Stress-Strain Modeling of 270 ksi Low-Relaxation Prestressing Strands

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A stress-strain relationship for Grade 270 low-relaxation prestressing strands is presented. It is based on recent extensive testing by the authors requested by the PCI Industry Handbook Committee. The testing has resulted in refined constants of the previously developed power formula which has been shown in several studies to predict prestressing steel stress for a given strain to within 1 percent error of any prescribed experimental value. Tables and stress-strain graphs for other common types of prestressing steel are reproduced here, from an earlier study, for the convenience of readers.

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Skogman et al.[1] presented a formula for predicting the stress-strain relationship for any type of prestressing steel, the so-called "power formula." Its general form is:

$$f_{ps} = \varepsilon_{ps} E \left[Q + \frac{1-Q}{\left\{ 1 + \left(\frac{E\varepsilon_{ps}}{Kf_{py}} \right)^{R} \right\}^{1/R}} \right] \tag{1}$$

where f_{ps} is the stress corresponding to a given strain ε_{ps}; E, Q, K and R are curve fitting constants, and f_{py} is the stress at 1 percent strain.

The stress f_{py} may be taken from experimental results or to comply with ASTM minimum standards. For example, ASTM A-416[2] specifies that minimum f_{py} for Grade 270, low-relaxation steel be equal to 0.9 of the breaking stress, i.e., 0.9(270) = 243 ksi (1676 MPa).

A simple procedure for calculating the four power formula constants to provide a close fit of the power formula to a prescribed experimental curve is given by Mattock.[3] He showed that the theoretical curve can be made to produce stresses within 1 percent of the prescribed values. Other independent studies by Skogman et al.,[1] Naaman,[4] Harajli and Naaman,[5] and Menegotto and Pinto[6] have confirmed the great accuracy and versatility of the power formula.

As part of the work of Skogman et al.,[1] constants for the commonly used types of prestressing steel were developed. These constants were based on a number of actual stress-strain curves supplied by steel manufacturers. Use of the so-called "typical" curves was avoided since they generally were supplied as theoretical curves coinciding with ASTM minimums.

Due to the scarcity of the available actual curves, the constants developed in Ref. 1 were based on conservative assumptions. The prediction curves were in some instances significantly lower than the lowest available experimental curves.

The PCI Industry Handbook Committee requested that the University of Nebraska conduct additional experimental work to serve two purposes:

(a) Provide for a larger population of experimental stress-strain curves and thus more viable statistical lower bound analysis.

(b) Investigate the influence of the variability of stress-strain curves supplied by manufacturers, which were derived from tests conducted on different types of steel and various testing machines by different operators.

Twenty-eight strands supplied directly by precast concrete producers, from four different strand manufacturing sources, were tested. All strands were Grade 270 low-relaxation strands as this type appeared to be the most widely used by producers in the United States. The number of specimens representing other grades was not large enough to allow useful statistical analysis. No attempt was made to obtain specimens directly from strand manufacturers. Producer-supplied specimens were believed to be a more accurate representation of the steel actually used in concrete products.

The results of the 28 tests conducted at Nebraska were compared with those of 28 others conducted by manufacturers. Almost equal levels of stresses were observed in the Nebraska tests. The 56 curves were combined and a statistical lower bound curve was derived. The power formula constants were then developed such that the prediction curve would meet two requirements: (a) as close a fit as possible to the experimental lower bound, and (b) predicted stress at 1 percent strain is equal to the ASTM minimum $f_{py} = 0.9 f_{pu} = 243$ ksi (1676 MPa).

An upper limit was also placed on f_{ps} equal to ASTM minimum specified $f_{pu} = 270$ ksi (1862 MPa). Both ASTM f_{py} and f_{pu} values were found to be significantly lower than the experimental lower bound values. Thus, a reassessment of the ASTM A-416 specification may result in an upward revision of these minimums. However, until such revision is made, the authors recommend that the current minimums not be exceeded. This recommendation is not followed in the current PCI Design Handbook formula,[7] which predicts

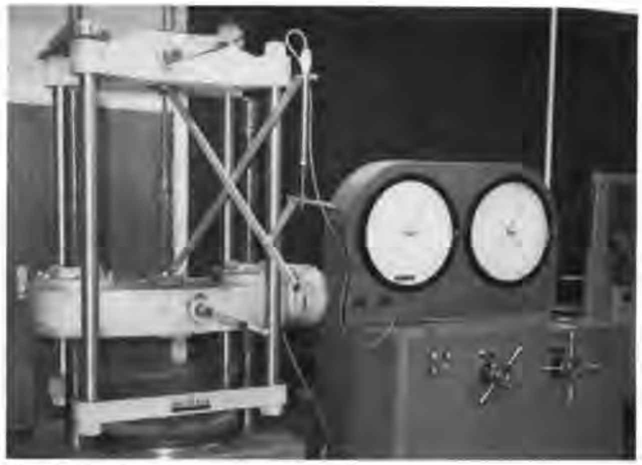

Fig. 1. Testing setup with a failed specimen.

