Relaxation of Steel in Prestressed Concrete

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A steel tendon stretched between two fixed points gradually loses a part of its tension due to creep. The loss in tension under constant strain, as in a test in which the length of the tendon is maintained constant after stretching, will be referred to as the intrinsic relaxation.

The amount of intrinsic relaxation depends on the ratio:

$$\lambda = \frac{f_{psi}}{f_{psu}} \tag{1}$$

where

f_{psi} = initial stress immediately after stretching (at time t_i)

f_{psu} = strength of prestressed steel

When λ is smaller than 0.5 the intrinsic relaxation is negligible, but its value increases rapidly as λ approaches 1.

A tendon in a prestressed concrete member loses a part of its initial tension due to the combined effect of creep and shrinkage of concrete and steel relaxation; the loss in tension is associated with shortening of the tendon. The reduction in tension caused by creep and shrinkage has the same effect on the magnitude of the relaxation as if the initial stress were smaller.

Thus, the value of relaxation to be used in predicting prestress loss and the associated deformations of prestressed concrete members should be smaller than the intrinsic relaxation obtained from a constant length relaxation test. Hence, one can write:

$$\bar{L}_r = \chi_r L_r \tag{2}$$

where

L_r = intrinsic relaxation which occurs in a constant length tendon; units of L_r are force/length^2

$\bar{L}_r$ = a reduced relaxation value to be used in predicting prestress loss and deformation of concrete members

χ_r = relaxation reduction coefficient, a dimensionless value smaller than unity

The purpose of this paper is to derive an expression for the relaxation reduction coefficient χ_r and demonstrate how it can be used in practice for calculating the loss of prestress and determining the stress and strain after loss in a prestressed concrete cross section.

Ghali et al[6] used a step-by-step computation procedure to derive values for the relaxation reduction coefficient. The present paper offers a more accurate evaluation of χ_r based on an equation for the intrinsic relaxation adopted from the CEB-FIP Model Code for Concrete Structures, 1978 (MC-78).[2] A graph and an equation are presented herein for the coefficient χ_r.

The relaxation reduction coefficient χ_r is intended for use in practice as a multiplier to the intrinsic relaxation value, L_r. The latter value may be based on test results, often reported by steel suppliers, or calculated by empirical equation.[8]

Most codes recognize the fact that the magnitude of relaxation of a tendon in a prestressed concrete member increases with the increase in initial steel stress and decreases with the increase of loss due to creep and shrinkage. Some codes give empirical expressions for the steel relaxation as a function of the above mentioned parameters.

The equation presented here for the relaxation reduction coefficient χ_r includes all the necessary parameters and is reached by rational derivation; hence it is more accurate. The equation is simple to use without complicating the design calculations. A numerical example is included in the paper, while the derivation of the coefficient χ_r is given in an appendix.

Synopsis

Relaxation of prestressed steel in a concrete member is of a smaller magnitude than the intrinsic relaxation which occurs in a tendon stretched between two fixed points. A reduced relaxation value should be employed in the calculation of prestress loss and the corresponding deformation.

In this paper a graph and an equation are presented for a relaxation reduction coefficient to be employed as a multiplier to the intrinsic relaxation for use in prestressed concrete design. A numerical example is included to show how the method can be applied.

SIGN CONVENTION

Tensile stress or tensile force in steel or concrete is assumed positive. The symbol ΔP represents a force increment in concrete or in steel. A positive ΔP indicates an increase in tension or a reduction in compression. Loss of tension in prestressed tendons due to the combined effects of creep, shrinkage and relaxation, ΔP_{ps}, is generally a negative value.

The same effects produce a change, ΔP_c, in the resultant of the stress on concrete; ΔP_c generally represents a reduction in compression, hence a positive quantity. Similarly, relaxation in prestressed steel is a reduction in tension, hence a negative value.

INTRINSIC RELAXATION

The magnitude of intrinsic relaxation depends on the value of λ and also on the quality of steel. MC-78[2] refers to two groups of steel. The first group with higher intrinsic relaxation includes cold-drawn wires and strands. The second group, with low relaxation, includes quenched and tempered wires and cold-drawn wires and strands which are treated (stabilized) to achieve low relaxation.

In the absence of relaxation tests, MC-78 gives a table which may be used to determine the value of the intrinsic relaxation as a function of λ (Fig.

