

Cracks and Crack Control in Concrete Structures



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The material presented in this paper is based on more than 30 years of research, observations and experience concerning causes, control, and consequences of cracking in concrete structures. This extensive background was helpful in the preparation of this paper which deals with questions of concrete cracking.

The presence of cracking does not necessarily indicate deficiency in strength or serviceability of concrete structures. While currently available design code provisions lead to reasonable control of cracking, additional control can be achieved by understanding the basic causes and mechanisms of crack-

ing in concrete structures. In this paper, causes of concrete cracking are discussed, including tensile strength of concrete, temperature, shrinkage and creep effects. Recommended crack widths are presented along with design methods for sizing reinforcement to control crack widths.

CAUSES OF CRACKING

Concrete can crack due to a number of causes. Some of the most significant causes are discussed in detail.

Tensile Strength of Concrete

The tensile strength of concrete is a widely scattering quantity. Cracking occurs when tensile stresses exceed the tensile strength of concrete. Therefore, to control concrete cracking, the tensile strength of concrete is of primary im-

Note: This paper is a revised and updated version of an article originally published in the Proceedings of the International Association for Bridge and Structural Engineering (IABSE), Zurich, Switzerland, 1987, p. 109.

portance. Laboratory test data conducted by H. Rüscher were analyzed statistically. As presented in Ref. 1, this analysis furnished the following relationships for the mean direct tensile strength, f'_{tm} , related to the 28-day compressive cylinder strength f'_c of concrete:

$$f'_{tm} = 2.1 (f'_c)^{2/3} (\text{psi})$$
$$f'_{tm} = 0.34 (f'_c)^{2/3} (\text{N/mm}^2)$$

The statistical analysis indicated that the coefficient in this equation can be modified to 1.4 (0.22) and 2.7 (0.45) to obtain the 5 and the 95 percentiles, respectively, of the tensile strength, f'_t . The tensile strength of concrete is slightly higher in flexure. However, it is recommended that values for direct tension be used in practice. Concrete cracks when the tensile strain, ϵ_{ct} , exceeds 0.010 to 0.012 percent. This limiting tensile strain is essentially independent of concrete strength.

The 5 percentile of the tensile strength, f'_{t5} , should be used in design to locate areas in the structure that are likely to crack by comparing calculated stresses with the expected concrete strength. The 95 percentile, f'_{t95} , should be used to obtain conservative values for restraint forces that might occur before the concrete cracks. These restraint forces are used to calculate the amount of reinforcement needed for crack width control.

Causes of Cracking During Concrete Hardening

Concrete cracking can develop during the first days after placing and before any loads are applied to the structure. Stresses develop due to differential temperatures within the concrete. Cracking occurs when these stresses exceed the developing tensile strength, f'_t , of the concrete as indicated in Figs. 1 and 2. Differential temperatures are mainly due to the heat of hydration of

Synopsis

Simple design rules are presented to control cracking in concrete structures. Causes of cracking and its effect on serviceability and durability are discussed. The paper is primarily applicable to large structures such as bridges. However, general concepts presented are applicable to any concrete structure. Prestressing forces are considered. A numerical example showing application of the method and use of simple design charts is included.

cement during concrete hardening. This effect is usually neglected except in massive structures as indicated in Ref. 2. However, depending on cement content and type of cement, the temperature within concrete members with dimensions of 12 to 36 in. (30 to 91 cm) can increase approximately 36°F to 108°F (20°C to 60°C) during the first 2 days after casting.

If concrete members are allowed to cool quickly, tensile stresses may reach values higher than the developing tensile strength of the concrete. Even if this process results only in microcracking, the effective tensile strength of the hardened concrete is reduced. However, very often wide cracks appear even when reinforcement is provided. In addition, the reinforcement may not be fully effective since bond strength is also developing and is yet too low. It is necessary to minimize such early cracks by keeping temperature differentials within the concrete as low as possible. This can be done by one or more of the following measures:

1. **Choice of cement** — A cement with low initial heat of hydration should be selected. Table 1 shows that there is a significant variation in heat develop-

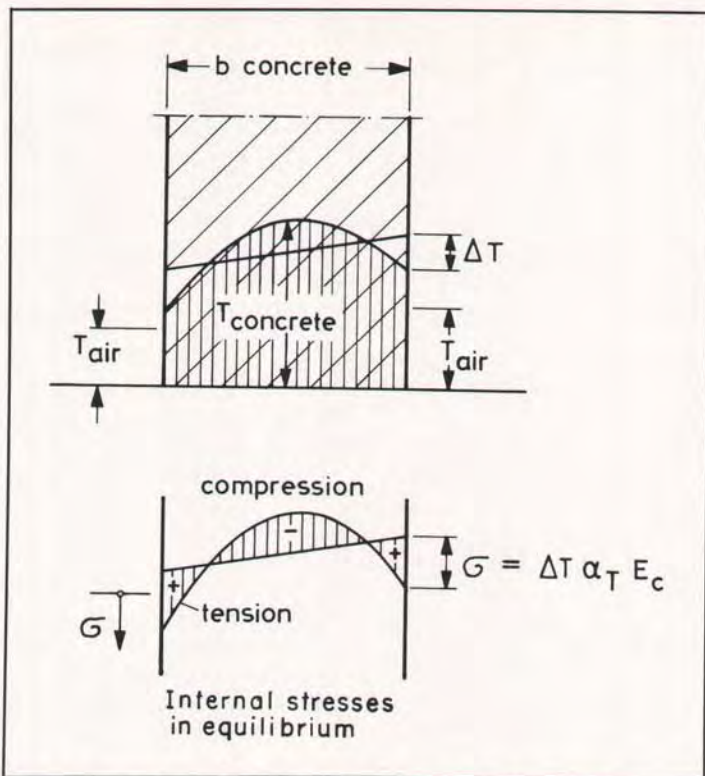


Fig. 1. Temperature distribution due to heat of hydration and internal stresses caused by outside cooling in a free standing concrete block.

ment among different types of cements. The cement content of concrete should be kept as low as possible by good grading of the aggregates. Heat development can also be reduced by adding fly ash or using slag furnace cement.

2. Curing — Evaporation of water must be prevented by using curing compounds or by covering the concrete with a membrane. Rapid evaporation can lead to plastic shrinkage cracking.

3. Curing by thermal insulation — Rapid cooling of the surface must be prevented. The degree of thermal insulation depends not only on the climate, but also on the thickness of the concrete member and on the type of cement used. Spraying cold water on warm young

concrete, as it was done years ago, is not recommended.

4. Precooling — This is a necessity for large massive concrete structures such as dams. For more usual structures, in which shortening after cooling can take place without creating significant restraint forces, precooling is expensive and unnecessary. In this case, thermal insulation is preferable and it also has the benefit of accelerating concrete strength development. An exception may be made in very hot climates since precooling can keep concrete workable for a longer period of time.

Often shrinkage is considered as a cause of early cracking. However, this is not true under normal climatic conditions. Shrinkage needs time to produce a

Table 1: Heat of hydration of various types of cements.*

Type of cement†	Heat of hydration (Btu/lb)			
	1 day	3 days	7 days	28 days
I	92	144	157	167
II	76	115	135	148
III	139	184	194	205
IV	50	81	94	117
V	58	88	101	124

*Data obtained from *Concrete Manual*, U.S. Bureau of Reclamation, 1975, pp. 45-46.

†Federal Specifications SS-C-192G, including Interim Amendment 2, classified the five types according to usage as follows: Type I for use in general concrete construction when Types II, III, IV, and V are not required; Type II for use in construction exposed to moderate sulfate attack; Type III for use when high early strength is required; Type IV for use when low heat of hydration is required; and Type V for use when high sulfate resistance is required.

Note: 1.0 Btu/lb = 2.32 J/g.

shortening as high as the tensile rupture strain. Only in very hot and dry air shrinkage can cause early cracks in young concrete, if measures against evaporation are not applied.