$f_{py} = 0.92 f_{pu}$. Additional comparisons between the proposed prediction formula and other equations are made in a separate section of this paper.

TESTING SETUP

A Tinius Olsen testing machine was used. An LVDT was connected to the specimen to measure elongation using an aluminum bracket system (see Fig. 1). Aluminum angles were placed between the strand and the machine grips to prevent the grip threads from "biting" on the strand wires and causing premature stress concentration failure.[8] This arrangement proved satisfactory and strand breakage took place away from the grips, with the classical wire "necking" before breakage. The specimen length, loading rate and other details were in accordance with ASTM.[9]

DISCUSSION OF TEST RESULTS

Fig. 2 shows plots of the test results supplied by strand manufacturers. It represents 14 specimens from Deriver, seven from Springfield Industries, three from Sumiden Wire Products Corporation, two from Florida Wire and Cable Company, one from Shinko Wire America Inc., and one from ARMCO. Most of the stress-strain relationship data were given in detail up to a 1.5 percent strain. The breaking load was given for all specimens with ultimate elongation.

Fig. 3 shows the results of the 28 tests conducted by the authors. The testing setup produced full stress-strain diagrams and no extrapolation had to be made. The specimens obtained from various precast concrete producers were traced back to the following sources: eight from Florida Wire and Cable Company, three from Union Wire Rope, nine from Shinko Wire America Inc., and eight from American Spring Wire Corp.

The results in Fig. 3 indicate a yield strength much higher than the ASTM value of $0.9 f_{pu}$. Also, the modulus of elasticity was higher than the current typical value of 28,000 ksi (193,060 MPa). Hence, a value of 28,500 ksi (196,508 MPa) was used to derive the proposed curve. More details of the test results are provided in Ref. 10 of this paper.

The two groups of tests are combined in Fig. 4. The results submitted by manufacturers appear to be consistent

