fig. 3. Recovery parameter μ_o versus ξ.

laxation as compared to the intrinsic relaxation, L_r.

This is so because the intrinsic relaxation L_r depends to a large extent on the stress level in the steel which may be expressed as the ratio β equals the initial steel stress, f_{pso} divided by the ultimate strength.

The method presented in this paper adopts a more accurate equation:

$$L_{ps} = s\,E_s + \psi\,L_r + (\nu - \mu)\,n\,f_{co} \quad (3)$$

In the presence of non-prestressed steel, f_{co} is the initial concrete stress at eccentricity e, the level of the centroid of the total steel area:

$$A_s = A_{ps} + A_{ns} \quad (4)$$

The factor ψ (<1.0) (Fig. 2) accounts for the reduction of steel relaxation.[7] The parameter μ accounts for the strain recovery. Its value is equal to the ratio of the instantaneous plus creep strain caused by the prestress loss to the instantaneous strain of concrete at prestress transfer; both strains are at the level of the centroid of A_s.

The graphs in Fig. 3 can be used to obtain the recovery parameter, μ. They are derived in Appendix B by a step-by-step procedure similar to what is recommended by the PCI Committee

on Prestress Losses. The use of the graphs eliminates the need for lengthy incremental computations.

The strain recovery described above has a significant effect on the deflection and to a lesser degree on the axial shortening. The same parameter, μ, will be used to calculate the time-dependent axial strain, ϵ, and the curvature, ϕ.

Design Procedure

Because of the presence of non-prestressed reinforcement, the loss in prestressing force in steel, $L_{ps}A_{ps}$ is less than the absolute value of the loss of the compressive prestressing force on the concrete, ΔP_c. The difference is a compressive force picked up by the non-prestressed steel.

The effective stress in concrete is of primary importance in design. This is equal to the initial concrete stress minus the stress caused by ΔP_c applied on the concrete section at eccentricity e.

The values of L_{ps}, ΔP_c, the axial strain ϵ and the curvature ϕ are calculated in three steps:

Step 1:

Calculate the concrete stress at eccentricity e immediately after transfer:

$$f_{co} = (\overline{\alpha} P_{co} + N - Me/r^2)/A_c \quad (5)$$

where A_c and r are the area and the radius of gyration of the concrete section, and:

$$\overline{\alpha} = 1 + (ee_{ps}/r^2) \quad (6)$$

The term P_{co} is the compressive force on the concrete immediately after transfer (assumed for simplicity to be acting at eccentricity e_{ps}). The value of P_{co} is to be calculated (as given in the following section) from the known value of the tension applied before transfer in pretensioned members or the initial prestressing force at transfer in

post-tensioned members.

The recovery parameter μ_o is read from Fig. 3 which should be entered by the creep coefficient v and the steel area parameter:

$$\xi = A_c/(\alpha n A_s) \quad (7)$$

where:

$$\alpha = 1 + e^2/r^2 \quad (8)$$

The value μ_o obtained from the graph corresponds to the situation when the shrinkage and the relaxation are zero. For the actual condition, the recovery parameter μ, is a larger value to be determined in Step 2.

Step 2:

Compute the shrinkage-relaxation parameter defined as follows:

$$\omega = \left(s E_s + \psi L_r \frac{A_{ps}}{A_s} \right) /(n f_{co}) \quad (9)$$

At this stage, the relaxation reduction factor ψ is not known, and a simple iteration is needed. At first, an estimated value of ψ (I) is used in the equation. The value of ψ can be between 0.0 and 1.0, and for most practical cases, a value of 0.7 leads to the accurate ψ value after a single iteration.

Now, the recovery parameter is calculated from:

$$\mu = \mu_o + \frac{(1 + 0.6\,v)\omega}{1 + 0.6\,v + \xi} \quad (10)$$

A first estimate of L_{ps} is calculated from Eq. (3). The accuracy of the assumed value of $\psi = \psi$ (I) is now examined by using the graph of Fig. 2, which is entered by β and the following parameter:

$$\Omega = (L_{ps} - L_r)/f_{pso} \quad (11)$$

If the value of ψ obtained from the graph is different from the assumed value, Step 2 is repeated using $\psi = \psi$ (II), the last value obtained from the graph. This repetition if needed, will in

most cases, give accurate values of the recovery parameter ψ and prestress loss L_{ps}, and no further iteration will be necessary.