Causes of Cracking After Concrete Hardening

Tensile stresses due to dead and live loads cause cracking. Normal rein-

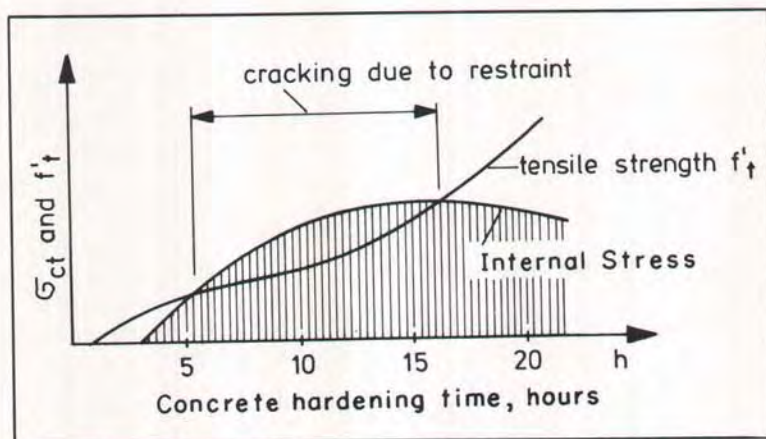


Fig. 2. Development of the tensile strength and stresses due to nonlinear temperature distribution within the concrete.

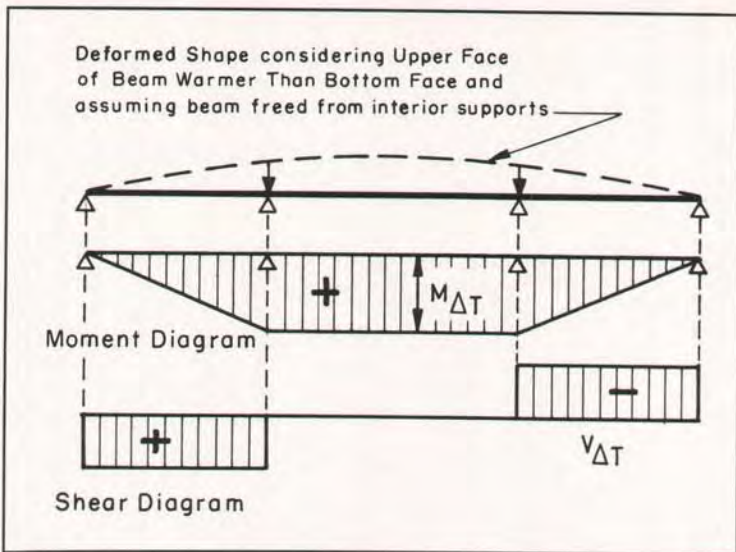


Fig. 3. Forces in a concrete beam due to a temperature rise ΔT at the upper face of the beam and external restraint provided by interior supports.

forcement or prestressing should be designed to provide required strength and keep crack widths within permissible limits. Tensile stresses due to service loads can be controlled by prestressing. The degree of prestressing can be chosen based on structural or economic considerations. Normally, partial prestressing leads to better serviceability than full prestressing.

Cracks can also be initiated by tensile stresses due to restrained deformations from temperature variations or from shrinkage and creep of concrete. Imposed deformations such as differential settlement between foundations can also cause cracks.

There are two types of restraint which cause stress in concrete members, namely, internal restraint as shown in Fig. 1, and external restraint in indeterminate structures, as shown in Fig. 3. Restrained deformations caused cracking in concrete bridges and it was primarily due to temperature differences produced by heating under the sun and cooling during the night. Ex-

treme temperatures that occur at 20 to 50-year intervals must be considered. As indicated in Refs. 3, 4, 5 and 6, temperatures in bridge structures were measured in several countries. Recently, the U.S. Transportation Research Board published in Ref. 7 temperature data for bridge design.

Temperature differentials should be considered along with recommended mean temperatures, T_m , used for calculating maximum and minimum changes in the lengths of structural members. In Central Europe values for T_m are specified for concrete bridges as varying from $+68^\circ\text{F}$ to -22° ($+20^\circ\text{C}$ to -30°C).

The temperature distribution over a beam cross section can be subdivided into three parts as shown in Fig. 4. The constant part, ΔT_1 , causes axial forces if overall length changes are restrained. The linear part, ΔT_2 , causes restraint forces, $M_{\Delta T}$ and $V_{\Delta T}$, in indeterminate structures as shown in Fig. 3 for a three span continuous beam. The nonlinear part, ΔT_3 , causes stresses, which are in

Table 2. Recommended cross section temperature differentials for bridge design in Europe.

Type of cross section and exposure	Box girder		T-beams	
	Maritime	Continental	Maritime	Continental
Top of cross section warmer than bottom ($^{\circ}\text{F}$)	18	27	14.4	21.6
Bottom of cross section warmer than top ($^{\circ}\text{F}$)	9	14.4	7.2	10.8

Note: $1.0\Delta^{\circ}\text{F} = (9/5)\Delta^{\circ}\text{C}$.

equilibrium over the cross section and produce no action forces. These stresses, which also exist in statically determinate structures, can be calculated by imposing equilibrium conditions and considering that:

$$f_{CT} = \Delta T_3 \alpha_T E_C$$

where α_T is equal to $6 \times 10^{-6}/^{\circ}\text{F}$ ($10^{-5}/^{\circ}\text{C}$), the coefficient of thermal ex-

pansion for concrete. Cooling causes tensile stresses in areas near extremities of the section.

For bridges in Europe, the ΔT values given in Table 2 are recommended. In addition to temperature, restrained concrete creep and shrinkage can cause stresses. Shrinkage often leads to cracks between connected members of significantly different sizes. Stress due to restrained creep and shrinkage can be cal-

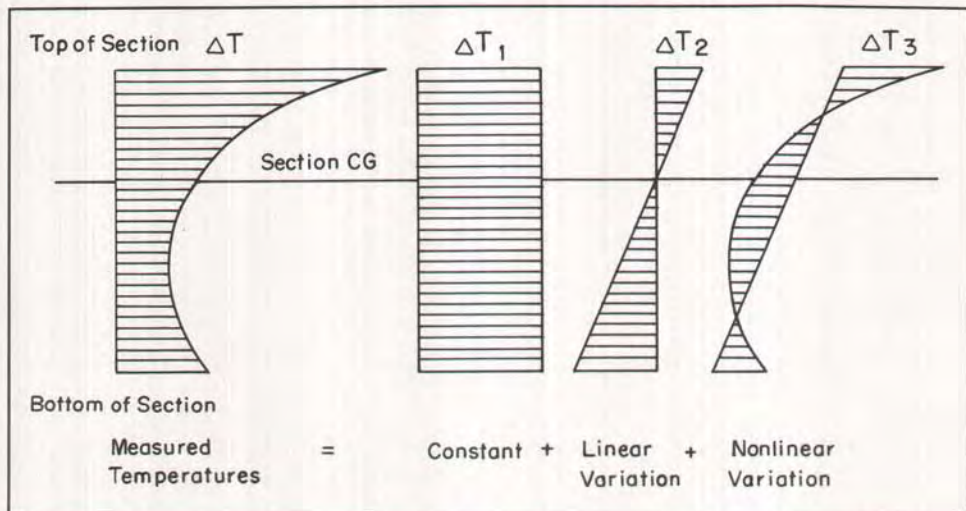


Fig. 4. Division of temperature diagram into its constant, linear and nonlinear parts.

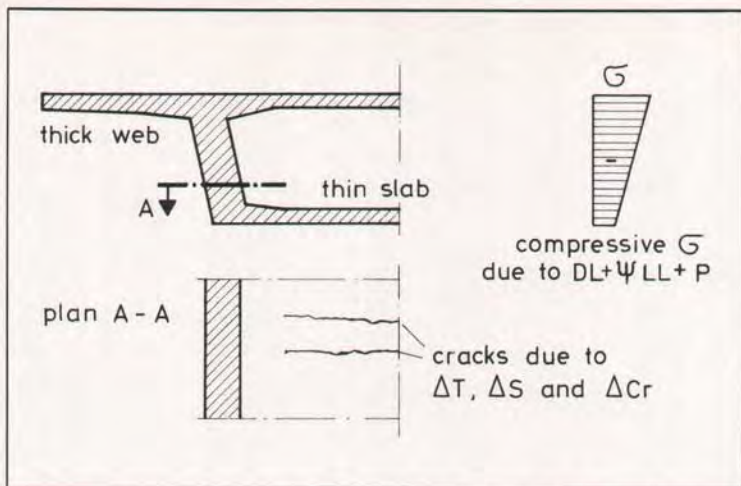


Fig. 5. Transverse cracks in thin bottom slab of box girder due to differential temperature, creep and shrinkage despite prestressing.

culated in the same way as stresses due to temperature.