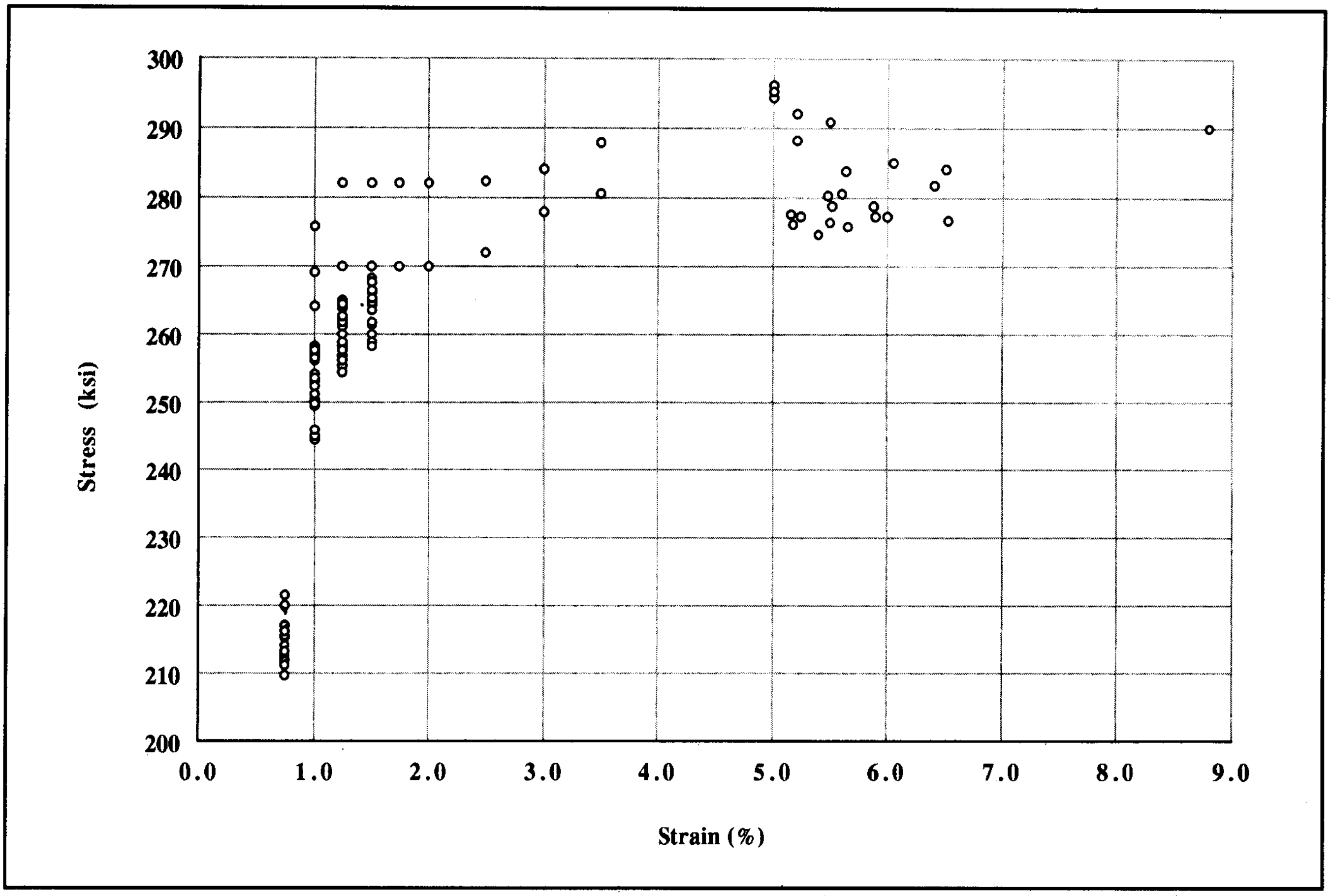

Fig. 2. Test data supplied by manufacturers (28 curves).

