Impact of LRFD Specification on Load Distribution of Prestressed Concrete Bridges

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The live load fraction carried by a beam (girder) in a bridge superstructure is usually determined using the simplified equations from current AASHTO standards. Many of these formulas are known to be conservative. In 1993, the AASHTO Subcommittee on Bridges approved a substantial change in the way future bridges will be designed. The subcommittee adopted the LRFD Specification, which drastically affects the loading and procedures with which live loads are distributed to various beams. The new specification offers two valid approaches: the simplified method and a variety of approved refined procedures. This paper examines the impact of the LRFD Specification on the design of prestressed concrete I-beams and spread-box beams in rectangular bridge superstructures. Significant reductions in the distribution factor for interior beams are found in both the simplified and refined procedures.

When the current AASHTO (American Association of State Highway and Transportation Officials) simplified method[1] of lateral load distribution is used, the resulting load share that is carried by a beam* is usually conservative. Furthermore, most state departments of transportation do not allow the use of multi-lane reduction factors (for three or more design lanes) unless refined methods of analysis are used. In the AASHTO procedure, multi-lane reduction factors are 0.90 for three loaded lanes and 0.75 for four (or more) lanes.

In the LRFD (Load and Resistance Factor Design) Specification,[2] the multiple presence factor, m, for three loaded lanes is 0.85, while m is reduced to 0.65 for four (or more) lanes. In using the multi-lane reduction factors, the extreme live load force effect is determined by considering each possible combination of several loaded lanes multiplied by the corresponding factor m.

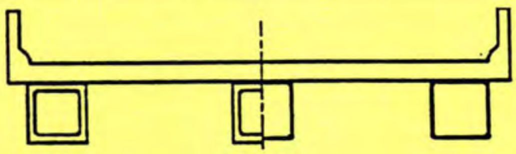
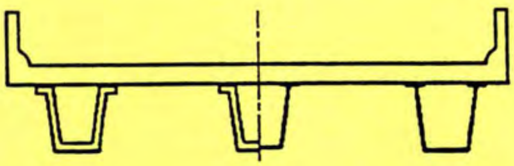
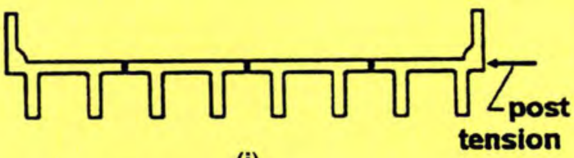
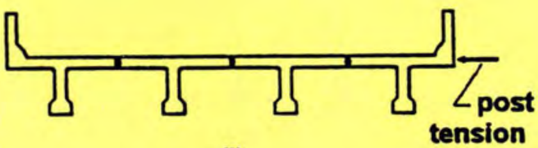
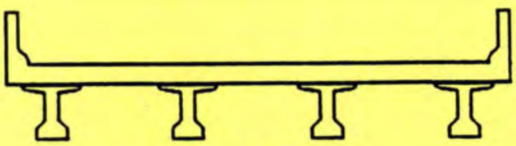
AASHTO standards[1] require that ex-

* Typically, long-span members used in bridges are referred to as "girders," but for consistency in notation the term "beam" is used throughout this paper.

Table 1. Distribution of live loads per lane for moment in interior beams, for use with U.S. customary units (Ref. 2).

Type of beam	Applicable cross section from Table 4.6.2.2.1-1	Distribution factors	Range of applicability
Concrete deck on concrete spread-box beams	b, c	One design lane loaded: $\left(\frac{S}{3.0}\right)^{0.35}\left(\frac{S\cdot d}{12.0L^2}\right)^{0.25}$ Two or more design lanes loaded: $\left(\frac{S}{6.3}\right)^{0.6}\left(\frac{S\cdot d}{12.0L^2}\right)^{0.125}$	$6.0 \le S \le 11.5$ $20 \le L \le 140$ $18 \le d \le 65$ $N_b \ge 3$
Concrete deck, filled grid, or partially filled grid on steel or concrete beams; Concrete T-beams, T- and double T-sections	k	One design lane loaded: $0.06+\left(\frac{S}{14}\right)^{0.4}\left(\frac{S}{L}\right)^{0.3}\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1}$	$3.5 \le S \le 16.0$ $4.5 \le t_s \le 12.0$ $20 \le L \le 240$ $N_b \ge 4$
	i, j If sufficiently connected to act as a unit	Two or more design lanes loaded: $0.075+\left(\frac{S}{9.5}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{12.0Lt_s^3}\right)^{0.1}$	

Table 2. Common deck superstructures (Ref. 2).

Supporting components	Type of deck	Typical cross section
Closed steel or precast concrete boxes	Cast-in-place concrete slab	 (b)
Open steel or precast concrete boxes	Cast-in-place concrete slab, precast concrete deck slab	 (c)
Precast concrete double tee section with shear keys and with or without transverse post-tensioning	Integral concrete	 (i)
Precast concrete tee section with shear keys and with or without transverse post-tensioning	Integral concrete	 (j)
Precast concrete I- or bulb tee sections	Cast-in-place concrete, precast concrete	 (k)

terior stringers (beams) have at least the carrying capacity of an interior stringer. In practice, most designs for prestressed concrete I-beams and spread-box superstructures are controlled by the interior beams. This paper will, therefore, concentrate on these types of members.

The NCHRP (National Cooperative Highway Research Program) Project 12-26 report,[3] developed by Imbsen & Associates, that was adopted by the LRFD Specification was designed to be a complete overhaul of AASHTO's distribution factors. Its proposed formulas were checked using finite element analyses and similar refined approaches and are, therefore, considered quite reliable.