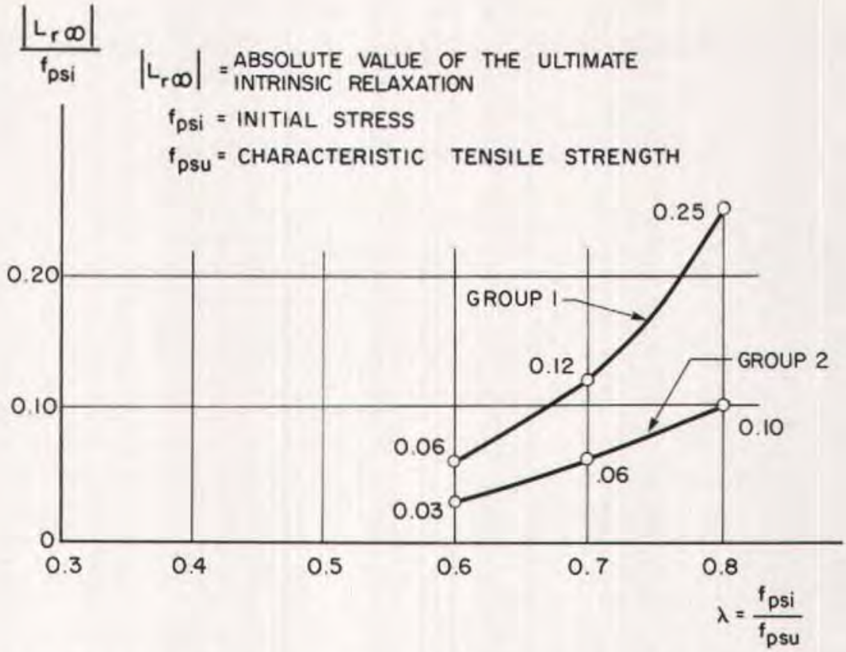

Fig. 1. Very long-term intrinsic relaxation of prestressing steels according to the CEB-FIP Code.[2]

1). The values in the graph correspond to the two steel groups mentioned above and to a constant length relaxation over a period of 0.5 x 10^6 hours. After this period the ultimate relaxation is considered to have been reached.

The following equations closely approximate the MC-78 values for the intrinsic relaxation:

$$\left.\begin{array}{ll} \dfrac{L_{r\infty}}{f_{psi}} = -\eta_\infty (\lambda - 0.4)^2 & \text{when } \lambda \geq 0.4 \\ \text{and} & \\ L_{r\infty} = 0 & \text{when } \lambda < 0.4 \end{array}\right\} \quad (3)$$

where

$L_{r\infty}$ = value of the intrinsic relaxation at time infinity

η_∞ = 1.5 or (⅔) for steels of Groups 1 and 2, respectively

When the value of the intrinsic relaxation is known for a particular value of λ, Eq. (3) may be used to derive a value of η_∞ for the type of steel considered. Subsequent use of the same equation varying λ gives the intrinsic relaxation for any initial stress value.

The intrinsic relaxation at any instant τ depends upon the length of the period $(\tau - t_i)$; where t_i is the time at which the initial tension is applied.

For any type of steel or initial tension, the intrinsic relaxation at any time τ may be expressed as a product of the ultimate intrinsic relaxation $L_{r\infty}$ and a dimensionless function of the period $(\tau - t_i)$ in hours:

$$L_r(\tau) = L_{r\infty}\left[\frac{1}{16}\ln\left(\frac{\tau - t_i}{10} + 1\right)\right] \quad (4)$$

$$\text{when } 0 \leq (\tau - t_i) \leq 1000$$

$$L_r(\tau) = L_{r\infty}\left[\left(\frac{\tau - t_i}{0.5 \times 10^6}\right)^{0.2}\right] \quad (5)$$

$$\text{when } 1000 < (\tau - t_i) \leq 0.5 \times 10^6$$

and

$$L_r(\tau) = L_{r\infty} \quad (6)$$
$$\text{when } (\tau - t_i) > 0.5 \times 10^6$$

Results of intrinsic relaxation tests are usually reported for a period $(\tau - t_i) =$ 1000 hours. Eqs. (4) to (6) may be applied to derive the relaxation value corresponding to $(\tau - t_i) = \infty$ or to any other period of time. The equations are based on experimental values reported in Reference 4.

REDUCED RELAXATION

Compare two tendons of the same steel quality: the first in a constant length relaxation test and the second used in prestressing a concrete member. Assume that the initial stress σ_{psi} at time t_i in the two tendons is the same. Because of creep and shrinkage of concrete, the stress at any instant τ is smaller in the second tendon compared to the first. Thus, the relaxation in the second tendon should be smaller than the first.