Transverse cracks due to temperature, creep and shrinkage effects are frequently found in the relatively thin bottom slabs of box girders despite the fact that calculations show considerable longitudinal compressive stresses due to prestressing. Compressive stresses tend to shift towards the thick webs which undergo less creep and shrinkage strains as illustrated in Fig. 5.

Box sections are indeterminate structures. Therefore, restraint moments are developed when the section is heated on one side by the sun. This leads to vertical cracking in bridge piers and tower shafts as shown in Fig. 6. Ref. 8 shows examples of temperature cracks in prestressed concrete structures.

Determination of Areas Likely to Crack

Cracking occurs whenever the principal stresses due to service loads or due to restraint forces or due to a combination of service loads and restraints exceed the tensile strength of concrete. These stresses can be calculated using

the linear theory of elasticity, considering the structure initially uncracked. In these calculations, f'_{ts} should be taken as the tensile strength of the concrete. In the tension side of a beam, cracking will occur in areas where bending moments due to service loads and restraint cause stresses in the extreme tensile fiber above f'_{ts} . As bending increases, the depth of cracking can be calculated by considering a maximum concrete tensile strain of 0.015 percent as shown in Fig. 7.

Calculation of possible maximum bending moments due to restraint should be based on f'_{ts} . As shown in Fig. 8, consideration of such moments increases the areas in which cracking may be expected to occur.

Bending moments due to restraint define only the location and quantity of reinforcement or prestressing necessary to limit the crack width for serviceability purposes. As proven long ago by Priestley,⁴ and illustrated in Fig. 9, these moments do not decrease the ultimate strength of the structure because they are reduced and finally disappear due to cracking and plastic deformation as service loads are increased until the

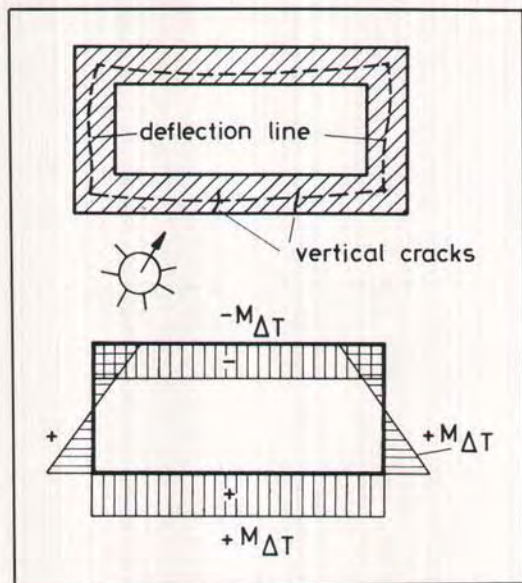


Fig. 6. Bridge pier cross section and moments due to temperature rise on one side of the pier.

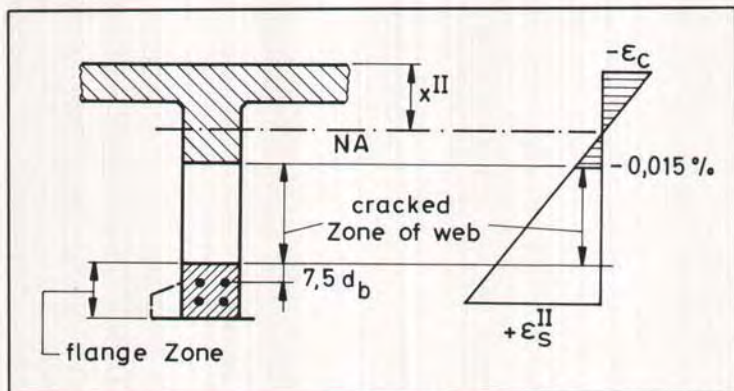


Fig. 7. Cracked zones in webs of beams under combined moment due to dead load, live load and restraint.

limit state is reached. However, the structure must be checked for possible brittle failure of the compression zone if a relatively high degree of prestressing is used, especially for continuous T-beams. Therefore, to satisfy strength requirements, bending moments due to restraint should not be added to mo-

ments due to service loads in sizing of main reinforcement. It must, however, be observed that restraint due to prestressing does not decrease on the way up to limit state.

Restraint forces decrease beginning with the first crack since the stiffness of the structure is progressively reduced

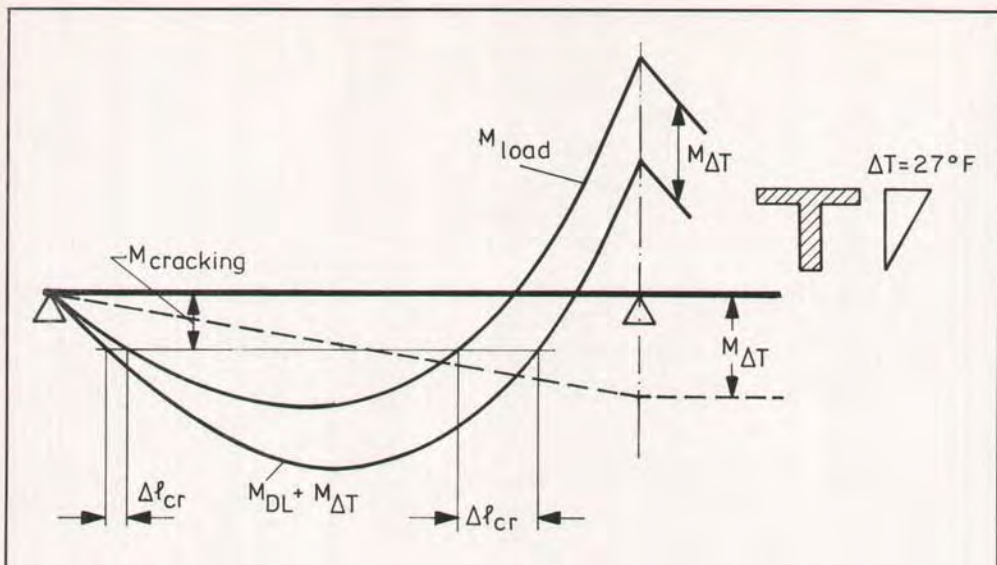


Fig. 8. Increased cracked area due to restraint moment.

with each crack that occurs. Steel stresses due to restraint are highest when the first crack occurs and decrease with each further crack. This tends to reduce crack widths. Fig. 9 shows the effect on moment due to reduction of restraint.

Evaluation of Cracks

As indicated in Fig. 10, crack widths are greater at the surface and decrease towards the reinforcement. Long years of research reported in Refs. 9 and 10, and experience indicate that crack

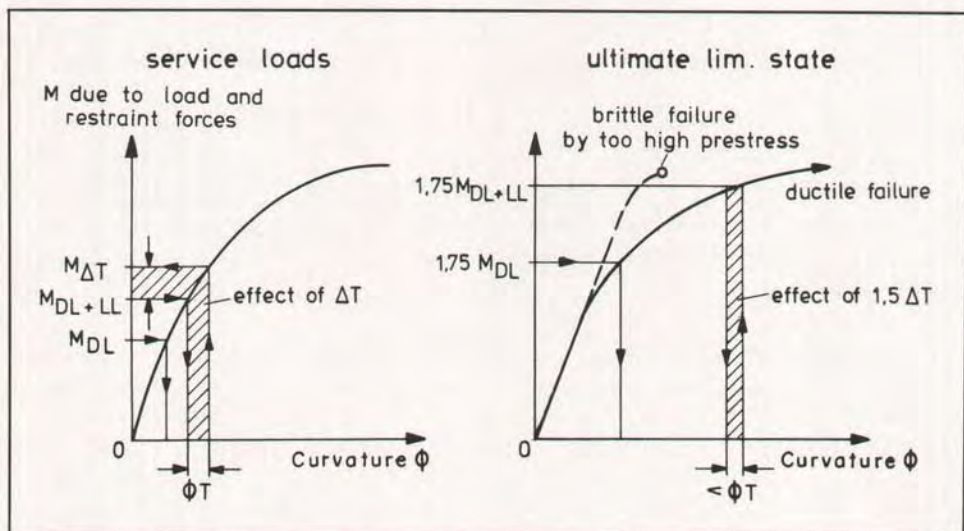


Fig. 9. Illustration of reduction of restraint forces as the limit state is approached.