The distribution factor (D.F.) is generally expressed by an exponential formula. As an example, for adjacent multi-beam decks using shear keys between the boxes, the distribution factor for bridges with two or more lanes is given by the following (simplified) formula:

$$D.F. = k_1 (b/305)^{0.6} (b/12L)^{0.2} (I/J)^{0.06} \quad (1)$$

where

b = width of prestressed concrete beam (in.)
L = span length (ft)
N_b = number of longitudinal beams
I, J = moment of inertia and St. Venant torsion constant, respectively
$k_1 = 2.5N_b^{-0.2} \geq 1.5$

Although the LRFD's simplified formulas are quite reliable and generally on the conservative side, they are not considered a part of the refined methods.

To attain the most efficient design, refined methods of analysis are needed. The use of three-dimensional finite element methods and two-dimensional grillage analogy are allowed by the LRFD Specification, as long as some general guidelines are used. The specification also allows the series-harmonic method, as commonly used by the Ontario Highway Bridge Design Code.[4]

The advantages of using refined methods include:

1. The analysis usually yields smaller distribution factors for live loads (so-called Strength I and Service III combination).

2. The LRFD Specification allows a multi-lane reduction factor for three or more loaded lanes. As stated earlier, for three loaded lanes, the reduction is 15 percent (Strength I and Service III combination). The reduction applies only if three loaded lanes produce larger stresses (after reduction) than two loaded lanes.

3. Additional savings are obtained, in some cases, when analysis for permit loads is required.

4. The final solution is more economical and may require a shallower section and/or a reduced number of strands.

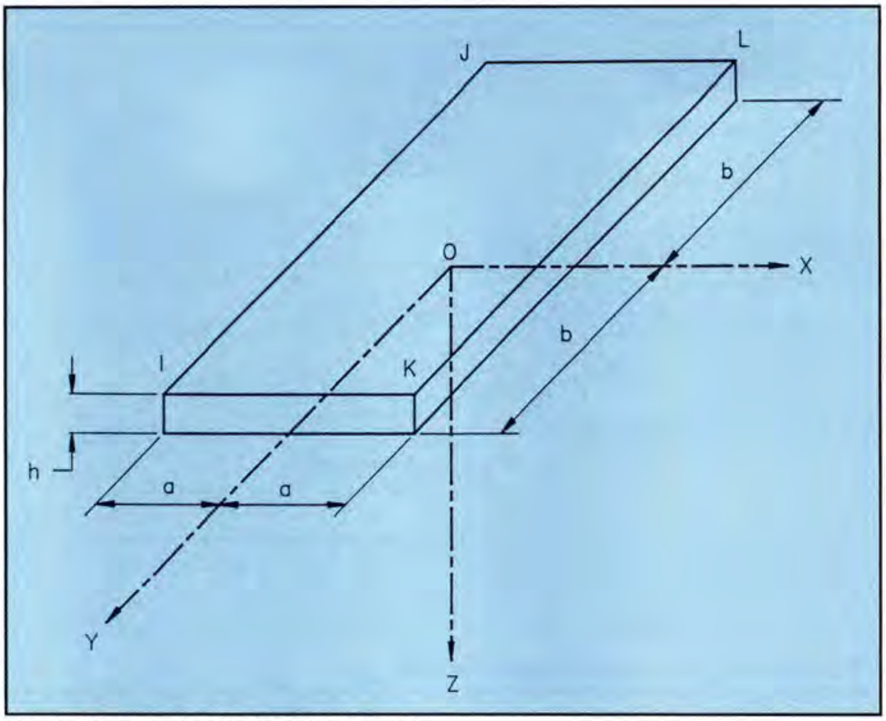


Fig. 1. Rectangular shell (slab) element.

OBJECTIVES

The objectives of this paper include:

1. A concise introduction to the AASHTO and LRFD simplified formulas for moment distribution factors pertaining to prestressed concrete I-beams and spread-box superstructures.

2. Explanation of the finite element modeling of ten selected superstructures and the derivation of related distribution factors by this refined method.

3. A comprehensive comparison of the current AASHTO procedures,* the LRFD simplified approach, and the refined method for selected cases.

* The term "current" refers to the presently used AASHTO Specification (1989/1992 versions), not to the new LFRD Specification (issued in August 1994).

CURRENT SIMPLIFIED AASHTO FACTORS

Moment Distribution to Interior Beams for Beam-and-Slab Bridges

The AASHTO formula for moment distribution, in cases of multi-lane loading, is given by $S/11$ (per lane) for prestressed concrete beam bridges with spacing, S, up to 14 ft (4.3 m). When the beam spacing is larger than 14 ft (4.3 m) — a rare occurrence — simple beam distribution can be used to calculate the distribution factors. A multiple lane reduction factor is not built into the AASHTO equation.

Moment Distribution to Interior Beams for Spread Box Bridges

Research at Lehigh University[5] in the late 1960s led to the following formula for load distribution due to multi-lane loading (per lane):

$$D.F. = (N_L/N_b) + 0.5k_2(S/L) \quad (2)$$

where

N_L = number of design lanes
N_b = number of beams
S = average beam spacing
L = span length