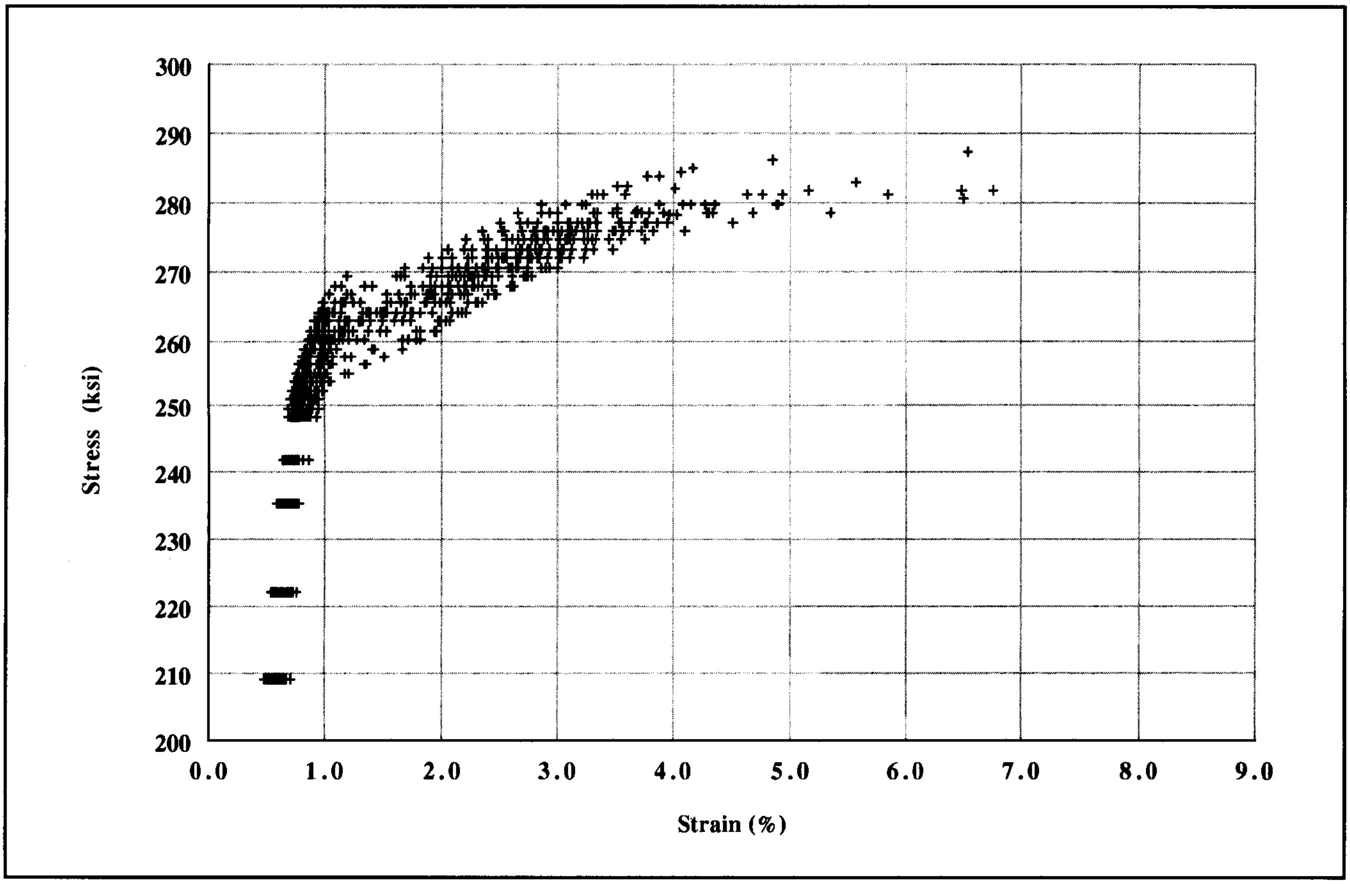

Fig. 3. Test results obtained by authors (28 curves).