The change in the prestressing compressive force in the concrete at age t_k equals:

$$\Delta P_c = -A_s \left[s\,E_s + \psi\,L_r\frac{A_{ps}}{A_s} + (v-\mu)\,n\,f_{co} \right] \qquad (12)$$

The negative sign outside the bracket indicates a reduction in the compressive force.

Step 3:

Calculate the values of the axial strain ϵ and the curvature ϕ at age t_k.

$$\epsilon = s + \frac{P_{co}+N}{A_c\,E_c}(1+v) - \frac{f_{co}}{\alpha E_c}\mu \qquad (13)$$

$$\phi = \frac{M-P_{co}\,e_{ps}}{r^2\,A_c\,E_c}(1+v) + \frac{ef_{co}}{\alpha r^2\,E_c}\mu \qquad (14)$$

Pretensioned Members

The transfer of the prestress in a pretensioned member reduces the tensile stress in A_{ps} and induces compression in A_{ns}. This is equivalent to a reduction in tensile stress in the total steel by the amount:

$$L_{cs} = nf_{ci}/\left(1+\frac{1}{\xi}\right) \qquad (15)$$

where f_{ci} is the concrete stress which would occur at the centroid of the total steel area had there been no instantaneous loss. Its value can be calculated by the equation:

$$f_{ci} = (\overline{\alpha}\,P_{psi} + N - Me/r^2)/A_c \qquad (16)$$

where P_{psi} is the force in the prestressed steel immediately before transfer.

Immediately after transfer, the com-

pression in the concrete is:

$$P_{co} = P_{psi} - A_s nf_{ci}/\left(1+\frac{1}{\xi}\right) \qquad (17)$$

The above equations are derived in Appendix C from the conditions that the instantaneous change in strain in the non-prestressed or prestressed steel is the same as that of the concrete at the level of the centroid of the total steel area, and that after transfer the tension in the prestressed steel is equal to the sum of the compression in the concrete and the non-prestressed steel.

Post-tensioned Members

With post-tensioning, only the non-prestressed steel, A_{ns}, restrains the deformation of the concrete. Thus, the prestressing force P_{pso} is larger than the compression transferred to the concrete which is calculated by:

$$P_{co} = P_{pso} - A_{ns}\,n\,f_{ci}/\left[1+\left(1+\frac{e_{ns}^2}{r^2}\right)\frac{n\,A_{ns}}{A_c}\right] \qquad (18)$$

where f_{ci} is the concrete stress which would occur at eccentricity e_{ns} in the absence of the non-prestressed steel:

$$f_{ci} = \frac{1}{A_c}\left[\left(1+\frac{e_{ns}\,e_{ps}}{r^2}\right)P_{pso} + N - \frac{Me_{ns}}{r^2}\right] \qquad (19)$$

The second term in Eq. (18) is the compression picked up by the non-prestressed steel. The derivation of Eq. (18) is included in Appendix C.

EXAMPLE 1

It is required to find the effective prestressing force in the concrete, the axial strain and curvature at the mid-span cross section of the pretensioned double-T simple beam "10DT32" of the

PCI Design Handbook,[5] given the following data (Figs. 4a and 4b).

$A_c = 615$ in.²; $r = 9.85$ in.; $A_{ps} = 2.14$ in.²; $e_{ps} = 18.48$ in.; $A_{ns} = 1.22$ in.²; $e_{ns} = 19.48$ in.; $M = 5553.6$ in.-kips; $P_{psi} = 404$ kips; $E_c = 3640$ ksi; $E_s = 28,000$ ksi; $v = 1.88$; $s = 546 \times 10^{-6}$; $L_r = 19.1$ ksi; $f_{psu} = 270$ ksi.

Eqs. (17) and (5) give $P_{co} = 376.8$ kips and $f_{co} = 1.05$ ksi. The total steel area $A_s = 3.36$ in.², its eccentricity $e = 18.84$ in. and $\xi = 5.11$. With $\xi = 5.11$ and $v = 1.88$, Fig. 3 gives $\mu_o = 0.555$.