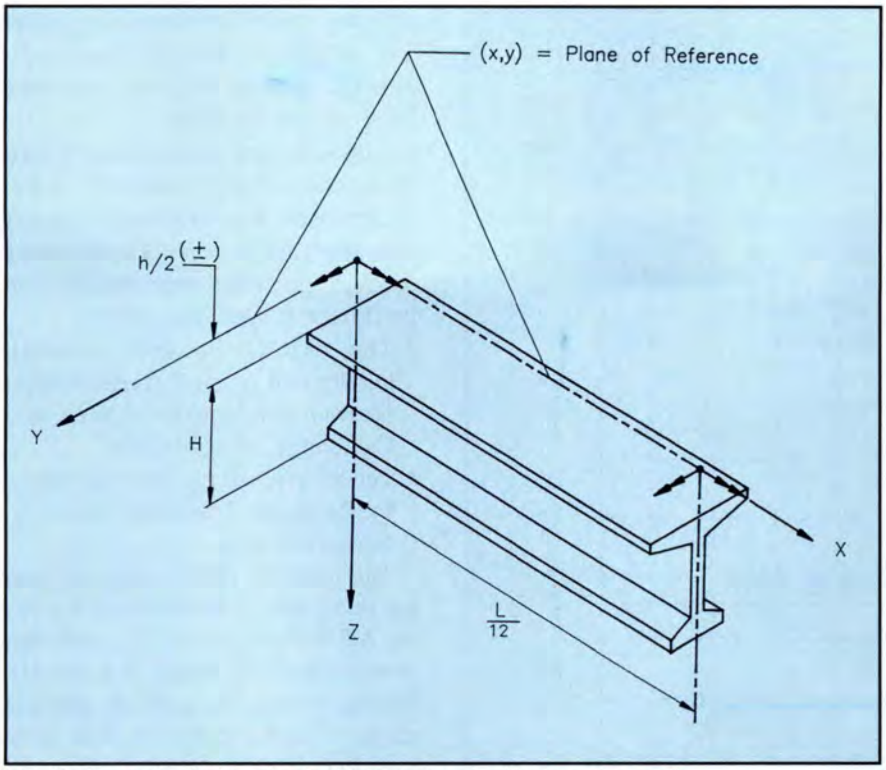


Fig. 2. Eccentrically attached stiffener element ("beam element").

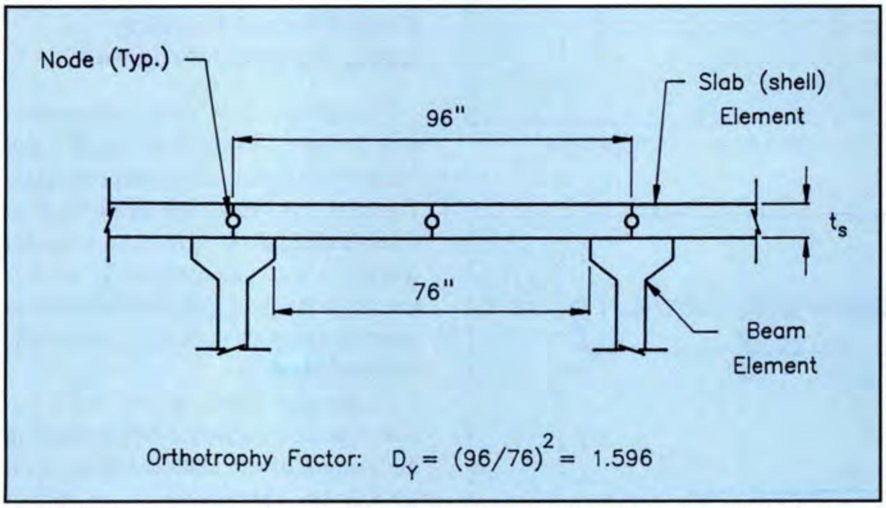


Fig. 3. Two-plate mesh discretization example and orthotropy factor (Ref. 8).

$k_2 = 0.07W_c - N_L(0.1N_L - 0.26) - 0.2N_b - 0.12$, in which W_c = roadway width between curbs (ft)

SIMPLIFIED METHOD IN LRFD SPECIFICATION

The simplified formulas for lateral distribution of live loads (per lane) for moment in interior beams are given in Table 1. The applicable cross sections are shown in Table 2. Multiple lane reduction factors are built into the formulas.

The following mathematical symbols are used in Table 1:

A = area of stringer, beam, or girder (sq in.)
b = width of beam (in.)
d = depth of girder or stringer (in.)
I = moment of inertia of basic beam (in.^{4})
J = St. Venant's torsional constant (in.^{4})
K_g = longitudinal stiffness parameter
L = span of beam (ft)
N_b = number of beams, stringers, or girders
N_L = number of specified design lanes
S = spacing of beam or webs (ft)
t_s = depth (thickness) of concrete slab (in.)

The longitudinal stiffness parameter, K_g, is taken as:

$$K_g = n(I + Ae_g^2) \qquad (3)$$

where

n = modular ratio between beam and deck materials
e_g = distance between centers of gravity of basic beam and deck (in.)

For closed thin-walled shapes (such as box sections):

$$J \approx 2A_o^2/\Sigma(s/t) \qquad (4)$$

where

t = thickness of plate-like element (in.)
A_o = area enclosed by centerlines of elements (sq in.)
s = length of a side element (in.)

The transverse post-tensioning shown for some cross sections in Table 2 is intended to make the units act together. This type of construction acts as a monolithic unit if sufficiently interconnected. The interconnection is enhanced by either transverse post-tensioning or by a reinforced structural overlay.

These equations are the so-called "Imbsen" formulas and were borrowed from an earlier NCHRP study[3] developed by Imbsen and Associates. The multi-beam stemmed deck equations are derived from NCHRP Report 287.[6] Although they are more complicated than past AASHTO equations, they were chosen for their accuracy.

Different distribution factor equations are specified for shear and exterior beams. The design for shear, though vastly changed, is probably considered secondary when viewed from the perspective of span capability, production constraints, and economy of prestressed concrete beams.