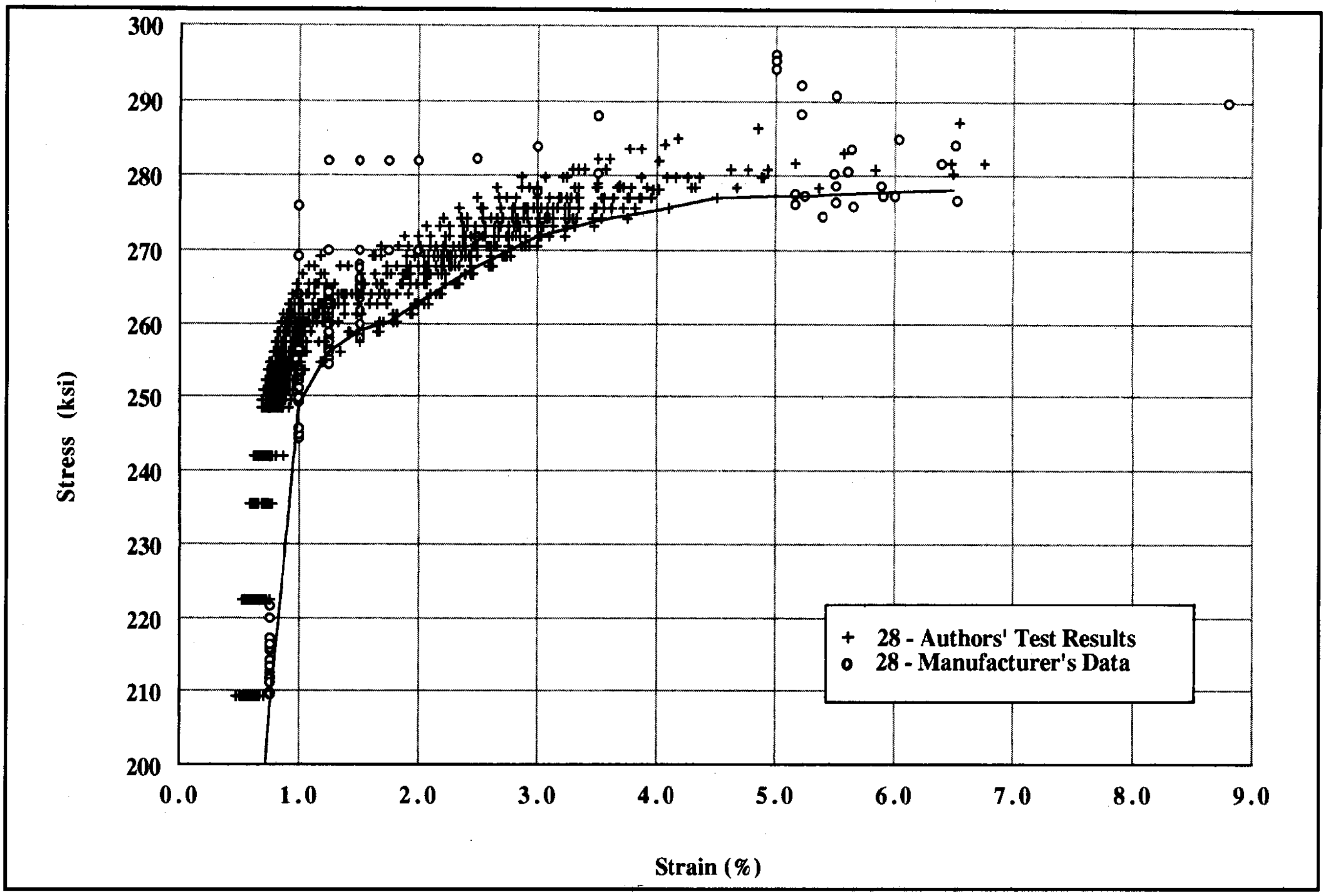

Fig. 4. Lower bound of 56 actual curves.

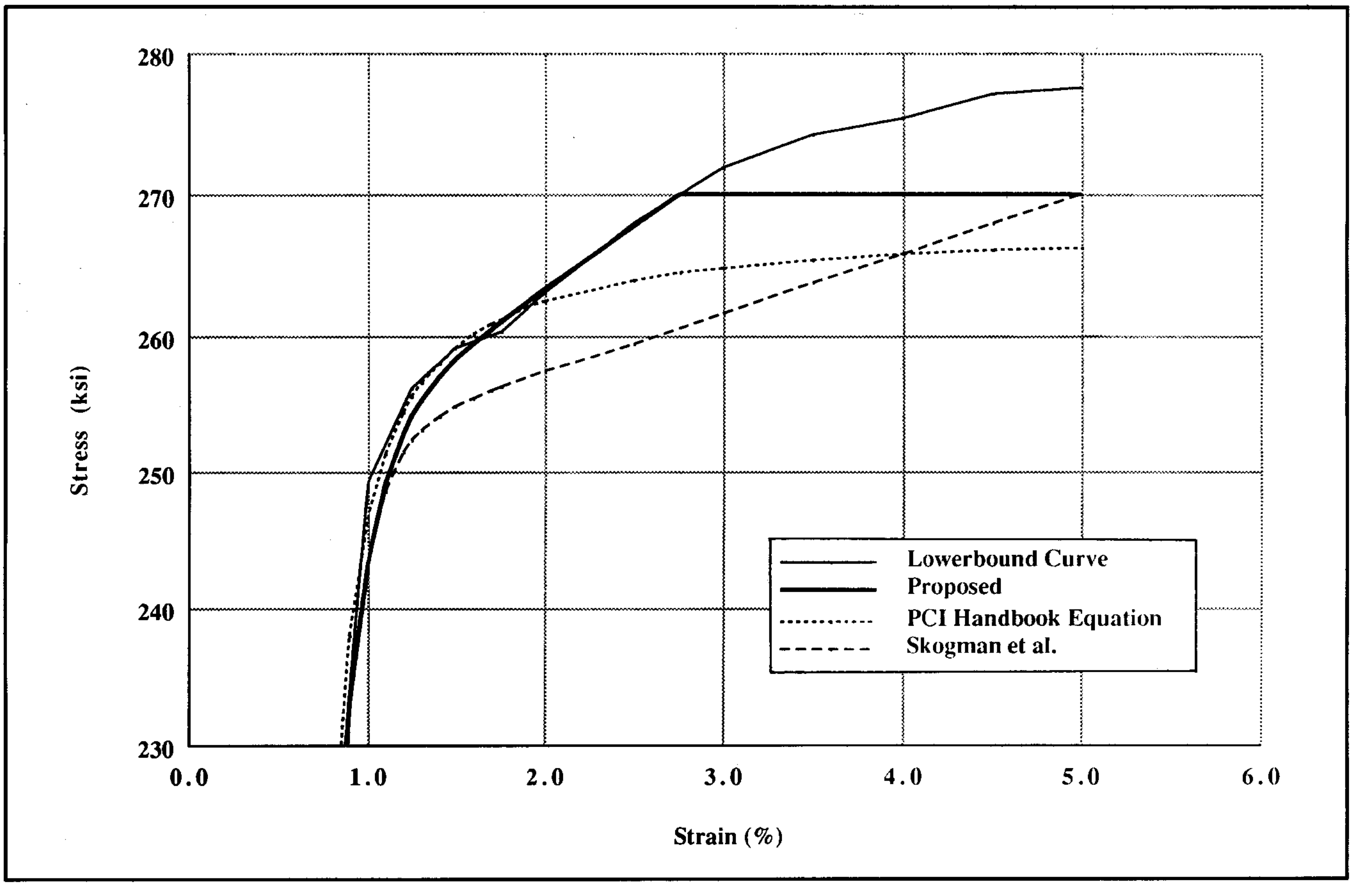

Fig. 5. Comparison of lower bound with other prediction curves.