For a first estimate, take $\psi = \psi$ (I) = 0.7. Substituting into Eq. (10) gives a value for the recovery parameter μ (I) = 1.49. A first estimate of the prestress loss L_{ps} (I) can then be obtained from Eq. (3); $L_{ps}(I) = 32.4$ ksi. An improved value for the relaxation reduction factor, ψ, is now read from Fig. 2 by entering $\beta = 0.669$ and $\Omega = 0.074$ giving $\psi(II) = 0.73$.

Therefore, μ (II) = 1.43 and L_{ps} (II) = 32.9 ksi. Using $\mu = \mu$ (II) = 1.43 in Eqs. (13) and (14), the axial strain

$\epsilon = 9.42 \times 10^{-4}$ and the curvature $\phi = -0.126 \times 10^{-5}$ in.$^{-1}$

Eq. (12) gives the reduction in the prestressing compressive force in the concrete, $\Delta P_c = -93.5$ kips. The stress distribution immediately after transfer, the loss and the effective stress in concrete are shown in Fig. 4c. The effect of the non-prestressed steel can be seen by comparison with Fig. 4d which corresponds to $A_{ns} = 0$; other data are unchanged.

Deflection Calculation

The deflection of a member supported at its two ends depends upon the variation of the curvature along the member. The curvature ϕ calculated by Eq. (14) is the sum of the following curvatures:

$$\phi_{\text{dead load}} = \frac{M}{r^2 A_c E_c} (1 + v) \quad (20)$$

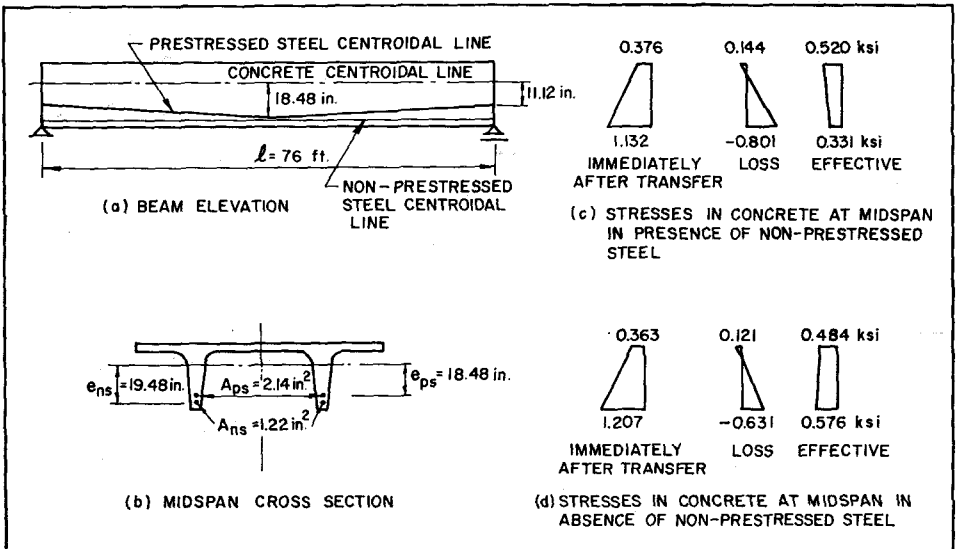$$\phi_{\text{prestressing}} = -\frac{P_{co} e_{ps}}{r^2 A_c E_c} (1 + v) \quad (21)$$
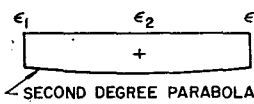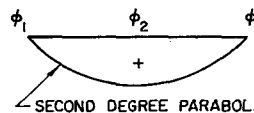