REFINED METHODS OF ANALYSIS

Section 4.6.3 of the LRFD Specification[2] allows the use of refined methods for bridge analysis. Three of these methods are:

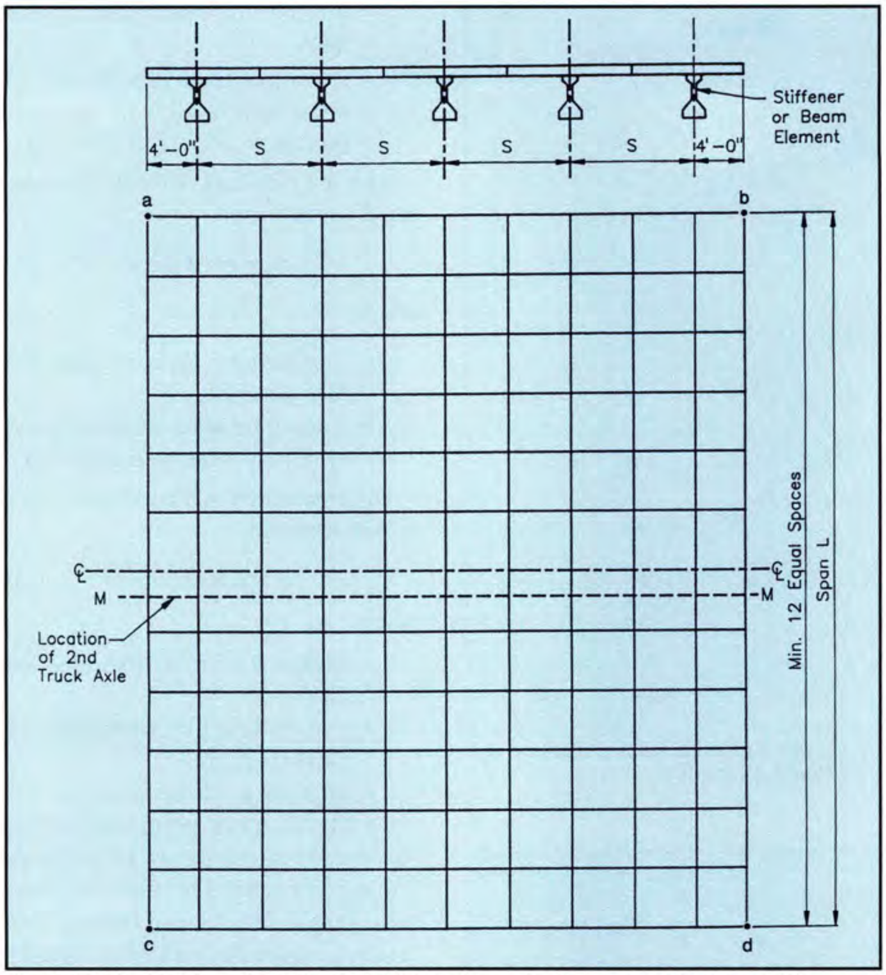


Fig. 4. Typical bridge discretization (dimensions shown as an example only).

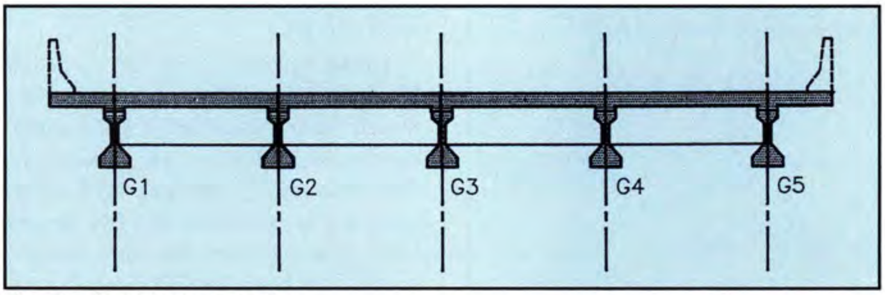


Fig. 5a. Actual cross section.

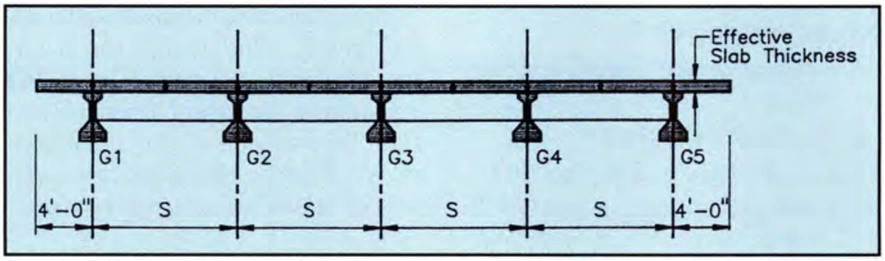


Fig. 5b. Idealized cross section.

- Finite element method
- Grillage analogy method
- Series or harmonic method

When such methods are used, consideration must be given to the number of nodes per span, aspect ratios of plate/shell-like elements, and maintenance of the relative vertical distances between various elements (i.e., plates and stiffeners). The St. Venant torsional constant may be evaluated using rational methods.[7]

Although access to advanced software for refined methods is not widespread, this situation is slowly changing. In this paper, the following software programs were used in conjunction with hand calculations:

- The ADINA program, a well-known and general finite element program developed by ADINA Inc., Cambridge, Massachusetts.
- The STAAD-III program, developed by Research Engineers, Inc., Orange, California.

The majority of the computer runs for this paper were conducted using the ADINA program. A few computer runs were done using STAAD-III. The sections below highlight features of the ADINA program that were used in the linearly elastic analyses of the superstructures.

Finite Element Modeling Using the ADINA Program

The bridge deck structural system was modeled using both "shell" and "beam" (stiffeners) elements (see Figs. 1 and 2). A standard quadrilateral (four-noded) shell element of constant thickness was incorporated in modeling the horizontal slab. Stiffeners were described using a standard isoparametric beam element.

Composite action of the beam and slab was accomplished by connecting the centers of the slab and beam with rigid links. This produced the correct constraint relations for displacements of the slab and beam.

Material properties required to describe the linear, isotropic material behavior were the modulus of elasticity and Poisson's ratio (μ = 0.20). Because the slab was modeled separately from the beam, it was possible to use different Young's moduli, E_1 and E_2, for each structural element.