This may be accounted for empirically by considering that the relaxation in the second tendon to be the same as the intrinsic relaxation for a reduced initial tension equal to the actual tension minus a fraction of the total loss of prestress due to the combined effect of creep, shrinkage and relaxation. This fraction is 0.3 according to MC-78.[2]

A more accurate approach is to multiply the intrinsic relaxation by a reduction coefficient χ_r to obtain a reduced relaxation value for a tendon in a prestressed member [Eq. (2)]. For practical application, the value χ_r may be read from Table 1 or the graph in Fig. 2 or calculated by a closely fitting equation:

$$\chi_r = e^{(-6.7 + 5.3\lambda)\Omega} \quad (7)$$

where

$$\Omega = \frac{|L_{ps}| - |L_r|}{f_{psi}} \quad (8)$$

$|L_{ps}|$ is the absolute value of change of stress in the prestressed steel due to the combined effect of creep, shrinkage and relaxation. $|L_r|$ is the absolute value of the intrinsic relaxation.

The parameter λ is the ratio of the initial tension at transfer, f_{psi}, to the ultimate strength, f_{psu}. With pretensioning, f_{psi} is the stress after elastic shortening. The relaxation occurring during the period between jacking and transfer [given by Eq. (4)] takes place without change in tendon length and thus should not be subject to any reduction. With post-tensioning, the value f_{psi} is the initial stress at any section after a reduction of the loss due to friction and anchor set.

The relaxation reduction coefficient χ_r calculated by Eq. (7) or read from Fig. 2 applies for any type of prestressing steel.

PRESTRESS LOSS

Fig. 3 represents a prestressed concrete cross section of a member with prestressed and nonprestressed reinforcement. At time t_i prestress is introduced simultaneously with the load due to the self weight of the member. The instantaneous stress and strain distributions at t_i can be determined by conventional equations.

Due to creep and shrinkage of concrete and relaxation of prestressed steel, concrete loses a part of its compression and prestressed steel loses a part of its tension. The nonprestressed steel generally picks up some compression. For any period $(t - t_o)$, where $t > t_o$, the sum of the force increments in the three materials must be zero; thus:

$$\Delta P_c + \Delta P_{ps} + \Delta P_{ns} = 0 \quad (9)$$

The symbol ΔP represents a force increment; when positive it indicates an increase in tension (or reduction in compression). The subscripts c, ps and ns refer to concrete, prestressed steel and nonprestressed steel, respectively.

$$\Omega = \frac{|\text{TOTAL PRESTRESS LOSS}| - |\text{INTRINSIC RELAXATION}|}{\text{STEEL STRESS IMMEDIATELY AFTER TRANSFER}}$$

$$\lambda = \frac{\text{STEEL STRESS IMMEDIATELY AFTER TRANSFER}}{\text{ULTIMATE TENSILE STRENGTH}}$$

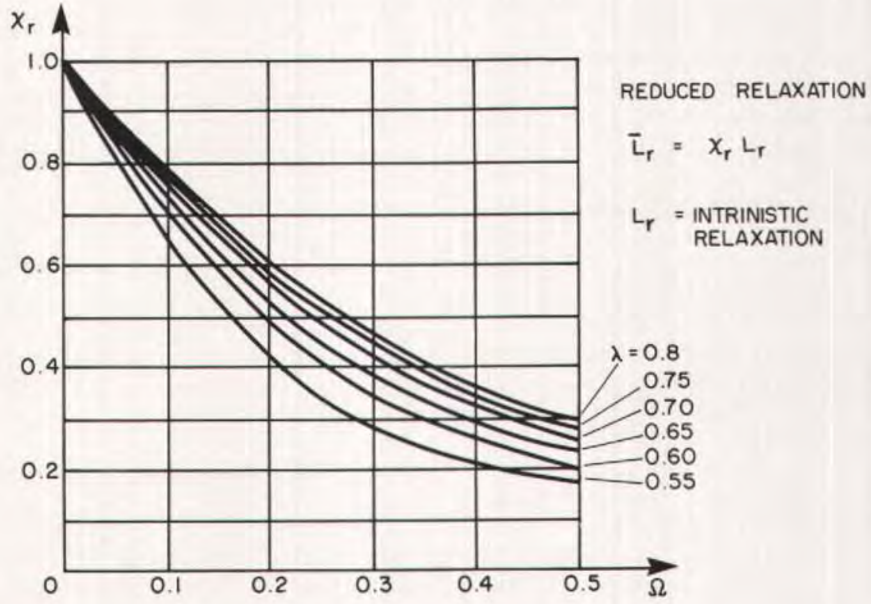

Fig. 2. Relaxation reduction coefficient χ_r.

In the common case discussed above ΔP_c is positive while ΔP_{ps} and ΔP_{ns} are negative quantities.