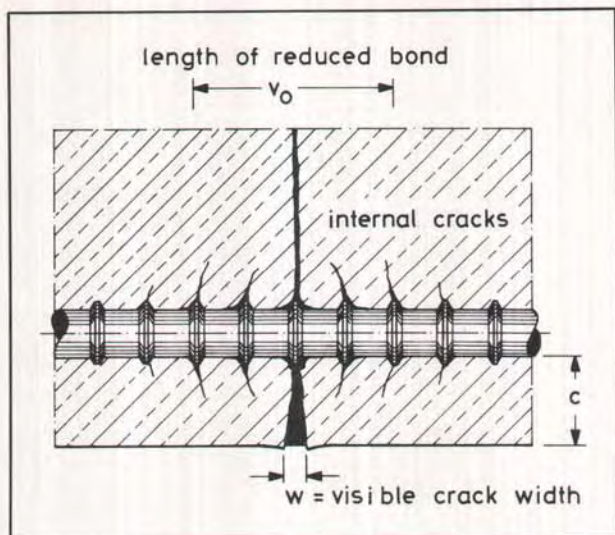


Fig. 10. Crack width at the surface is used as a measure of the effect of cracking on concrete members.

Table 3. Allowable crack widths.

Ambient condition of exposure†	w_{90} † (in.)	Maximum w permitted (in.)	Crack appearance
Mild	0.012	0.020	Easily visible
Moderate	0.008	0.016	Difficult to see with the naked eye
Severe	0.004	0.0012	

* w_{90} denotes the 90 percentile of the crack width, w .

† Defined as indicated in the CEB-FIP Model Code:

Mild exposure

- The interiors of buildings for normal habitation or offices.
- Conditions where a high level of relative humidity is reached for a short period only in any one year (for example 60 percent relative humidity for less than 3 months per year).

Moderate exposure

- The interior of buildings where the humidity is high and where there is a risk for the temporary presence of corrosive vapors.
- Running water.
- Inclement weather in rural or urban atmospheric conditions, without heavy condensation of aggressive gases.
- Ordinary soils.

Severe exposure

- Liquids containing slight amounts of acids, saline or strongly oxygenated waters.
- Corrosive gases or particularly corrosive soils.
- Corrosive industrial or maritime atmospheric conditions.

Note: 1 in. = 2.54 cm.

widths up to 0.016 in. (0.4 mm) do not significantly harm the corrosion protection of the reinforcement furnished by the concrete, provided the cover is sufficiently thick and dense. However, to avoid undue concern by casual observers, crack widths should be limited to 0.008 in. (0.2 mm) at surfaces which are often seen from a short distance.

Polluted air containing CO_2 (which causes carbonation), and SO_2 (which forms acids), or chlorides from deicing salts, can cause damage to concrete structures. Having cracks or not, concrete structures must be protected against such attacks.

Despite the evidence that crack widths up to 0.016 in. (0.4 mm) do not significantly affect the corrosion protection of reinforcement, different levels of environmental exposure and different

sensitivity to corrosion of various types of reinforcement led to different requirements for concrete cover. It is prudent to vary crack width limitations depending on environmental conditions.

For the environmental criteria of CEB and Eurocode No. 2, crack widths can be defined as presented in Table 3. These values are valid for a concrete cover, c , of 1.18 in. (30 mm) and for bar diameters, d_b , smaller than $c/1.2$ but not greater than 1 in. (25 mm). For larger cover, the allowable crack width should be increased to $c/30$ (c in mm). For cover greater than 2 3/8 in. (60 mm) and bar diameters of main reinforcement greater or equal to No. 10 (32 mm), small diameter and closely spaced reinforcement should be provided within the cover to control crack widths.

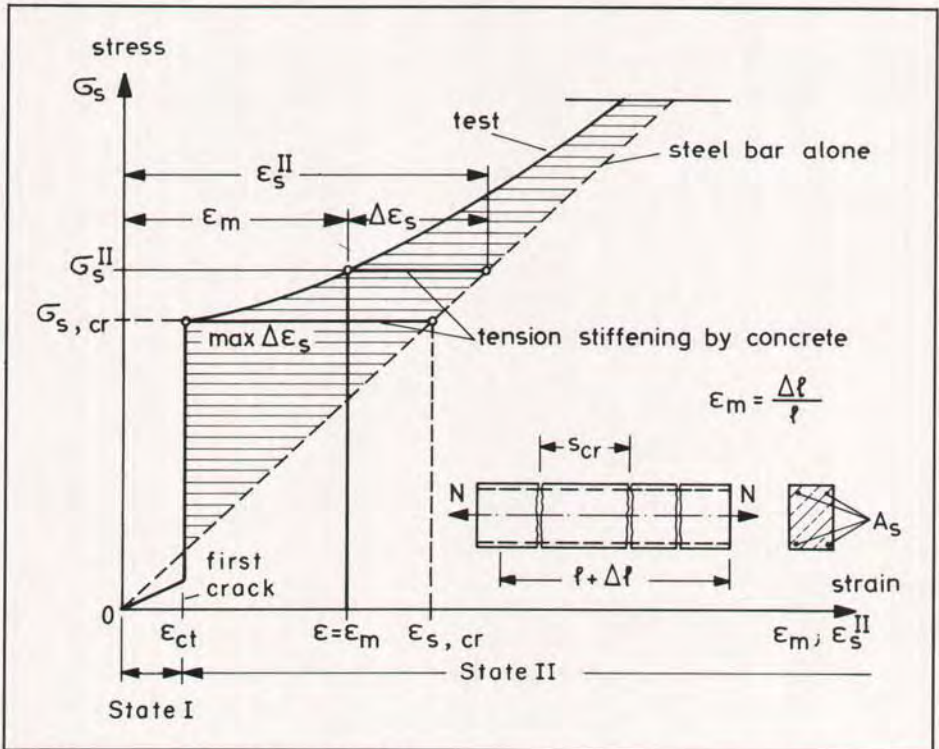


Fig. 11. Stress-strain diagram of a reinforced concrete member under direct tension.

DESIGN OF REINFORCEMENT

Reinforcement can be designed to control crack widths using information presented in the following sections.

Basic Analysis

The following presentation follows the 1978 CEB-FIP Model Code and the 1983 CEB Manual. The material is based on theoretical considerations and experimental results.

Fig. 11 shows a plot of steel stress versus longitudinal strain over a given length, l , of a reinforced concrete element in direct tension. As the load increases, cracks are assumed to occur within this length. The crack spacing and the longitudinal mean strain define the mean crack width:

$$w_m = s_{crm} \epsilon_m \quad (1)$$

where

- w_m = mean crack width
- s_{crm} = mean crack spacing
- ϵ_m = mean strain = $\Delta l/l$

As the load increases, reinforcement stress at a potential crack location varies linearly. When the crack occurs, reinforcement stress at the crack, σ_s , increases suddenly without a significant change in the mean strain. As the load continues to increase and more cracks appear, the relationship between the mean strain, ϵ_m , and reinforcement stress at the crack, σ_s , approaches that of the reinforcement alone, as indicated in Fig. 11. Conditions before cracking will be referred to as State I and conditions assuming the reinforcement working in a cracked section will be referred to as State II:

$$\epsilon_m = \epsilon_s^{\text{II}} - \Delta \epsilon_s \quad (2)$$

where

- ϵ_s^{II} = steel strain in State II
- $\Delta \epsilon_s$ = strain reduction by concrete in

tension between cracks, referred to as tension stiffening effect (see Fig. 11)

As indicated in Ref. 9, $\Delta \epsilon_s$ can be expressed as:

$$\Delta \epsilon_s = (1/E_s) (\sigma_{scr}^2 / \sigma_s^{\text{II}}) \quad (3)$$

where

- σ_{scr} = reinforcement stress immediately after cracking
- σ_s^{II} = steel stress in cracked state
- E_s = Young's modulus for the reinforcement

The strains ϵ_s^{II} and $\Delta \epsilon_s$ are significantly affected by concrete strength and reinforcement ratio.