This procedure was advantageous because, typically, the concrete strength for the cast-in-place slab is lower than that in the precast concrete beams. The St. Venant torsional constant, J, for the basic beam was calculated using an advanced method based on the finite difference procedure.[7]

To better represent the structural behavior of the deck slab, it was modeled as an orthotropic plate, as recommended by Kulicki et al.[8] To accomplish this, an orthotropy factor, D_y, based on the ratio of center-to-center spacing, S, to the clear span, was introduced (see Fig. 3). Its value is:

$$D_y = \left(\frac{\text{Beam spacing}}{\text{Clear slab span}} \right)^2 \qquad (5)$$

In the ADINA program, this can be done easily by multiplying Young's modulus of elasticity in the transverse direction by D_y.

Support for the structure consisted of a roller at each end of the beams. This roller provided resistance to vertical (z-direction) movements only. The beams were, therefore, free to rotate about the transverse axis at their ends but were assumed to be torsionally restrained.

To maintain structural stability, no x-displacement was allowed at Points a and b (Fig. 4) and hinge support was applied at Points c and d. The finite element mesh was proportioned so that the maximum aspect ratio of the quadrilateral elements always remained at about 2 to 1, or less.

Typical discretization of the bridge deck structure is shown in Figs. 4 to 6. There were 12 (or more) subdivisions in the longitudinal direction. The slab ("shell") elements were $S/2$ wide in the transverse direction, where S is the beam spacing.

The finite element program ADINA consists of three parts: ADINA-IN (preprocessor), ADINA (main program), and ADINA-PL (post processor). ADINA-IN was used to prepare the input data, and ADINA-PL to plot, scan and analyze the numerical results.

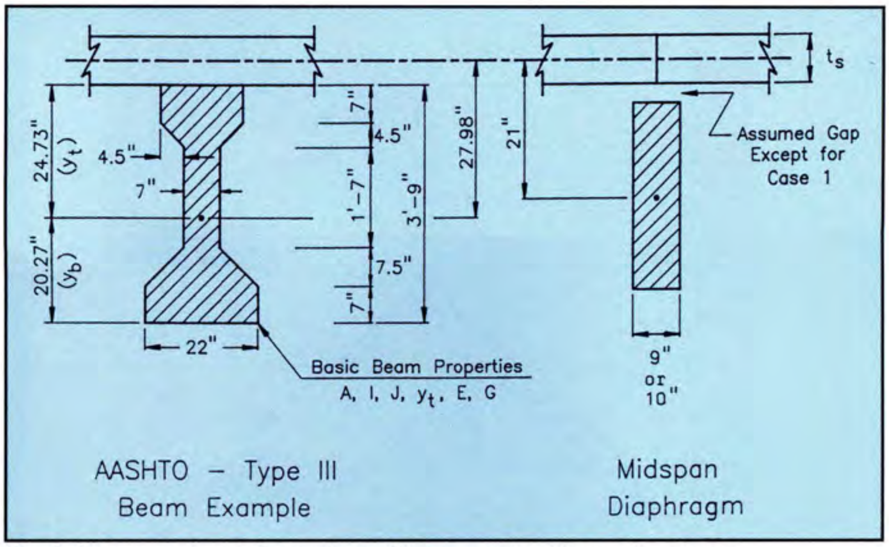


Fig. 6. Cross-sectional dimensions of beams and midspan diaphragm (I-beam example).

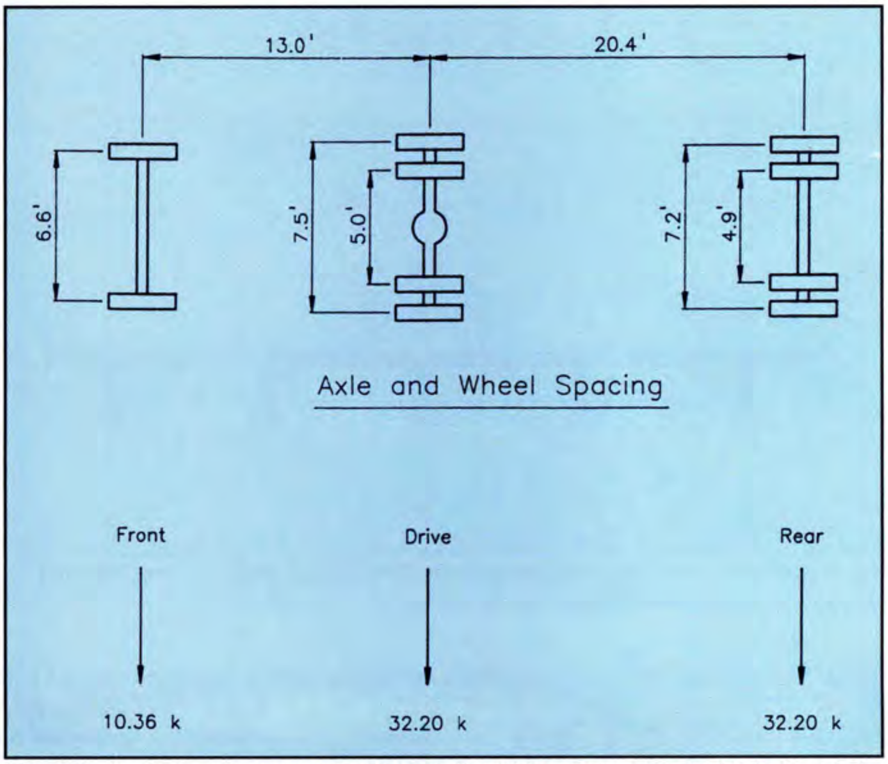


Fig. 7. Test vehicle and axle loads, Bartonsville Bridge, Pennsylvania (Refs. 8 and 9).

Computation of the Composite Beam Moment

The ADINA program requires the input of the basic beam ("stiffener") properties: A, I, J, E, and G (see Fig. 6) in addition to the slab ("shell") properties. The output then lists the axial force, P, and moment, M_b, that pertain to the beam element. From these values, the stress computation at the centerline of the bottom flange can be calculated:

$$f_b = (P/A) + (M_b/S_b) \qquad (6)$$

where S_b is the non-composite section modulus at the bottom fiber.