The change in force on concrete, ΔP_c, during the period $(t - t_i)$ due to creep, shrinkage and relaxation may be calculated by the equation (derived in Ref. 7):

$$\Delta P_c = -\beta[C f_{cei}\, n A_{st} + s E_s A_{st} + \bar{L}_r A_{ps}] \quad (10)$$

where

$$\beta = \left[1 + \frac{\alpha n A_{st}}{A_c}(1 + \chi C) \right]^{-1} \quad (11)$$

in which the total steel area, A_{st}, is the sum of the areas of prestressed steel, A_{ps}, and nonprestressed steel, A_{ns}:

$$A_{st} = A_{ns} + A_{ps} \quad (12)$$

and

$$\alpha = 1 + \frac{e^2}{r^2} \quad (13)$$

in which

e = eccentricity, i.e., the distance from Point O, the centroid of concrete area (A_c) to the centroid of A_{st}

$r^2 = I_c/A_c$ = square of radius of gyration of area, A_c

I_c = moment of inertia of the cross section area of concrete about an axis through its centroid

In Eqs. (10) and (11) the following additional symbols are explained:

f_{cei} = instantaneous stress at $y = e$ (Fig. 3) due to prestressing and dead load applied at time t_i

E_s = modulus of elasticity of the reinforcement, assumed the same for prestressed and nonprestressed steels

s = free (unrestrained) shrinkage

n = $E_s/E_c(t_i)$, where $E_c(t_i)$ = modulus of elasticity of concrete at time t_i

C = creep coefficient = ratio of creep of concrete to instantaneous strain due to a stress introduced at time t_i and sustained, without change in magnitude, up to time t

χ = aging coefficient depending upon the ages of concrete at t_i and t; its value generally ranges between 0.6 and 0.9. Tables and graphs for the value of χ are available.[1,3,5] Note that χ is used as a multiplier to C, when a stress increment is gradually introduced during the period t_i to t.

$\bar{L}_r$ = reduced relaxation = intrinsic relaxation multiplied by the relaxation reduction coefficient χ_r [Eq. (2)]

Eq. (10) expresses the loss of compression in the concrete as the sum of three terms inside the square brackets which represent, respectively, the effects of creep, shrinkage and relaxation. The dimensionless coefficient β accounts for the fact that the loss due to each of the three causes depends upon creep and the cross section areas and locations of the prestressed and nonprestressed steels.

Post-tensioned and pretensioned members differ only in the calculation of f_{cei}. With post-tensioning, the area of cross section to be employed in calculating f_{cei} includes the cross section areas of the nonprestressed steel and of concrete, excluding the area of the prestressing duct. With pretensioning, the cross section to be used is composed of the areas of concrete and of prestressed and nonprestressed steels.

The value of the relaxation reduction coefficient χ_r depends upon the total loss, L_{ps}, due to the combined effects of creep, shrinkage and relaxation. Since this is generally not known before Eq.

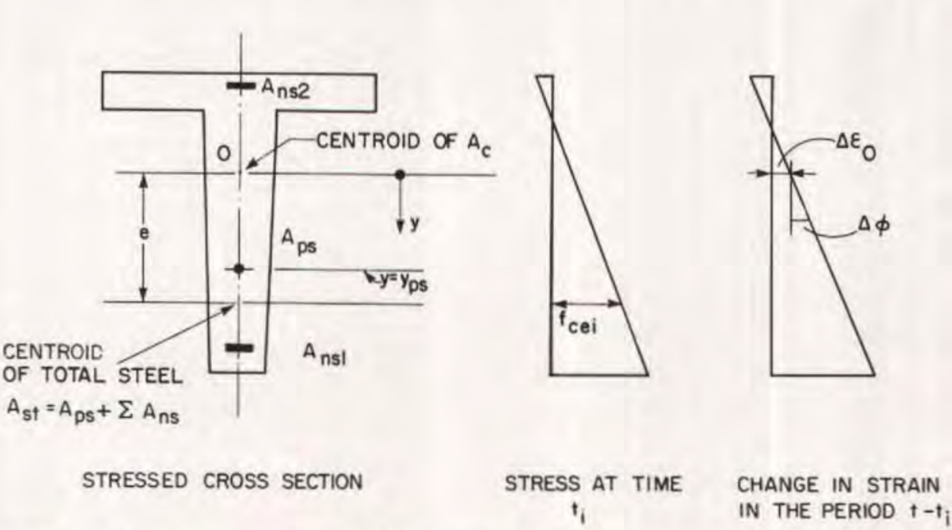

Fig. 3. Definition of symbols used in Eqs. (10) to (18).

(10) is applied, a value for χ_r must first be assumed (for example 0.7) and later corrected by iteration. A single iteration is sufficient in most cases (see Example).