Fig. 4. Pretensioned beam considered in Example 1.
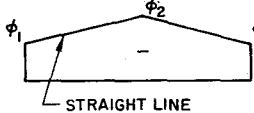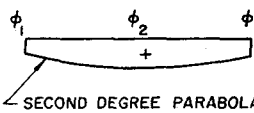
			without non-prestressed steel	with non-prestressed steel
TOTAL AXIAL SHORTENING $\Delta \ell = \frac{\ell}{3}(\epsilon_1 + 2\epsilon_2)$	$\epsilon_1 \quad \epsilon_2 \quad \epsilon_1$ + SECOND DEGREE PARABOLA	$\epsilon_1 \times 10^4$:	9.554	9.030
		$\epsilon_2 \times 10^4$:	9.706	9.418
		$\Delta \ell$ (in.) :	0.881	0.847
DEAD LOAD DEFLECTION $\delta_{dead\ load} = \frac{\phi_2 \ell^2}{9.6}$	$\phi_1 \quad \phi_2 \quad \phi_1$ + SECOND DEGREE PARABOLA	$\phi_1 \times 10^5$ (in.$^{-1}$) :	0	0
		$\phi_2 \times 10^5$ (in.$^{-1}$) :	7.358	7.358
		$\delta_{dead\ load}$ (in.) :	6.376	6.376
PRESTRESS DEFLECTION $\delta_{prestress} = (\phi_1 + 2\phi_2)\,\ell^2/24$	$\phi_1 \quad \phi_2 \quad \phi_1$ − STRAIGHT LINE	$\phi_1 \times 10^5$ (in.$^{-1}$) :	−5.610	−5.213
		$\phi_2 \times 10^5$ (in.$^{-1}$) :	−9.444	−9.210
		$\delta_{prestress}$ (in.) :	−8.490	−8.190
PRESTRESS LOSS DEFLECTION $\delta_{prestress\ loss} = (\phi_1 + 5\phi_2)\,\ell^2/48$	$\phi_1 \quad \phi_2 \quad \phi_1$ + SECOND DEGREE PARABOLA	$\phi_1 \times 10^5$ (in.$^{-1}$) :	0.928	1.632
		$\phi_2 \times 10^5$ (in.$^{-1}$) :	1.363	1.729
		$\delta_{prestress\ loss}$ (in.) :	1.342	1.778
TOTAL DEFLECTION $\delta_{total} = \delta_{deadload} + \delta_{prestress} + \delta_{prestress\ loss}$		δ_{total} (in.) :	−0.772	−0.036

Fig. 5. Comparison of axial shortening and midspan deflection in a beam with and without non-prestressed steel.

$$\phi_{\text{prestress loss}} = \frac{e\,f_{co}}{\alpha\,r^2\,E_c}\,\mu \qquad (22)$$

The variations of the curvatures due to the dead load or due to the prestressing are similar to the bending moments due to the same causes.

It is reasonable to assume parabolic variation for the prestress loss curvature. The equations included in Fig. 5 may be used to calculate the central deflection for linear and for parabolic curvature variations.

EXAMPLE 2

It is required to calculate the axial shortening and the midspan deflection of the beam of Example 1.

The dead load moment will cause the initial prestressing force immediately after transfer to be somewhat less at the beam ends than at midspan. Thus, a smaller intrinsic relaxation loss L_r is to be expected.[6] A value $L_r = 17.5$ ksi

should be used for the end sections in this example (based on Magura et al.[9]).

The axial strain at age t_k is calculated at the ends and at midspan. The axial shortening is calculated in Fig. 5 assuming a parabolic variation of ϵ along the beam length.

Eqs. (20)-(22) give the curvatures at the ends and at midspan due to dead load, prestressing and prestress loss. The variations of ϕ are given in Fig. 5. The deflections due to each of the three causes are given in the same figure together with the equations used.

A comparison is made of the axial shortening and the midspan deflection of the same beam with and without non-prestressed steel. The presence of the non-prestressed steel reduces the axial strain. Its effect on the instantaneous camber is a slight reduction.

However, the downward deflection caused by prestress loss is significantly increased from 1.342 to 1.778 in. This

Table 1. Comparison of Abeles and Kung's test results[1] with theoretical values.

$\frac{A_{ns}}{A_{ps}}$	ΔP_c (kips)		Midspan Deflection (in.)	
	Experimental	Theoretical	Experimental	Theoretical
1	-5.73	-5.35	-0.454	-0.516
2.67	-9.23	-8.33	-0.475	-0.475
3.90	-9.21	-9.49	-0.398	-0.402
5.22	-9.94	-10.44	-0.340	-0.315

indicates that a designer can adjust the amounts of A_{ns} and A_{ps} to control the camber or the downward deflection.