The moment, M_c, carried by one composite cross section is given by:

$$M_c = M_b' + \int_o^b M_{slab} dl \qquad (7)$$

in which b is the effective width of the slab, M_{slab} is the slab moment, and M_b' is the beam moment referenced to a plane within the slab.

It is usually very tedious and difficult to calculate the integral term in Eq. (7) unless the reference plane is set at the level of the slab compression resultant. In that case, the integral becomes zero. The location of the resultant is not known a priori. However, because of the general trapezoidal shape of the stress diagram, it is reasonable to assume the plane is somewhere between

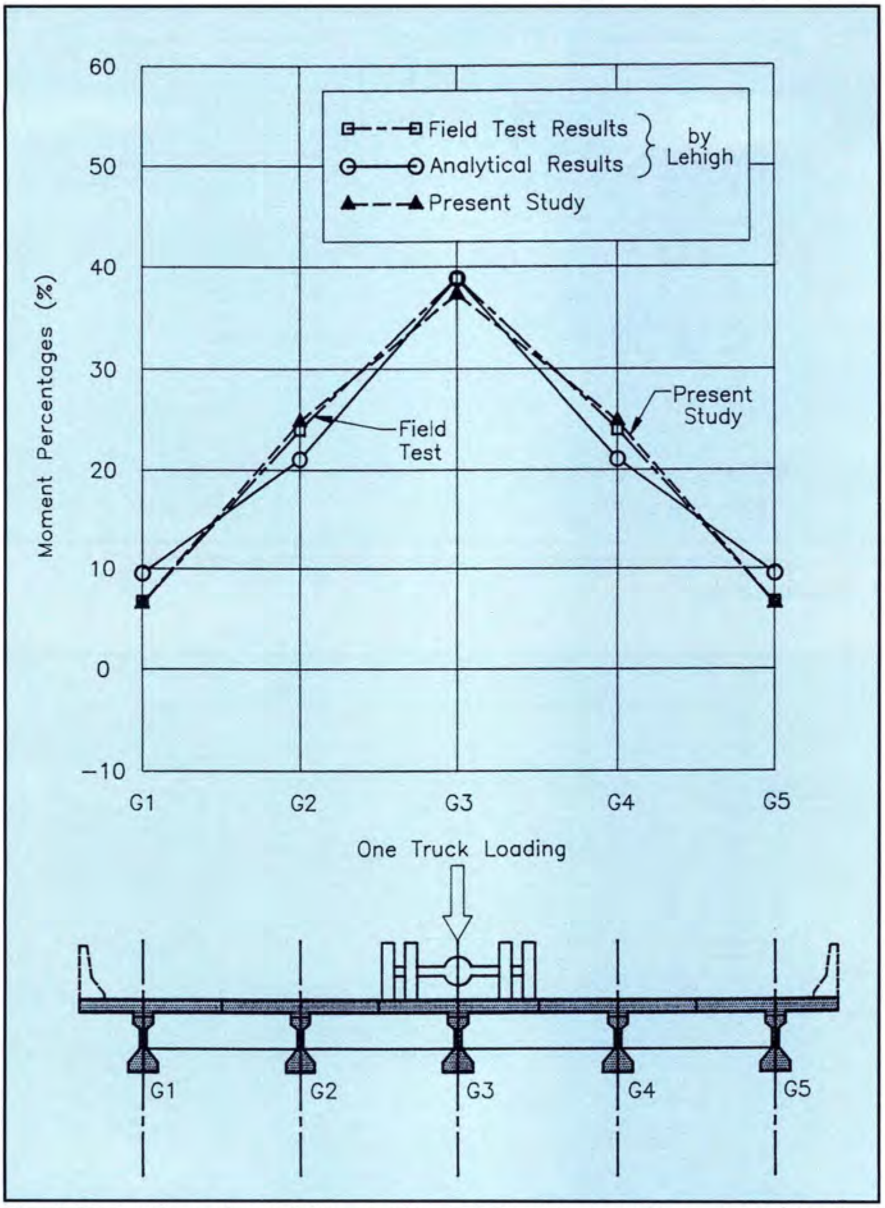


Fig. 8. Comparison of moment percentages derived from analyses and field test results — Bartonsville Bridge (Refs. 8 and 9).

$0.66t_s$ and the middle plane, say at $0.60t_s$ from the top of the basic beam, where t_s is the slab thickness. Therefore:

$$M_c = M_b' \approx M_b + P(y_t + 0.60t_s) \quad (8)$$

where y_t is the distance from the basic beam centroid to its top fiber.

Another way of computing M_c is to use the moment formula from beam theory:

$$M_c = S_{bc} f_b \quad (9)$$

where S_{bc} is the composite section modulus for the bottom fiber.

The composite section includes an effective flange, b, with due consideration of the shear lag effect. In general, however, b equals the beam spacing, unless S exceeds approximately 11 ft (3.35 m).

Quality Control Checks for ADINA Program

Finite element programs are notorious for generating stacks of printout and a multitude of results. It is essential that the designer conduct some checks by independent means to detect any gross errors that may be introduced into the analysis through incorrect input data. To achieve this objective, three types of checks or safe measures were used:

1. Checking the general adequacy of the ADINA prediction by comparing the results to previous tests and analyses by Kulicki et al. on the Bar-

Table 3. Distribution factors of simplified and refined models (applicable to lane loads and interior beams).