The mean crack spacing can be expressed as:

$$s_{crm} = 2(c + s/10) + k_1 k_2 d_b / \rho_e \quad (4)$$

where

- c = concrete cover in mm
- s = bar spacing in mm
- k_1 = 0.4 for deformed reinforcement, considering bond strength
- k_2 = 0.125 for bending members, considering shape of ϵ diagram
= 0.25 for members under direct tension,
< 0.125 for combined bending and compression
- d_b = bar diameter
- ρ_e = effective reinforcement ratio, A_s/A_{ce} , where A_{ce} is the effective concrete area around the bar defined as indicated in Fig. 13.

Using the equations presented, the mean crack width, w_m , can be calculated. The 90 percentile of w can be assumed to be:

$$w_{90} = k_4 w_m \quad (5)$$

where k_4 is given in the Eurocode as 1.3 and 1.7 for restraint forces and service loads, respectively. The author recommends $k_4 = 1.5$ for all cases. The effect of repeated loads can be considered by reducing the value of $\Delta \epsilon_s$ in Eq. (2):

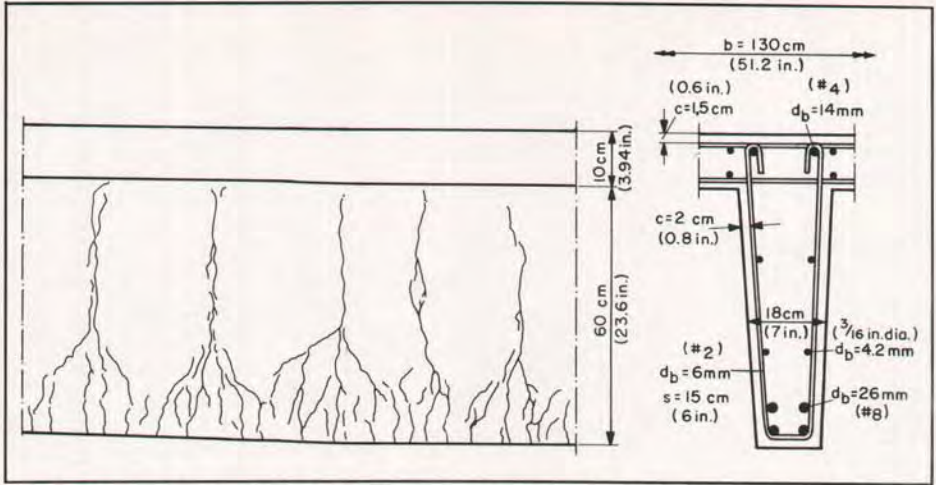


Fig. 12. Crack pattern on T-beam with large bars near extreme tension fiber and light reinforcement in the stem.

$$\Delta \epsilon_s = k_5 (1/E_s) (\sigma_{scr}^2 / \sigma_s^{II})$$

where k_5 varies from 0.4 to 0.8 depending on frequency of load repetition as indicated in Ref. 11.

If the direction of the reinforcement is not normal to the crack, as in the case of shear or torsion, the crack width can be multiplied by a factor k_a which can be taken as 1.0 for angles up to 15 degrees from the normal with the crack direction and 2.0 for a 45 degree angle. This factor can be interpolated for angles between 15 and 45 degrees.

Fig. 12 shows that the reinforcement controls crack width only within a small area around the bars. This area is defined in the CEB-FIP Model Code as the effective area, A_{ce} , shown in Fig. 13.

Sizing Reinforcement for Crack Control

For practical design, the use of simple charts is recommended to obtain the necessary crack control reinforcement area. Design charts are presented in Section 2.42 of the 1983 CEB Manual. Use of these charts will be explained in the following sections.

Sizing Reinforcement for Direct Tension

The diagram in Fig. 14 can be used to obtain the required reinforcement area for a given bar diameter, specified crack width limit, and given concrete strength. The diagram applies for the case of axial tension under service loads or restraint forces. The full lines in Fig. 14 refer to a characteristic strength of the concrete C 20, the dotted lines to C 40. For other strengths, the factor k_c must be used.

For crack control one should always choose the concrete class above the one specified for ultimate strength of the structure. Bar diameter should be chosen for obtaining small bar spacings as indicated in Section 5.5. Fig. 14 also shows steel stress at first crack and minimum reinforcement percentage required to avoid reinforcement yielding when first crack occurs. Steel stress at first crack is given by:

$$\sigma_{scr} = f_{tm} / \rho_e = 2.1 (f'_c)^{2/3} / \rho_e$$

For reinforcement with a yield strength of 60,000 psi (413 N/mm²) and 3000 and 5000 psi (21 and 34 N/mm²)

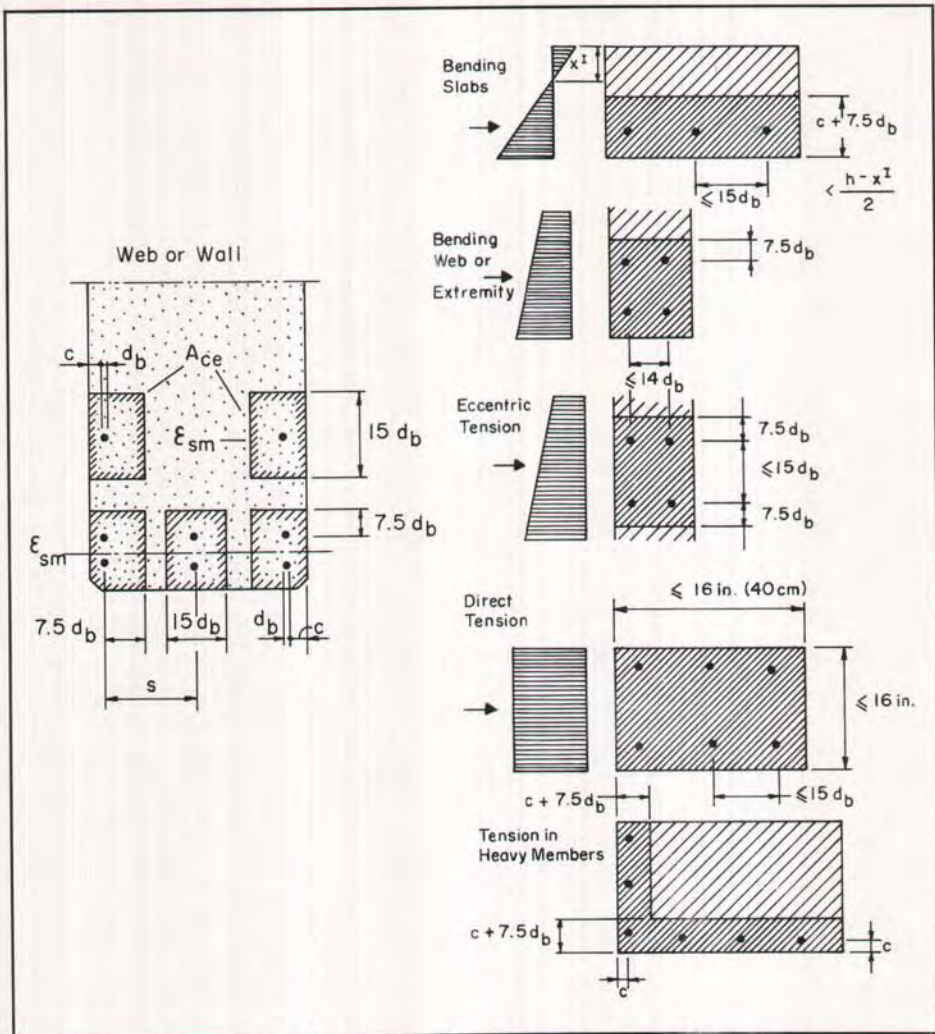


Fig. 13. Definition of the effective concrete area according to CEB-FIP 1978 Model Code.

concrete strength, the minimum reinforcement percentage can be calculated as 0.57 and 0.81 percent, respectively.