Table 1. Power formula constants for various steel types.

Steel type	f_{py}/f_{pu}*	A	B	C	D
270 ksi strand	0.90†	887	27613	112.4	7.360
	0.85	756	27244	117.3	6.598
250 ksi strand	0.90	384	27616	119.7	6.430
	0.85	689	27311	126.7	5.305
250 ksi wire	0.90	435	28565	125.1	6.351
	0.85	734	28266	132.5	5.256
235 ksi wire	0.90	403	28597	133.1	5.463
	0.85	682	28318	141.0	4.612
150 ksi bar	0.85	467	28533	225.2	4.991
	0.80	629	28371	239.3	4.224

* ASTM minimum specification.
† Proposed curve; E = 28,500 ksi (196,508 MPa).

with those developed by the authors. The only exception is one curve exhibiting unusually high stresses. Otherwise, the band of stress range is relatively narrow, indicating a high degree of confidence in accurately predicting stresses. Fig. 4 also shows the statistical lower bound with a 99 percent confidence level.[11] This level statistically assures that with a chosen probability of 0.95, all test data lie above the lower bound value.

PROPOSED PREDICTION FORMULA CONSTANTS

A simplified form of Eq. (1) is:

$$f_{ps} = \varepsilon_{ps}\left[A + \frac{B}{\left\{1+\left(C\varepsilon_{ps}\right)^{D}\right\}^{1/D}}\right] \leq f_{pu} \qquad (2)$$

The constants A, B, C and D for the proposed 270 ksi low-relaxation formula were found to be equal to 887, 27613, 112.4 and 7.360, respectively. These constants were obtained by fitting the power formula to the lower bound curve of Fig. 5. The following constraints were imposed: stress at 1 percent strain equal to 243 ksi (1676 MPa) and maximum stress equal to 270 ksi (1862 MPa).

The procedure outlined in Appendix B shows how to determine the power formula constants to accurately predict the stress-strain relationship for any given experimental curve. These constants, along with constants for other steel grades which were earlier developed by Skogman et al.,[1] are given in Table 1. It should be noted that the large number of significant digits presented here is advisable to be used because the values of f_{ps} are sensitive to these constants.

Table 2 represents the stress at various strain levels for each steel shown in Table 1. Table 2 could thus be conveniently used as a design aid for designers who do not wish to substitute their parameters into the power formula.

CALCULATION EXAMPLE

Required:

Calculate the steel stress at ultimate flexure f_{ps} for the hollow-core section 4HC8 shown in Fig. 6 using Eq. (2) and compare the results with those of Table 2.

Given:

Concrete: f'_c = 5000 psi (34.5 MPa)
Prestressing steel: Use six ½ in. (12.7 mm) diameter Grade 270 ksi (1862 MPa), low-relaxation strands
A_{ps} = 0.918 in.^{2} (592 mm^2)
Effective prestress f_{se} = 162 ksi (1117 MPa)

Solution:

Using the iterative strain compatibility method, such as

Table 2. Tendon steel stress-strain relationship (strain is in in./in.; stress is in ksi).