Comparison With Test Results

Abeles and Küng[1] tested a series of simple beams with the ratio A_{ns}/A_{ps} varying between 1.00 and 5.22. By measurement of the strain in the non-prestressed and the prestressed steel they derived the loss of prestress in concrete, ΔP_c at midspan.

The creep coefficient v and the free shrinkage s are not given in Reference 1; but using the reported concrete composition, the member dimensions and the relative humidity, it is possible to calculate values $v = 2.25$ and $s = 310 \times 10^{-6}$ using the recommendations of the CEB-FIP Committee.[10]

The method presented was used to calculate the reduction of the compressive force ΔP_c and the midspan deflection. Table 1 compares the theoretical and experimental results.

Effect of Varying A_{ns} on ΔP_c

The method of calculation presented above is used to study the effect of varying the non-prestressed steel area on the loss of compressive prestress force on the concrete.

Define a parameter λ as the ratio of the values of ΔP_c in the presence and in the absence of non-prestressed steel.

Fig. 6 shows the variation of λ for the beam in Figs. 4a and 4b as v varies between 0 and 4 and changing A_{ns} such that $A_{ns}/A_{ps} = 0$ to 2.0. The graph indicates that the loss of prestressing in concrete is substantially increased by the presence of A_{ns}; e.g., with $v = 2$ and $A_{ns}/A_{ps} = 1.0$ the loss is increased by 36 percent.

Note that this conclusion can also be reached by noting the increase in loss with A_{ns}/A_{ps} in Abeles and Küng's results in Table 1.

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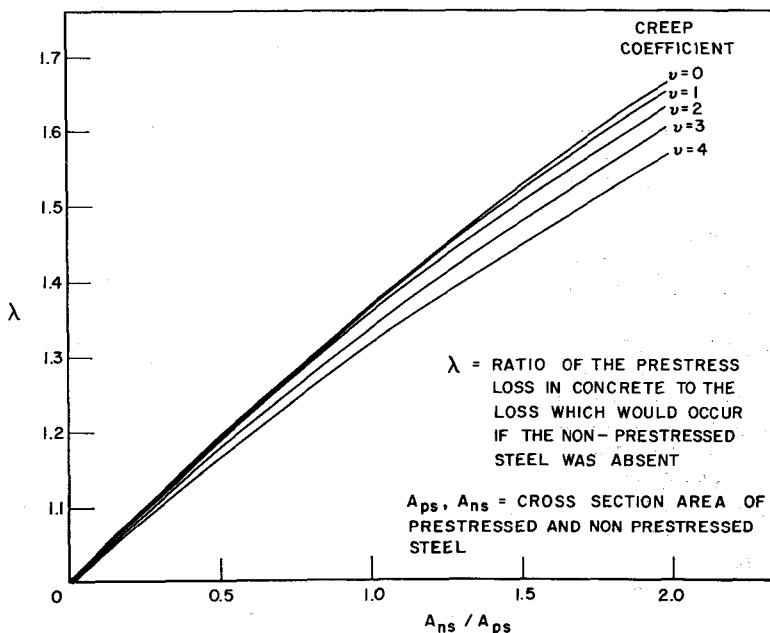
Fig. 6. Effect of the presence of non-prestressed steel on loss of prestress in concrete (midspan section of beam of Example 1).

A_c = cross-sectional area of concrete beam

A_{ns} = cross-sectional area of non-prestressed steel

A_{ps} = cross-sectional area of prestressed steel

A_s = total steel area $(A_s = A_{ps} + A_{ns})$

E_c = modulus of elasticity of concrete at age t_o

E_s = modulus of elasticity of steel

e = eccentricity of total steel area measured downwards from centroid of concrete

e_{ns} = eccentricity of non-prestressed steel measured downwards from centroid of concrete

e_{ns} = eccentricity of prestressed steel measured downwards from centroid of concrete

f_{ct} = hypothetical value of concrete stress [Eqs. (16) and (19) for pretensioned and post-tensioned members, respectively]

f_{co} = compressive stress in concrete at eccentricity e immediately after transfer

f_{ps} = tensile stress in prestressed steel

L_{os} = instantaneous change of steel stress at transfer; value is equal to the reduction in tension in prestressed steel (in pretensioned members). It is also equal to the increase in compression in non-prestressed steel (in both pretensioned and post-tensioned members)