The force ΔP_c acts on the concrete cross section at eccentricity e; it produces increments in normal strain at Point O and in curvature. Adding the effects of ΔP_c to creep and shrinkage gives the total change in strain at Point O during the period $(t - t_o)$:

$$\Delta\epsilon_O = C\,\epsilon_O(t_o) + s + \frac{\Delta P_c}{\bar{E}_c A_c} \quad (14)$$

Similarly, the change in curvature over the same period is:

$$\Delta\phi = C\,\phi(t_o) + \frac{\Delta P_c e}{\bar{E}_c I_c} \quad (15)$$

where $\bar{E}_c$ is the age-adjusted elasticity modulus of concrete:

$$\bar{E}_c = \frac{E_c(t_i)}{1 + \chi C} \quad (16)$$

The age-adjusted modulus represents the stress necessary to produce a total strain (instantaneous plus creep) of magnitude unity; the stress is here assumed to be gradually introduced over the period t_i to t.

The change in concrete stress at any fiber is:

$$\Delta f_c = \frac{\Delta P_c}{A_c} + \frac{\Delta P_c e}{I_c} y \quad (17)$$

where y is the coordinate of the fiber considered (measured downwards from Point O).

The change in stress in the prestressed steel is:

$$L_{ps} = E_s(\Delta\epsilon_O + y_{ps}\Delta\phi) + \bar{L}_r \quad (18)$$

where y_{ps} is the y coordinate of the prestressed steel.

The following numerical example illustrates the application of the proposed method.

EXAMPLE

Calculate the changes in stress and in strain which occur in a period $(t - t_i)$ in the concrete cross section shown in Fig. 4a which is post-tensioned at time t_i.

The following data are given:

$E_c(t_i) = 4500$ ksi; $E_s = 29 \times 10^3$ ksi; initial force in prestressed tendon = 315 kips; bending moment due to dead load introduced at the same time as the prestress = 3500 kip-in.; free shrinkage $s = -240 \times 10^{-6}$; creep coefficient $C = 3$; aging coefficient $\chi = 0.8$; intrinsic relaxation $L_r = -17$ ksi, strength of prestressed steel, $f_{psu} = 270$ ksi.

Fig. 4b shows the strain and stress distributions at time t_i, immediately after prestressing. These are calculated by considering the initial prestress and the dead load bending moment to be applied on a transformed section of modulus of elasticity $E_c(t_i)$ and composed of the area of concrete (less prestressing duct) plus n times the area of nonprestressed steel; where $n = 29 \times 10^3/4500 = 6.44$.

The centroids of A_c and A_{st} are determined (Fig. 4a) as well as the following geometric properties:

$A_c = 570.4$ in.^{2}; $I_c = 108000$ in.^{4}; $r^2 = 189.6$ in.^{2}; $\alpha = 1.36$; $A_{st} = 5.6$ in.^{2}; $A_{ps} = 1.7$ in.^{2}

Assume a value for the relaxation reduction coefficient: $\chi_r = 0.7$.

Thus, the reduced relaxation [Eq. (2)] is:

$$\bar{L}_r = 0.7(-17) = -11.9 \text{ ksi}$$

Eq. (11) gives:

$$\beta = \left[1 + \frac{1.36\,(6.44)\,(5.6)}{570.4}(1 + 0.8 \times 3)\right]^{-1} = 0.7738$$

The change in force on concrete during the period considered [Eq. (10)] is:

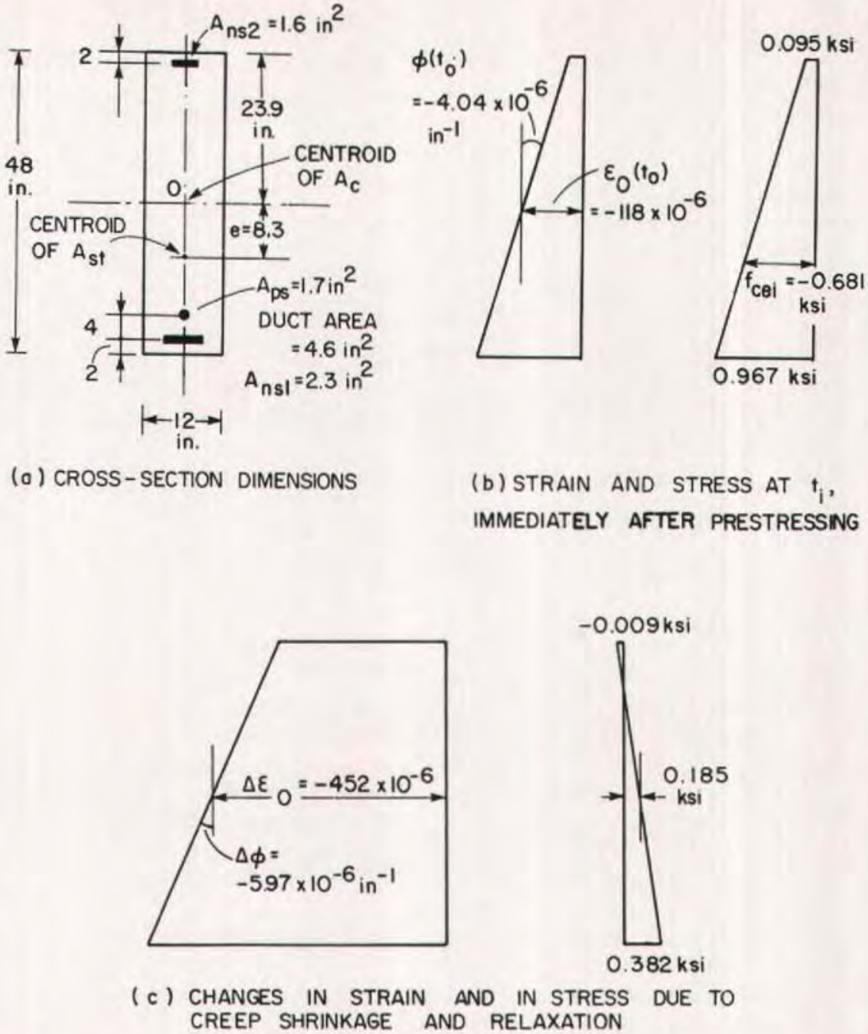

Fig. 4. Analysis of time-dependent strain and stress in a post-tensioned cross section.

$$\Delta P_c = -0.7738\,[3(-0.681)\,(6.44)\,(5.6) + (-240 \times 10^{-6})\,(29000)\,(5.6) + (-11.9)\,(1.7)] = 102.8 \text{ kips}$$

The age-adjusted modulus of elasticity [Eq. (16)] is:

$$\bar{E}_c = \frac{4500}{1 + 0.8 \times 3} = 1324 \text{ ksi}$$

The change in axial strain [Eq. (14)] is:

$$\Delta\epsilon_O = 3(-118 \times 10^{-6}) - 240 \times 10^{-6} + \frac{102.8}{1324\,(570.4)} = -458 \times 10^{-6}$$

and the change in curvature [Eq. (15)] is:

$$\Delta\phi = 3(-4.04 \times 10^{-6}) + \frac{102.8\,(8.3)}{1324\,(108000)} = -6.15 \times 10^{-6} \text{ in.}^{-1}$$

The change in stress in prestressed steel [Eq. (18)] is:

$$L_{ps} = 29000\,[-458 + 18.1\,(-6.15)] \times 10^{-6} - 11.9 = -28.4 \text{ ksi}$$

To calculate an improved value of the reduced relaxation, substitute f_{psi} = 315/1.7 = 185.3 ksi in Eqs. (1) and (8):

$$\lambda = \frac{185.3}{270} = 0.7$$

$$\Omega = \frac{28.4 - 17}{185.3} = 0.06$$

Table 1 or Fig. 3 gives $\chi_r = 0.85$.

Thus, a more accurate value of the reduced relaxation [Eq. (2)] is:

$$\bar{L}_r = 0.85(-17) = -14.4 \text{ ksi}$$

Substitution of this value in Eq. (10) gives ΔP_c = 106.1 kips. The corresponding changes in axial strain in curvature and in stress [Eqs. (14), (15) and (17)], are plotted in Fig. 4c. The change in stress in prestressed steel [Eq. (18)] is $L_{ps} = -30.7$ ksi. Further iteration would change these results only slightly.

It should be noted that the loss of tension in the prestressed steel (30.7 x 1.7 = 52.2 kips) is smaller in absolute value than the loss of compression in concrete (106.1 kips). The difference represents the compressive force picked up by the nonprestressed reinforcement.

CONCLUSION

A prestressed tendon exhibits relaxation in a concrete member of smaller magnitude than the intrinsic relaxation which would occur if the length of the tendon was maintained constant. The reduction in relaxation is caused by the shortening of the tendon due to shrinkage and creep of concrete. The relaxation coefficient χ_r can be used in prestressed concrete design as a multiplier to the intrinsic relaxation in the prediction of the prestress loss and the associated deformation.

ACKNOWLEDGMENT

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SI Conversion Factors

1 in.	= 25.4 mm
1 in.^{2}	= 645 mm^2
1 kip	= 4.448 kN
1 ksi	= 6.894 MPa
1 kip-in.	= 0.1130 kN-m

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NOTE: Discussion of this paper is invited. Please submit your comments to PCI Headquarters by May 1, 1986.