Case number	Shape	Length / Spacing	Current AASHTO D.F.*	LRFD simplified D.F.	Refined D.F. (Finite element analysis)	Refined with multi-lane D.F.†	Refined with multi-lane D.F. / LRFD simplified D.F.
I-beams							
1 (field test)	Type III	68.5/8	–	–	see text	–	–
2	Type III	78/7	0.636	0.603	0.551	0.468	0.78
3	Type IV	80/9	0.818	0.746	0.676	0.575	0.77
4	Type IV	85/8	0.727	0.684	0.612	0.520	0.76
5	Type IV	96/10	0.909	0.769	0.712	0.605	0.79
6a	Type V‡	96/10	0.909	0.800	0.717	0.609	0.76
6b	Type VI	96/10	0.909	0.800	0.774	0.658	0.82‡
7	Type VI	122/10	0.909	0.772	0.707	0.601	0.78
8	84 in., I	122/10	0.909	0.798	0.711	0.604	0.76
Boxes							
9	48 x 33 in.	78/7	0.574	0.516	0.526	0.447	0.87
10	48 x 48 in.	85/8	0.590	0.590	0.570	0.484	0.83

Note: 1 ft = 0.3048 m; 1 in. = 25.4 mm.

* Live load moment in this case is due to an HS-20-44 truck, which is lighter than the truck used in the LRFD Specification.

† A factor of 0.85 is used assuming that three loaded lanes controlled the design for bottom tension in the first interior beam.

‡ Slab orthotropy not considered (D_y = 1.0, conservative for interior beams, used for comparison only).

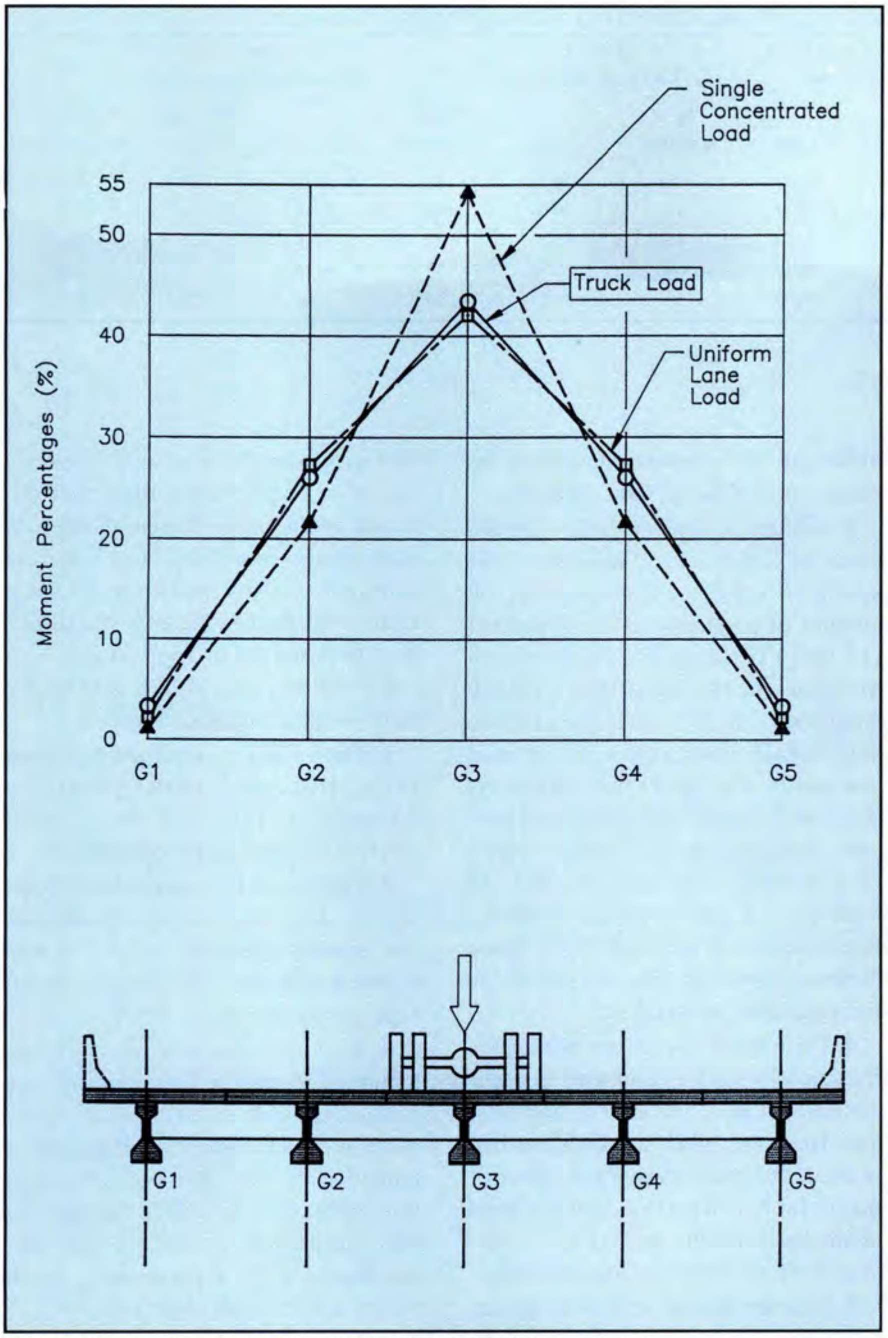


Fig. 9. Effect of type of load on lateral load distribution, Ref. 9.

tonsville Bridge.[8] This two-lane bridge consisted of five AASHTO Type III beams, 8 ft (2.44 m) on center, and a 7.5 in. (0.19 m) slab, as shown in Figs. 4 to 7. The loading vehicle approximated the HS-20 load very closely. The simple span was 68.5 ft (20.9 m). Fig. 8 shows the predicted values by the ADINA program and the actual field results for one loaded lane. The correlation is excellent.

2. Predicting the average tensile stress in the bottom fiber using the beam formula:

$$f_{avg} = \frac{\begin{pmatrix}\text{Number of}\\ \text{loaded lanes}\end{pmatrix}\begin{pmatrix}\text{Midspan moment}\\ \text{per lane}\end{pmatrix}}{(\text{Number of beams})(S_{bc})} \quad (10)$$

and comparing the value to the computed average from the ADINA program. As can be verified from Appendix A, the statics check shows a very small relative deviation (less than 1 percent).