L_{ps} = total loss of stress in prestressed steel in period $(t_k - t_o)$ excluding instantaneous loss at transfer

L_r = intrinsic relaxation loss of steel stress of tendon stretched between two fixed points

l = span length

Δl = axial shortening of member at time t_k

M = bending moment in section due to applied loads; positive M produces tension at bottom fibers of beam

N = normal force in section due to applied loads; positive N denotes compression

n = modular ratio at time $t_o = E_s/E_c$

P_c = compressive force in concrete caused by prestressing

P_{co} = compressive force on concrete immediately after transfer of prestress

P_{ps} = tensile force in prestressed steel

P_{ns} = tensile force in non-prestressed steel

r = radius of gyration of concrete section

s = free shrinkage of concrete in period $t_k - t_o$

t_o = age of concrete at prestress transfer, days

t_k = age of concrete at which prestress loss and displacements are required, days

$\bar{a}$ = dimensionless coefficient defined by Eq (8)

a = dimensionless coefficient defined by Eq. (6)

β = ratio of initial stress in prestressed steel to ultimate strength $= f_{pso}/f_{psu}$

Δ = used as a prefix to indicate an increment of value

δ = midspan deflection at time t_k

ϵ = axial strain at time t_k

λ = ratio of prestress loss in concrete in presence of non-prestressed steel to prestress loss in absence of non-prestressed steel

μ = recovery parameter, equals the ratio of the instantaneous plus creep strain caused by the prestress loss to the instantaneous strain at prestress transfer, both at eccentricity e [Eq. (10) and Fig. 3]

ξ = steel area parameter defined by Eq. (7)

ν = creep coefficient, equals ratio of creep at age t_k to instantaneous

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* This list is intended to comply in general with the notation adopted in the *PCI Design Handbook*.[5]

strain when a constant sustained load is applied at t_o

ψ = relaxation reduction factor (Fig. 2)

Ω = dimensionless value [Eq. (11)]

ϕ = curvature at time t_k

Subscripts

c, s, ps, ns = concrete, total steel, pre-stressed steel and non-prestressed steel, respectively

i, o = value immediately before and after transfer, respectively

u = ultimate strength

APPENDIX B—DERIVATION OF RECOVERY PARAMETER

Creep of concrete is assumed proportional to the stress and the superposition of stresses and strains is thus valid.

The service life of the member is divided into m discrete time intervals. A change in force on the concrete or the steel, $\Delta P(i)$, in an intermediate interval, is assumed to occur at the middle of that interval.

The strain in concrete at the level of prestressed steel at the end of interval i is:

$$\epsilon_{cp}(i + \tfrac{1}{2}) = \frac{f_{cpo}}{E_c}\, v\,(i + \tfrac{1}{2}, o) +$$

$$\sum_{j=1}^{i} \frac{\Delta P_{ps}(j)}{E_c(j)\,A_c}\left(1 + \frac{e_{ps}^2}{r^2}\right) \times$$

$$[1 + v\,(i + \tfrac{1}{2}, j)] +$$

$$\sum_{j=1}^{i} \frac{\Delta P_{ns}(j)}{E_c(j)\,A_c}\left(1 + \frac{e_{ns}\,e_{ps}}{r^2}\right) \times$$

$$[1 + v\,(i + \tfrac{1}{2}, j)] \qquad (23)$$

where:

f_{cpo} = stress in concrete at the same level due to initial prestressing and dead load;

$v\,(i + \tfrac{1}{2}, j)$ = ratio of the creep strain from the middle of interval j to the end of i, to the instantaneous strain caused by a constant sustained stress introduced at the middle of interval j;

$\Delta P_{ps}(j)$ = change in tension during the jth interval in the non-prestressed steel (value is usually negative)

$\Delta P_{ns}(j)$ = change in tension during the jth interval in the prestressed steel (value is usually negative)

The time-dependent strain in the concrete at the level of steel must be equal to that in steel:

$$\epsilon_{ps}\,(i + \tfrac{1}{2}) = -\,\epsilon_{cp}\,(i + \tfrac{1}{2}) \qquad (24)$$

where

$$\epsilon_{ps}\,(i + \tfrac{1}{2}) = \frac{1}{E_s\,A_{ps}}[P_{ps}\,(i + \tfrac{1}{2}) -$$

$$P_{pso} + A_{ps}\,\overline{L}_r\,(i + \tfrac{1}{2})] \qquad (25)$$

The term $\overline{L}_r\,(i + \tfrac{1}{2})$ is the reduced relaxation loss occurring between transfer and the end of interval i.