Steel type	270 ksi strand f_{py}/f_{pu}		250 ksi strand f_{py}/f_{pu}		250 ksi wire f_{py}/f_{pu}		235 ksi wire f_{py}/f_{pu}		150 ksi bar f_{py}/f_{pu}	
ε_{ps}	0.9	0.85	0.9	0.85	0.9	0.85	0.9	0.85	0.85	0.80
0.0000	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.0070	195.4	189.2	187.8	181.3	192.0	184.5	184.9	176.4	127.5	120.0
0.0080	217.0	207.9	205.5	195.9	208.3	197.9	198.0	187.5	129.1	121.9
0.0090	232.8	221.1	217.5	205.9	218.8	206.8	206.4	194.8	130.2	123.2
0.0100	243.0	229.5	225.0	212.5	225.0	212.5	211.5	199.7	130.9	124.2
0.0125	254.1	239.0	232.9	220.8	231.7	219.8	217.5	206.3	132.4	126.2
0.0150	258.3	242.8	235.7	224.6	234.2	223.3	220.0	209.7	133.7	127.9
0.0175	261.0	245.2	237.1	227.0	235.7	225.7	221.5	212.1	134.9	129.5
0.0200	263.3	247.3	238.3	229.0	236.9	227.8	222.7	214.1	136.0	131.1
0.0225	265.6	249.2	239.3	230.9	238.1	229.7	223.8	216.0	137.2	132.7
0.0250	267.8	251.1	240.3	232.7	239.2	231.6	224.9	217.8	138.4	134.3
0.0275	270.0	253.0	241.3	234.5	240.3	233.5	225.9	219.5	139.5	135.9
0.0300	270.0	254.9	242.2	236.2	241.4	235.3	226.9	221.2	140.7	137.4
0.0350	270.0	258.7	244.2	239.7	243.6	239.0	229.0	224.7	143.1	140.6
0.0400	270.0	262.5	246.1	243.1	245.7	242.7	231.0	228.1	145.4	143.7
0.0450	270.0	266.3	248.0	246.6	247.9	246.4	233.0	231.5	147.7	146.9
0.0500	270.0	270.0	250.0	250.0	250.0	250.0	235.0	235.0	150.0	150.0

Note: 1 ksi = 6.895 MPa.

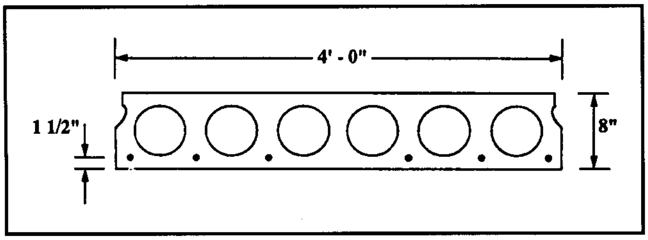

Fig. 6. Precast concrete hollow-core slab for calculation example.

that given in Ref. 1, $\varepsilon_{ps} = 0.017$. Details are not shown here for brevity.

1. Using Eq. (2), substitute the value of ε_{ps} and the constants A, B, C and D from Table 1 that represent low-relaxation strand:

$$f_{ps} = 0.017\left[887 + \frac{27{,}613}{\left\{1+\left(112.4(0.017)\right)^{7.360}\right\}^{1/7.360}}\right] \le 270$$

$$= 260.5 \text{ ksi (1796 MPa)}$$

2. Interpolating values from Table 2, f_{ps} = 260.6 ksi (1797 MPa).

CONCLUSIONS

Examination of Fig. 5 reveals the following observations:

1. The proposed curve is extremely close to the experimental lower bound curve. Yet, it satisfies the current ASTM A-416 minimums.

2. The proposed curve will give higher stresses than that of Skogman et al. It also gives higher stresses than the PCI Design Handbook formula for strains in excess of about 2 percent, which is common in double tees and hollow-core slabs.

3. ASTM minimums are considerably lower than the experimental lower bound values.

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APPENDIX A — NOTATION

f_{ps} = stress in prestressed reinforcement at ultimate flexure

f_{pu} = specified tensile strength of prestressing tendons

f_{py} = specified yield strength of prestressing tendons

$E, K, Q, R/A, B, C, D$ = constants used in power formula

ε_{ps} = strain in prestressed tendon reinforcement at ultimate flexure

ε_{pu} = ultimate strain in prestressing tendon

APPENDIX B — PROCEDURE TO DETERMINE POWER FORMULA CONSTANTS

For the convenience of readers, the procedure to calculate the power formula constants for prestressing steel is outlined here. The basic procedure is taken from Ref. 3. This procedure can be used for any type of prestressing steel. The procedure is also helpful to readers who wish to derive their own power formula constants to fit a given experimental curve. Fig. B shows the given experimental stress-strain curve for prestressing steel. It is required to represent this curve with the power formula.

1. Determine the modulus of elasticity of the steel, E, which is given by the slope of the first linear part of the curve.

2. Produce the two linear parts of the stress-strain curve until they meet. If the upper portion of the curve is not a straight line, use the closest straight line. The stress corresponding to the point of intersection is f_{so}. If the complete curve is not available, a reasonable value of f_{so} = 1.04 (f_{py}) can be assumed in the case of seven-wire strand.