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APPENDIX A — DERIVATION OF EXPRESSION FOR RELAXATION REDUCTION COEFFICIENT χ_r

The reduction coefficient for the relaxation of prestressed steel during any period $(t - t_i)$ may be expressed as:

$$\chi_r = \int_o^1 (1 - \Omega\xi) \left[\frac{\lambda(1 - \Omega\xi) - 0.4}{\lambda - 0.4} \right]^2 d\xi \tag{A1}$$

where ξ is a dimensionless time function defining the shape of the stress-time curve for a constant length tendon (Fig. A1).

The value ξ equals zero when $\tau = t_i$ and equals 1 when $\tau = t$; τ represents any instant between t_i and t. Thus, the intrinsic relaxation at any instant τ is:

$$L_r(\tau) = [L_r(t)]\,\xi \tag{A2}$$

Eq. (A1) is derived assuming that the prestress loss due to the combined effects of creep, shrinkage and relaxation varies with time according to the same shape function ξ. Thus:

$$L_{ps}(\tau) = [L_{ps}(t)]\,\xi \tag{A3}$$

At any instant τ (Fig. A1) the tendon in the prestressed concrete member exhibits relaxation as if its initial tension were:

$$\bar{f}_{psi}(\tau) = [f_{psi} - |L_{ps}(\tau) - L_r(\tau)|\,] \tag{A4}$$

The term in absolute value represents a reduction in tension caused by the shortening of the tendon; $\bar{f}_{psi}(\tau)$ is a reduced initial tension at the instant τ. Substitution of Eqs. (8), (A2) and (A3) into Eq. (A4) yields:

$$\bar{f}_{psi}(\tau) = f_{psi}(1 - \Omega\xi) \tag{A5}$$

Eqs. (3), (4) [or Eqs. (5) and (6)] and Eq. (A2) may be combined to express the intrinsic relaxation. At any instant:

$$L_r(\tau) = -\eta_t f_{psi}(\lambda - 0.4)^2\,\xi \quad \text{when } \lambda \geq 0.4 \tag{A6}$$

and

$$L_r(\tau) = 0 \quad \text{when } \lambda < 0.4 \tag{A7}$$

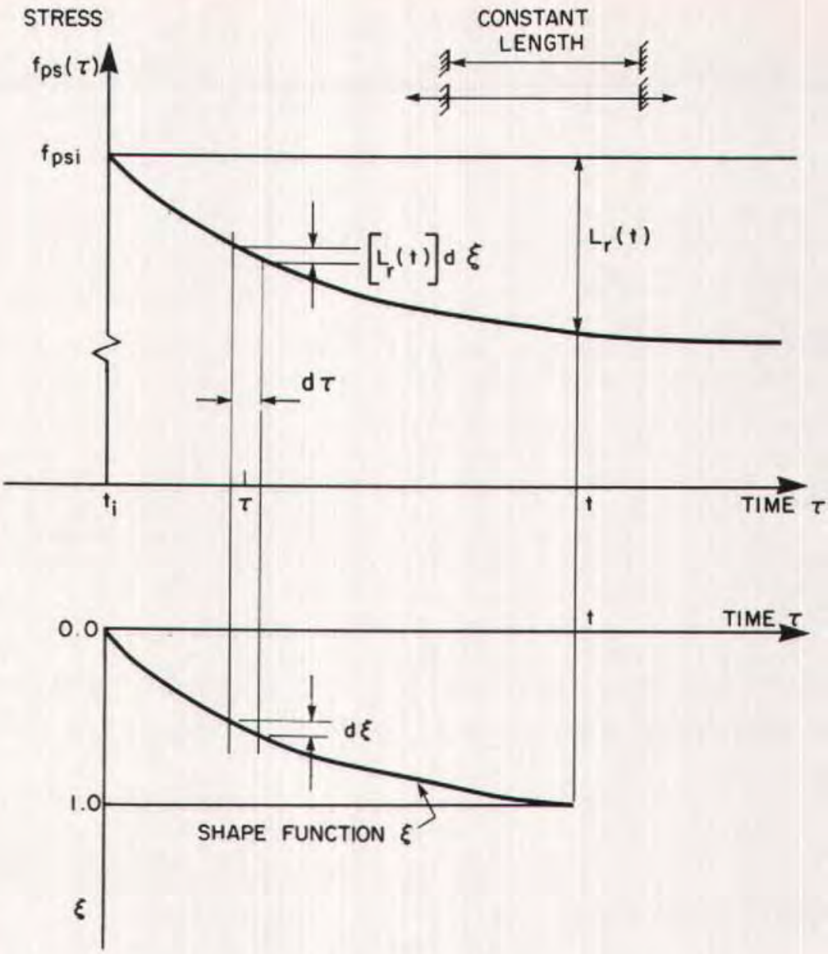

Fig. A1. Stress versus strain in a constant length relaxation test. Definition of the shape function ξ.

where $\eta_t = \eta_\infty$ multiplied by the value between square brackets in Eq. (4) or (5), with $\tau = t$. Note that when $(t - t_o) > 0.5 \times 10^6$, $\eta_t = \eta_\infty$.