Steel stresses at cracking can be higher than allowable stress under working stress provisions of design codes. This is acceptable for restraint forces because additional cracking will decrease stresses. For loads however, such high stresses will not occur if the structure meets the strength requirements of applicable codes. Normally,

the reinforcement area obtained to satisfy strength requirements will be sufficient for crack control requirements in the effective area.

If the load is high, then the steel stresses can rise above σ_{ser} and cause an additional crack width Δw . This Δw can be estimated using Eqs. (1) and (2) and obtaining the mean crack spacing for given d_b and ρ_e from Fig. 15. The Δw must then be subtracted from the specified w_{90} to read the higher ρ_e from

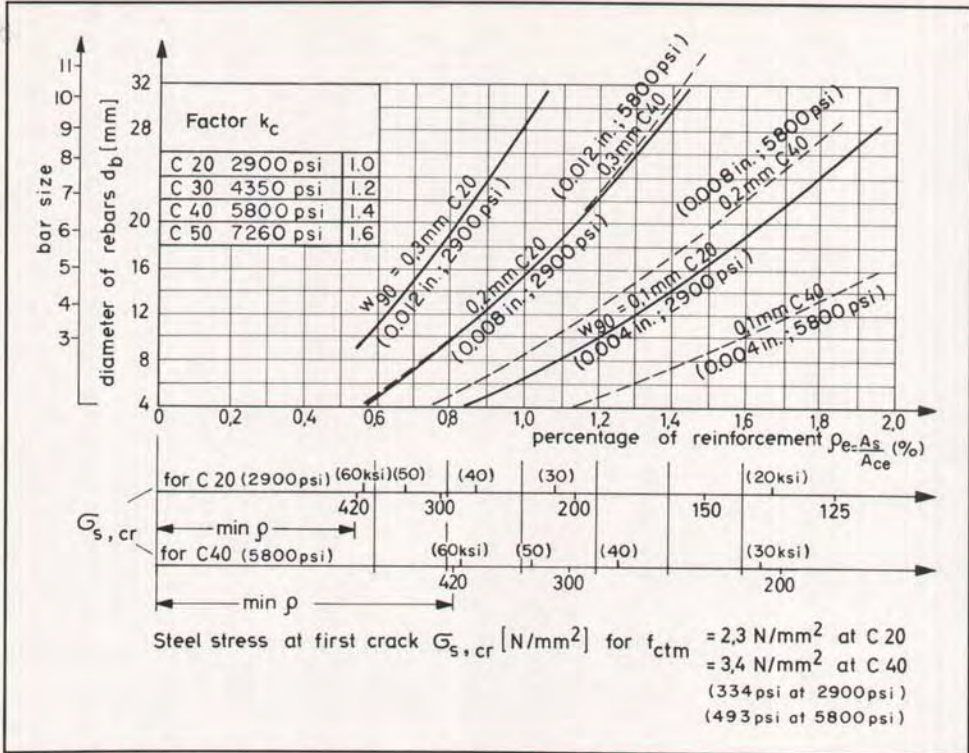


Fig. 14. Design chart ($\rho_e - d_b$ diagram) for determining direct tension.

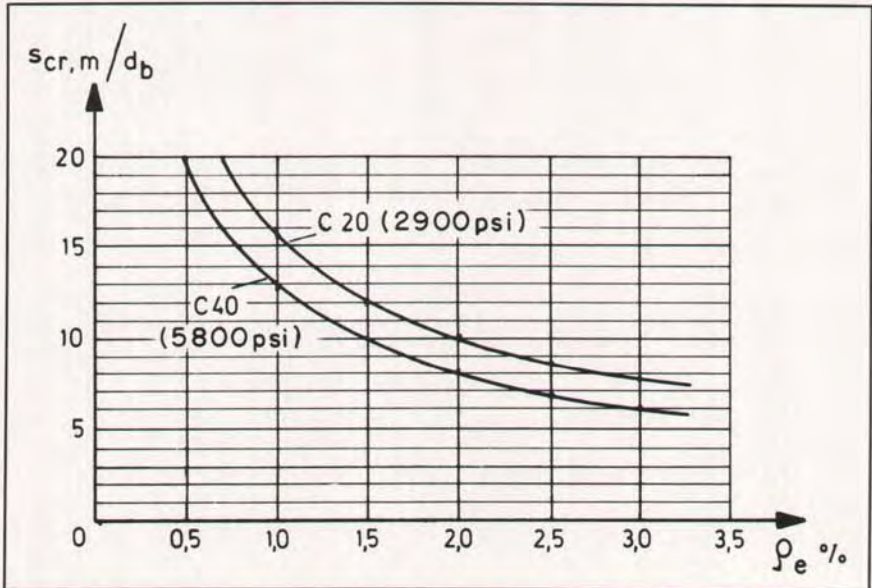


Fig. 15. Mean crack spacing for reinforced concrete member under direct tension.

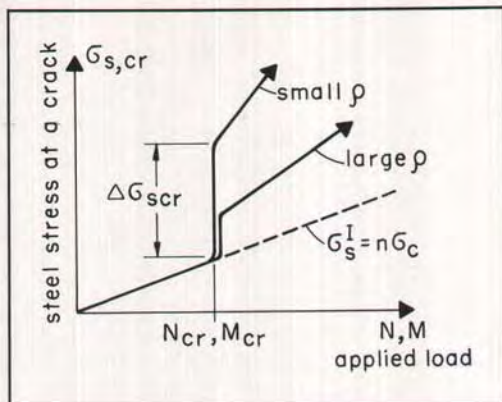


Fig. 16. Illustration of jump in steel stress at cracking for different reinforcement ratios.