Two equations similar to Eqs. (23) and (25) can be written for the strain change in the non-prestressed steel and in the concrete at the same level. Solving the four equations, together with the following equilibrium equation:

$$\Delta P_{ns}\,(i) + \Delta P_{ps}\,(i) = \Delta P_c\,(i) \qquad (26)$$

yields the expression:

$$\frac{P_{co} - P_c\,(i + \tfrac{1}{2})}{E_s\,A_s} = s\,(i + \tfrac{1}{2}) +$$

$$\left[\frac{f_{cp}\,A_{ps} + f_{cn}\,A_{ns}}{A_s}\right] \times$$

$$\frac{v\,(i + \tfrac{1}{2}, o)}{E_c} + SUM\,(i) \qquad (27)$$

where

$$SUM\,(i) = \sum_{j=1}^{i}\left\{\left[\Delta P_{ps}(j)\left\{A_{ps}\left(1 + \frac{e_{ps}^2}{r^2}\right) + \right.\right.\right.$$

$$A_{ns}\left(1 + \frac{e_{ps}\,e_{ns}}{r^2}\right)\Big\} +$$

$$\Delta P_{ns}(j)\left\{A_{ps}\left(1 + \frac{e_{ps}\,e_{ns}}{r^2}\right) + \right.$$

$$\left.A_{ns}\left(1 + \frac{e_{ns}^2}{r^2}\right)\right\}\right]\left(\frac{1 + v\,(i + \frac{1}{2}, j)}{E_c(j)\,A_c\,A_s}\right)\Big\} \tag{28}$$

The summation term $SUM\ (i)$ can be simplified by assuming that the steel stress increments $\Delta P_{ps}(j)/A_{ps}$ and $\Delta P_{ns}(j)/A_{ns}$ are equal. Therefore:

$$SUM\ (i) = \sum_{j=1}^{i} \frac{\Delta P_c(j)\,a}{E_c(j)\,A_c} \times$$

$$[1 + v\,(i + \frac{1}{2}, j)] \tag{29}$$

The value of $SUM\ (i)$ can be calculated by a step-by-step procedure for $i = 1, 2, \ldots, m$, where m is the total number of time intervals. Rewrite Eq. (29) in the form:

$$SUM\ (m) = -\mu f_{co}/E_c \tag{30}$$

where μ is the recovery parameter defined by the equation:

$$\mu = -\frac{E_c}{f_{co}} \sum_{j=1}^{m} \frac{\Delta P_c(j)\,a}{E_c(j)\,A_c} \times$$

$$[1 + v\,(m + \frac{1}{2}, j)] \tag{31}$$

The approximation involved in Eq. (29) is that the actual time-dependent change in stress in the prestressed and non-prestressed steel is replaced by one average value in evaluating the recovery parameter μ. The error is expected to be small particularly when the value of e_{ns} is close to that of e_{ps}.

A computer program in which the above approximation is avoided was developed and used for the following comparisons. The same beam of Example 1 was analyzed by the program for two locations of non-prestressed steel:

(a) in the same location as in Example 1 ($e_{ns} = 19.48$), and (b) $e_{ns} = 5.98$ in. The latter value corresponds to the case when the non-prestressed steel is equally divided between the top and bottom of the section.

Table 2 gives the percentage error resulting from the above-mentioned simplification. It is clear that no great loss in accuracy is involved even when there is a large difference in the eccentricities of the two steel areas (as in Case b considered).

Table 2. Error resulting from ignoring the spread of steel in a beam example.