Employing Eq. (A6) or (A7), a differential of the intrinsic relaxation may be expressed as:

$$dL_r = -\eta_t f_{psi} (\lambda - 0.4)^2 \, d\xi \quad \text{when } \lambda \geq 0.4 \quad \text{(A8)}$$

and

$$dL_r = 0 \quad \text{when } \lambda < 0.4 \quad \text{(A9)}$$

It can be seen from Eq. (A8) that the differential intrinsic relaxation depends upon the initial tension value f_{psi} and λ $(= f_{psi}/f_{psu})$. For the tendon in prestressed concrete, the effective initial tension at any instant is reduced by the factor $(1 - \Omega\xi)$. Thus, Eq. (A8) may be used to express the differential reduced relaxation by replacing f_{psi} by $\bar{f}_{psi}$ and substituting for the latter by Eq. (A5):

$$d\bar{L}_r = -\eta_t f_{psi} (1 - \Omega\xi) \times [\lambda(1 - \Omega\xi) - 0.4]^2 \, d\xi$$

Table A1. Relaxation reduction coefficient χ_r.

Ω	$\lambda = 0.55$	$\lambda = 0.60$	$\lambda = 0.65$	$\lambda = 0.70$	$\lambda = 0.75$	$\lambda = 0.80$
0.0	1.000	1.000	1.000	1.000	1.000	1.000
0.1	0.6492	0.6978	0.7282	0.7490	0.7642	0.7757
0.2	0.4168	0.4820	0.5259	0.5573	0.5806	0.5987
0.3	0.2824	0.3393	0.3832	0.4166	0.4425	0.4630
0.4	0.2118	0.2546	0.2897	0.3188	0.3429	0.3627
0.5	0.1694	0.2037	0.2318	0.2551	0.2748	0.2917

$$\text{when } \lambda\,(1 - \Omega\xi) \geq 0.4 \tag{A10}$$

or

$$d\bar{L}_r = 0 \qquad \text{when } \lambda\,(1 - \Omega\xi) < 0.4 \tag{A11}$$

Integration of each of Eqs. (A10) and (A8) and then division gives the relaxation reduction coefficient Eq. (A1). The equation applies when $\lambda(1 - \Omega\xi) \geq 0.4$. This restriction can be accounted for simply by replacing the upper limit of the integral in Eq. (A1) by the smaller of 1 and the value $[(\lambda - 0.4)/(\lambda\Omega)]$.

The values of the relaxation reduction coefficient χ_r in Table A1 or Fig. 2 are calculated by evaluating the integral in Eq. (A1) for various values of λ and Ω. A closed form expression resulting from integration of Eq. (A1) is rather lengthy. Instead, Eq. (7) (obtained by curve fitting) may be used for the relaxation reduction coefficient.

* * *

APPENDIX B — NOTATION

A = cross section area

C = creep coefficient = ratio of creep which occurs during a period $(t - t_i)$ to the instantaneous strain due to a stress introduced at t_i and sustained constant thereafter

E = modulus of elasticity

e = eccentricity of the centroid of total steel area measured downward from centroid of concrete area

f = stress

I = moment of inertia

L_r = intrinsic relaxation of prestressed steel = change in stress in a tendon stretched between two fixed points

$\bar{L}_r$ = reduced relaxation (for prestressing tendons in concrete)

n = $E_s/E_c\,(t_o)$

r^2 = I_c/A_c = square of radius of gyration of concrete area with respect to its centroid

s = strain due to shrinkage when it is free to occur without restrain

t = time

α = $1 + (e^2/r^2)$

Δ = an increment

ϵ = strain

ϕ = curvature

χ = aging coefficient for a specified period t_i to t. A value smaller than unity used as a multiplier to the creep coefficient when a stress increment is gradually introduced during the period $(t - t_i)$

χ_r = relaxation reduction coefficient

β = dimensionless coefficient defined by Eq. (11)

η = dimensionless coefficient employed as multiplier in Eq. (3) for the intrinsic relaxation. The value of η depends upon the steel quality and on the length of the period of relaxation.

λ = ratio of steel stress immediately after transfer to ultimate tensile strength

σ = stress

τ = any instant of time

ξ = dimensionless function varying between 0 and 1 (Fig. A1)

Ω = ratio of total prestress loss minus steel stress immediately after transfer to steel stress immediately after transfer

Subscripts

c = concrete

ps = prestressed steel

ns = nonprestressed steel

i = instant t_i, the time of introduction of prestressing

st = total steel area

u = ultimate strength

∞ = very long period (exceeding 0.5 x 10^6 hours)

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