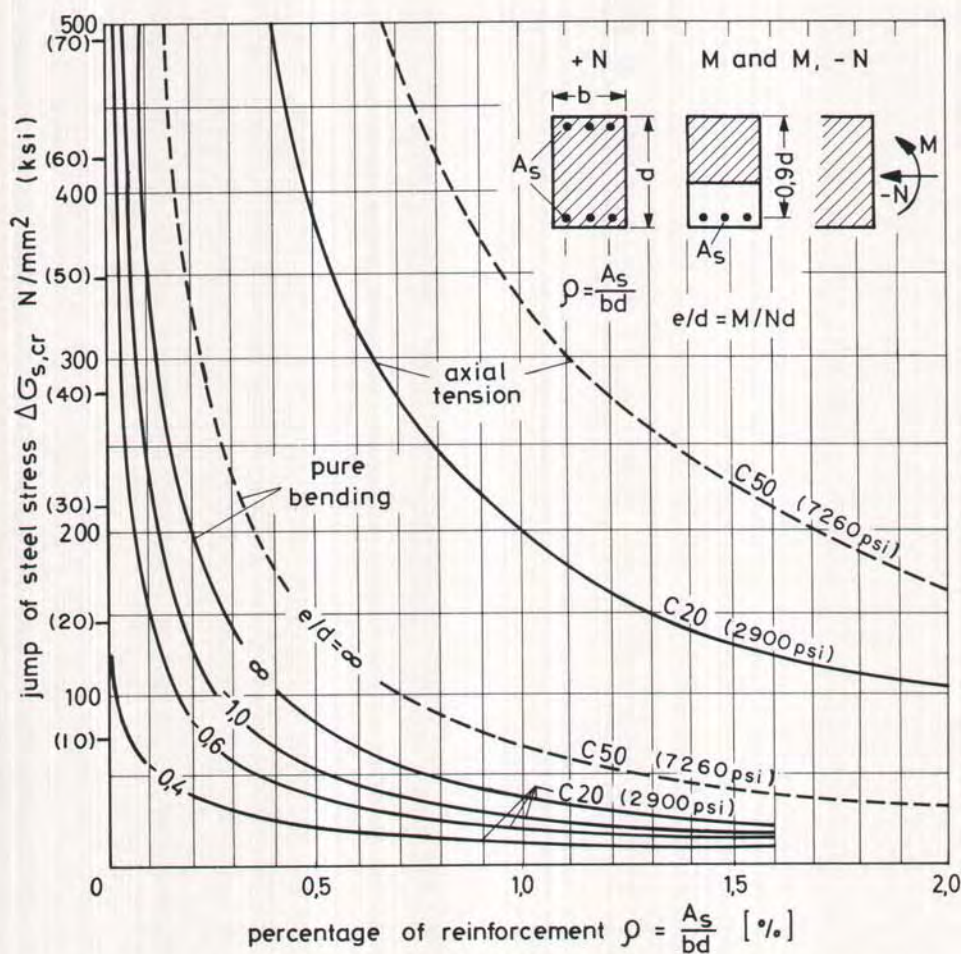


Fig. 17. Diagram to obtain increase of steel stress at cracking under tension, bending or bending with compression for varying eccentricity e/h of resultant for $f'_{tm} = 1.2 (f'_c)^{2/3}$.

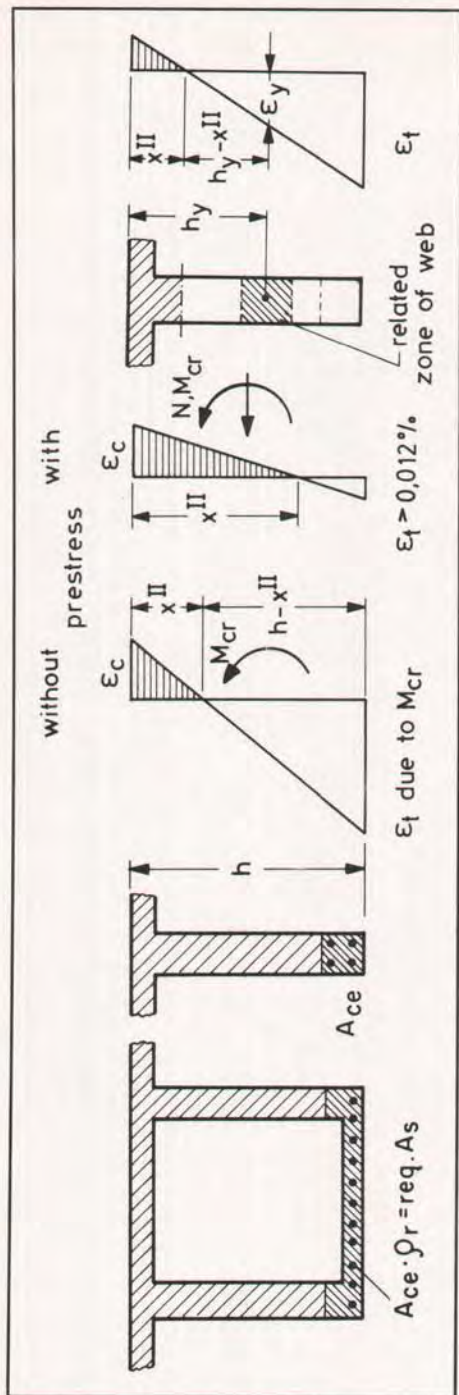


Fig. 18. Illustration of effective areas and associated strain distributions under cracking moment.

Fig. 14 along a line for $(w_{90} - \Delta w)$. Rough estimates are sufficient in practice.

Sizing Reinforcement for Bending and Combined Bending and Compression

Members subjected to bending plus compression require much less reinforcement for crack control than members under direct tension. This can be understood considering the sharp increase in steel stress that occurs at cracking. As indicated in Figs. 16 and 17, the increase in steel stress depends on concrete tensile strength, f'_t , reinforcement percentage, ρ , and type of stress distribution such as produced by direct tension, or combined bending and compression. It should be noted that in Fig. 17 the reinforcement ratio refers to the concrete area $A_c = bh$.

As can be seen from Fig. 17, there is a significant difference between the stress increase for tension compared to the increase for bending. For prestressed concrete structures, the stress increase is significantly reduced depending on the degree of prestressing. In Fig. 17 the values of $e/h = -1.0$ and -0.4 correspond to moderate and limited prestressing levels, respectively, $e/h = -0.17$ to full prestressing. Even moderate prestressing leads to low steel stresses at cracking and, therefore, small percentages are sufficient for crack control. For bending, and combined bending and compression, the diagram given in Fig. 14 can be used to obtain ρ_e applying the correction factor:

$$k_B = (h - x^{II})/h$$

where x^{II} is the depth of the neutral axis for State II and under the cracking moment considering axial loads from restraint and prestressing, and including reinforcement to satisfy strength requirements.

For sizing longitudinal crack control web reinforcement, the correction factor

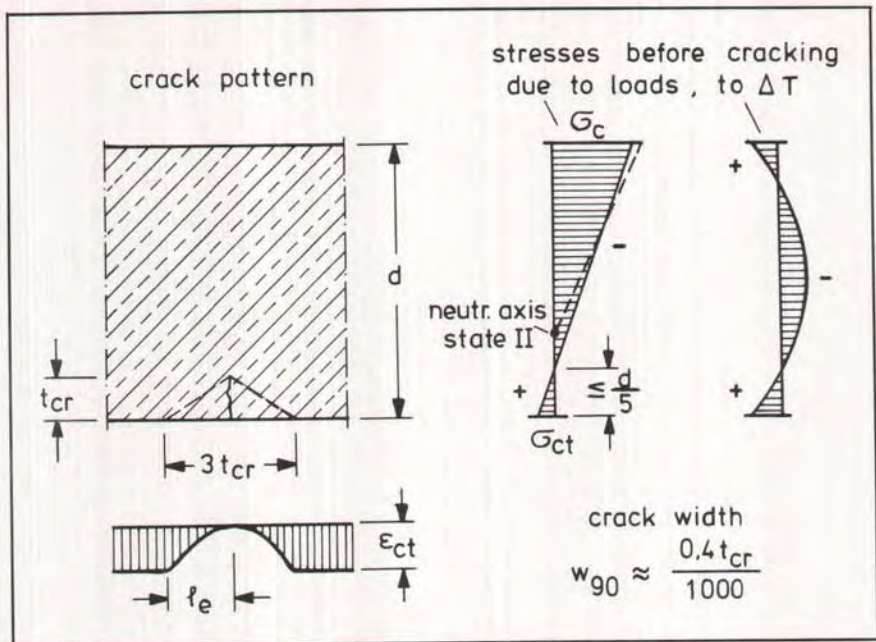


Fig. 19. Cracks and stresses in members without reinforcement.

becomes:

$$k_B = (h_y - x^{II})/h_y$$

where h_y is the depth to the reinforcement as shown in Fig. 18.

For crack control of members subjected to shearing or torsional stress, the equations given in Ref. 11 should be used.

Crack Control Without Reinforcement

In massive concrete structures or in moderately prestressed concrete structures subjected to tensile stresses due to temperature, as shown in Figs. 1 and 4, cracks that occur often remain as fine cracks having widths below w_{90} even without reinforcement. Larger cracks do not occur because the tensile strain in the concrete, ϵ_{ct} , is restrained by the adjoining zone under compression as shown in Fig. 19.

The width, w_{90} , of such cracks de-

pends on the possible depth t_{cr} of the crack and can be calculated from the maximum tensile strain of concrete $\epsilon_{ct,u} \leq 0.012$ percent with $k_4 = 1.6$. Thus:

$$w_{90} = 1.6 \cdot 2 t_{cr} \cdot \epsilon_{ct,u} \approx 0.4 (10^{-3}) t_{cr}$$

In a dry climate, shrinkage of the cracked zone should be taken as $\Delta \epsilon_{sh} \approx 0.01$ percent. Thus:

$$w_{90} = 1.6 \cdot 2 t_{cr} (\epsilon_{ct,u} + \Delta \epsilon_{sh}) \approx 0.6 (10^{-3}) t_{cr}$$

For $w_{90} = 0.004$ in. (0.1 mm), the depth of the cracks can be as large as 10 in. (25 cm). For restrained bending (e.g., unreinforced but moderately prestressed slabs or beams), the depth should remain below $t_{cr} \leq h/5$.