	Time-Dependent Value	Without Non-Prestressed Steel	Position of Non-Prestressed Steel					
			(a) Bottom			(b) Equally Divided Between Top and Bottom		
			Method* 1	Method* 2	Percentage Error	Method* 1	Method* 2	Percentage Error
Midspan Section	L_{ps} (ksi)	35.14	32.7	32.9	0.6	34.0	34.8	2.4
	ΔP_c (kip)	-75.2	-93.0	-93.5	0.5	-98.7	-100.6	1.9
	$\epsilon \times 10^4$	9.70	9.39	9.41	0.2	9.37	9.36	-0.1
	$\phi \times 10^5$ (in.$^{-1}$)	-0.72	-0.13	-0.13	0	-0.56	-0.68	21.4
End Section	L_{ps} (ksi)	40.6	37.6	37.6	0	39.4	38.8	-1.5
	ΔP_c (kip)	-86.9	-117.4	-115.0	2.0	-114.9	-116.6	1.5
	$\epsilon \times 10^4$	9.54	9.04	9.03	-0.1	9.15	9.23	0.9
	$\phi \times 10^5$ (in.$^{-1}$)	-4.65	3.57	-3.58	0.3	-4.43	-4.48	1.1

* Method 1 uses a computer program which accounts for the actual positions of A_{ps} and A_{ns} in all computations. Method 2 is detailed in the paper.

APPENDIX C—PRESTRESS FORCES AT TRANSFER

(a) Pretensioned members

Immediately before transfer, the force in the prestressed steel is P_{psi}. Transfer of prestress corresponds to the application of this force to a concrete-steel composite section.

The forces P_{pso}, P_{nso} and P_{co} in the three components of the section, A_{ps}, A_{ns} and A_c, respectively, immediately after transfer are given by:

$$P_{pso} = P_{psi} - L_{es} A_{ps} \qquad (32)$$

and

$$P_{nso} = - L_{es} A_{ns} \qquad (33)$$

The above involves the assumption that A_{ps} and A_{ns} can be replaced in the composite section by $A_s = A_{ps} + A_{ns}$ at eccentricity e.

From the equilibrium of forces:

$$P_{co} = P_{psi} - L_{es} (A_{ps} + A_{ns}) \qquad (34)$$

or

$$P_{co} = P_{psi} - L_{es} A_s \qquad (35)$$

The value of L_{es} can be obtained by equating the strain in concrete and in steel at eccentricity e:

$$\frac{1}{A_c E_c}(P_{psi}\,\bar{a} - L_{es} A_s\, a + N - Me/r^2)$$

$$= \frac{L_{es}}{E_s} \qquad (36)$$

The first two terms in Eq. (36) are the strain caused by P_{co}, which is equal to P_{psi} applied at e_{ps} minus $(L_{es}A_s)$ applied

at e. The dead load normal force N and bending moment M are assumed to come into effect at transfer.

Solving Eq. (36) for L_{es} gives Eq. (15).

(b) Post-tensioned members

With post-tensioned members the prestressed steel is not bonded to the concrete at transfer. It, therefore, does not restrain the deformation of the concrete. In other words, the stresses in concrete and non-prestressed steel at transfer are dependent only on the force and eccentricity of the prestressing steel, but not on its area.

The post-tensioning results in the following forces:

$$P_{nso} = - L_{es} A_{ns} \qquad (37)$$

$$P_{co} = P_{pso} - A_{ns} L_{es} \qquad (38)$$

Equating the strain in concrete and in steel at eccentricity e_{ns} gives:

$$\frac{1}{A_c E_c}\left[\left(1 + \frac{e_{ns}\,e_{ps}}{r^2}\right) P_{pso} - L_{es} A_{ns} \times \right.$$

$$\left.\left(1 + \frac{e^2_{ns}}{r^2}\right) + N - \frac{Me_{ns}}{r^2}\right] = \frac{L_{es}}{E_s} \qquad (39)$$

from which:

$$L_{es} = n f_{ci}/\left[1 + \left(1 + \frac{e^2_{ns}}{r^2}\right)\frac{n A_{ns}}{A_c}\right] \qquad (40)$$

where f_{ci} is the concrete stress which would occur at eccentricity e_{ns} in the absence of the non-prestressed steel [Eq. (19)].

Substitution of Eq. (40) in Eq. (38) gives Eq. (18).

Discussion of this paper is invited. Please forward your comments to PCI Headquarters by Sept. 1, 1977.