3. Calculate the constant C using the relation: $C = E/f_{so}$

4. Determine the constant A which is given by:

$$A = E\left(\frac{f_{pu} - f_{so}}{\varepsilon_{pu}E - f_{so}}\right)$$

5. Calculate the constant B from the equation: $B = (E - A)$

6. Finally, the constant D is determined solving the power formula with the stress equal to f_{py} at strain equal to ε_{py} = 0.01. This is a trial and error procedure.

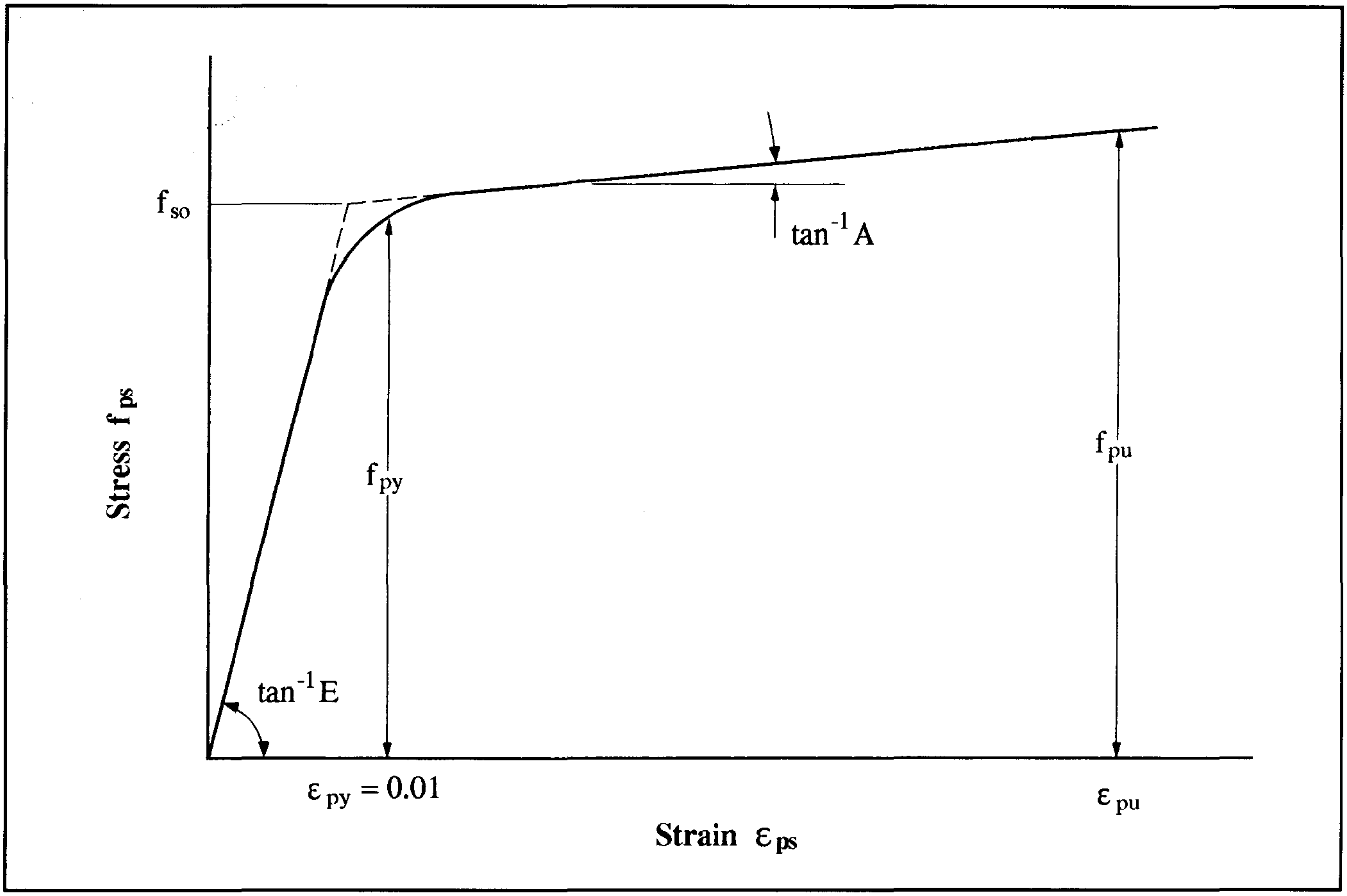

Fig. B. Typical stress-strain curve for prestressing steel represented by Eq. (2).