Recommendations for Size and Spacing of Reinforcement

Optimal crack control is obtained by choosing small bar diameters and a small spacing. Table 4 shows recom-

Table 4. Recommended upper limits for reinforcing bar spacing.

Allowable crack width, w_{90} (in.)	0.004	0.008	0.012
Direct tension	4	6	8
Bending ($\sigma_s^II = 36,000$ psi)*	4	6	8
Bending ($\sigma_s^II = 18,000$ psi)	6	8	12
Shear ($v_o = 290$ psi) Stirrups rectangular to axis of member	4	6	8
Shear ($v_o = 435$ psi) Stirrups rectangular to axis of member	2	4	6
Shear ($v_o = 435$ psi) Stirrups at 45 to 60 deg with axis of member	4	8	10
Torsion ($t_o > 290$ psi) Ties rectangular to axis of member	2	3	5
Torsion ($t_o > 290$ psi) Ties at 45 deg with axis of member	4	8	10

*The steel stress σ_s^II and the nominal vertical shear and nominal torsional stresses v_o and t_o , respectively, refer to the load specified for the serviceability limit state of crack control.
 Note: 1.0 in. = 2.54 cm; 1.0 psi = 6.89×10^{-3} N/mm².

mended upper limits for the spacing of reinforcing bars.

Minimum Reinforcement

Minimum reinforcement should meet the following conditions:

(a) **Satisfy strength requirements** — It is noted in passing that in most European codes not only a maximum concrete strain but also a maximum steel strain is specified for strength design.

(b) **Prevent sudden failure when cracking occurs** — This can occur when the force transferred from the concrete to the reinforcement is greater than the strength of the reinforcement. Cracking can be due to applied loads or restraint forces. The minimum amount of reinforcement can be calculated as follows:

Table 5. Minimum reinforcement percentage for $f_y = 60,000$ psi.

Concrete strength f'_c (psi)	3000	5000	Related area
Direct tension	0.76	1.08	$A_c = b h$
Bending	0.15	0.22	$A_c = b h$

Note: 1.0 psi = 6.8×10^{-3} N/mm².

a. Direct tension:

$$\rho_{min} = f'_{t95} / f_y = 2.7 (f'_c)^{2/3} / f_y$$

b. Bending:

$$\rho_{min} = 0.5 f'_{t95} / f_y$$

where ρ_{min} refers to the full cross-sectional area, $A_c = b h$ and f_y is the yield strength of the reinforcement. If cracking is caused by restraint forces due to temperature, creep, shrinkage or differential settlement, then the A_c can be limited to two to three times the effective area as indicated in Fig. 13. Table 5 presents minimum reinforcement requirements.

(c) **Satisfy serviceability requirements by controlling crack widths** — Minimum reinforcement should be used in all areas where in the cracked state concrete tensile stresses due to loads or restraint forces exceed f'_{ts} . In these areas, the minimum reinforcement can be obtained from Fig. 14 using the k_b factor according to Fig. 18.

ACKNOWLEDGMENT

The author wishes to express his appreciation to Dr. Walter Dilger and Eduardo A. B. Salse for reviewing the paper and for their helpful suggestions.

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APPENDIX A — NOTATION

<p>A_c = gross area of concrete cross section</p> <p>A_{ce} = effective area of concrete tensile zone surrounding reinforcing bar</p> <p>A_{ps} = area of prestressed reinforcement</p> <p>A_s = area of nonprestressed reinforcement</p> <p>b = width of member</p> <p>c = thickness of concrete cover</p> <p>d = effective depth of member</p> <p>d_b = bar diameter</p> <p>E_c = Young's modulus of concrete</p> <p>E_s = Young's modulus of steel</p> <p>e = eccentricity</p> <p>σ_t = tensile stress in concrete</p> <p>f'_{tp} = percentile, p, of direct tensile strength of concrete (percentile is that value of the quantity below or equal to which p percent of all measurements may be expected to fall)</p> <p>f'_t = strength of concrete in direct tension</p> <p>f'_{tm} = mean strength of concrete in direct tension</p> <p>σ_s = steel stress in cracked state</p> <p>σ_{scr} = steel stress at first crack immediately after cracking</p> <p>f_y = yield strength of steel reinforcement</p>	<p>h = total depth of member</p> <p>$k_1 \dots k_5$ = coefficients in crack width equations</p> <p>M = bending moment</p> <p>M_{cr} = cracking moment</p> <p>M_{DL} = moment due to dead load</p> <p>M_{LL} = moment due to live load</p> <p>N = normal force (positive if tension)</p> <p>n = modular ratio = E_s/E_c</p> <p>s = bar spacing</p> <p>s_{cr} = crack spacing</p> <p>s_{crm} = mean crack spacing</p> <p>T = temperature</p> <p>ΔT = temperature change</p> <p>w = crack width</p> <p>w_m = mean crack width</p> <p>w_{90} = 90 percentile of crack width</p> <p>x^{II} = depth of neutral axis in cracked sections</p> <p>α_T = coefficient of thermal expansion</p> <p>ϵ_{ct} = tensile strain in concrete</p> <p>ϵ_m = mean strain over an average length of cracked concrete</p> <p>ϵ_s^{II} = steel strain in cracked stage (neglecting tension stiffening)</p> <p>$\Delta \epsilon_s$ = reduction of steel strain by tension stiffening</p> <p>ρ = reinforcement ratio</p> <p>ρ_e = A_s/A_{ce} = reinforcement ratio in effective tension zone of concrete</p> <p>ρ_{min} = minimum reinforcement ratio</p>
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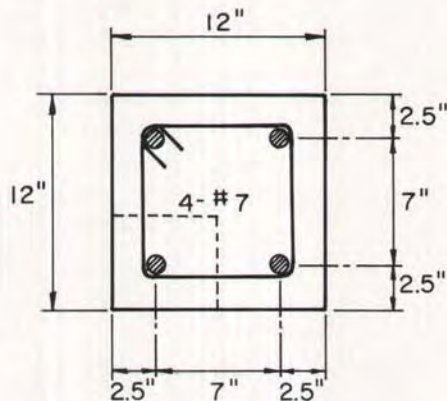
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NOTE: Discussion of this article is invited. Please submit your comments to PCI Headquarters by April 1, 1989.

APPENDIX B — DESIGN EXAMPLE

Given: A 12 in. (30.5 cm) square reinforced concrete section in direct tension.

$f'_c = 5800$ psi (40 N/mm²)
 $f_y = 60,000$ psi (413 N/mm²)
 $N = 100$ kips (445 N)
 Cover, $c = 2$ in. (5 cm)
 $d_b = 0.875$ in. (22 mm)



Required: Check the bar diameter to limit crack width at crack formation to 0.012 in. (0.305 mm).

Solution: Fig. 14 provides a chart relating bar diameter and effective reinforcement ratio for various limiting crack widths and concrete strengths.

Calculate effective area, A_{ce} :

$$A_{ce} = 6 \times 6 = 36 \text{ in.}^2 (232 \text{ cm}^2)$$

$$\rho_e = 0.6/36 = 0.0167 \text{ or } 1.67 \text{ percent}$$

Entering the chart in Fig. 14 with $\rho_e = 1.67$ percent and No. 7 bar size, a crack width, w_{90} , of approximately 0.007 in.

(0.178 mm) is obtained. Since this value is less than the 0.012 in. (0.305 mm) required crack width limit in this example, the No. 7 bars provided are considered adequate for crack width control in this case. Use of the chart is illustrated below.

From Fig. 14, a designer can also determine that the steel stress at cracking is approximately 30 ksi (207 N/mm²).