3. Computing the composite beam moment using both Eqs. (8) and (9) and selecting the largest of the two formulas for deriving the distribution factors. The relative difference between the two equations was found to vary between 1 to 1.5 percent, and Eq. (9) usually controlled.

SCOPE OF THE PARAMETRIC STUDY

Eight AASHTO-type shapes, ranging in depth from 45 to 84 in. (1.14 to 2.13 m), were included in the study as shown in Table 3. Span-to-precast depth ratios varied between approximately 17 and 21. Minimum and maximum spacings were 7 and 10 ft (2.13 and 3.05 m), respectively. The 84 in. (2.13 m) section is a standard Pennsylvania section.

Two spread-box superstructures were investigated. The first was a 48 x 33 in. (1.22 x 0.84 m) box spanning 78 ft (23.8 m); the second was a 48 x 48 in. (1.22 x 1.22 m) box spanning 85 ft (25.9 m).

Young's modulus of elasticity for the basic beams, E_1, ranged from 4769 to 5250 ksi (32.9 to 36.2 GPa), depending on span and shape, while the average slab modulus, E_2, was 3832 ksi (26.4 GPa).

Following the practice in Pennsylvania and some other states, the midspan diaphragm [10 in. (254 mm)] was assumed to be non-integral with the cast-in-place deck (see Fig. 6). This will result in a more flexible diaphragm and is a conservative assumption for lateral load distribution. The concrete barriers were also assumed to be non-integral with the deck and would result in a larger share of the load carried by interior beams.

For Case 1, Table 3, the slab thickness was set at 7.5 in. (190 mm), as assumed in Ref. 8. For the remaining cases, slab thickness varied between 8.5 and 9 in. (216 and 229 mm), following common practice and to partially account for the extra deck thickness when corrugated metal decks are used. Except for Case 6b, slab orthotropy was considered in the refined method of analysis. Bridge carriageway was designed for three traffic lanes (plus shoulders) except for Case 1, which had two lanes in the original design.

Table 4. Comparison of reinforcing and release strength requirement (f'_{ci}) by three different procedures.

Case number	AASHTO type	Span / Spacing (ft)	Simplified AASHTO/ HS-20 method		Simplified LRFD method		Finite element analysis (Refined LRFD method)			
			No. of strands*	f'_{ci} (psi)	No. of strands*	f'_{ci} (psi)	No. of strands*	f'_{ci} (psi)	$M_{u\ avail}/M_{u\ reqd}$	Final camber (in.)
4	IV	85/8	35	3440	36	3615	31	3060	1.38	0.44
	IV	95/8	46	4460	48	4675	42	4010	1.50	0.96
6a	V	96/10	46	3870	47	3920	42	3450	1.41	0.80
	V	106/10	59	4800	64	5050	55	4425	1.51	1.25
7	VI	118/10	63	4670	66	4920	59	4290	1.50	1.00

Note: 1 in. = 25.4 mm; 1 ft = 0.3048 m; 1 psi = 6.9 kPa.
* 0.5 in. (12.7 mm) diameter, low relaxation strands.

The use of a truck load or a uniform lane load will result in almost the same moment percentages to beams as can be seen from Fig. 9.[9] Therefore, HS-25 truck loads were used throughout the analyses to derive the distribution factors for live load.

RESULTS OF STUDY

Table 3 summarizes the distribution factors for live load moments by three approaches: the current AASHTO method, the simplified LRFD method, and the refined method (finite element analysis), with and without the multi-lane reduction.

Bottom tension controlled the design for the first interior I-beams. Table 4 shows, in a concise summary, the reinforcing steel and release strength requirements using the two different methods, plus the current simplified AASHTO procedure (HS-20 load). Through inspection, the following trends emerge:

1. The simplified LRFD method yields smaller distribution factors than current AASHTO rules for interior beams.

2. In the case of interior I-beams, the distribution factor obtained through the use of refined methods is consistently smaller — by 4 to 11 percent — than the LRFD simplified method, even without considering the multi-lane reduction. With a further 15 percent live load reduction for a three-lane bridge analyzed by the refined method, the total reduction in live load moment ranges from 18 to 24 percent. This will allow many three- and four-lane bridges, where a three-lane presence controls, to be designed with significantly less reinforcement than the current AASHTO method requires.

3. Using a refined method — in this case, the finite element analysis — resulted in significant savings in the amount of prestressing reinforcement (11 to 14 percent) and moderate reduction in the required release strength, f'_{ci} [470 to 660 psi (3.24 to 4.55 MPa)]. Furthermore, the refined analysis by the LRFD method always required less reinforcing steel and concrete strength than the current simplified AASHTO method for HS-20 loading — if full advantage of the refined method is allowed. Some jurisdictions, however, may not allow the full reductions to be taken.

4. For interior spread-box beams, the reduction in bending moments is negligible (less than 2 percent) or non-existent. However, when a refined method is used, the total cumulative effect of multi-lane reduction and refined method still results in a 13 to 17 percent decrease in the live load moment.

5. Exterior beams need special consideration. The refined method analysis shows consistently higher factors for the exterior beams — by 7 to 15 percent. The LRFD method has a simplified and conservative procedure for exterior beams. Its impact, however, has not been assessed.

CONCLUSIONS AND RECOMMENDATIONS

As expected, there is a substantial reduction in the distribution factor for interior beams between the current AASHTO and the LRFD Specification. In addition, the interior factor determined by the specification's simplified procedure is generally close to, but more conservative than, that obtained by the finite element method. In wide bridges with three- and four-lane carriageways, the multi-lane presence factor will further increase the difference between the two approaches.

The authors would like to offer the following recommendations:

1. The LRFD's simplified equations for distribution factors should be adopted in place of the current AASHTO simplified procedures.

2. The use of refined methods is encouraged, as they are more accurate and usually offer substantial live load moment reductions for three (or more) lane carriageways.

3. Exterior beams deserve special attention. The present finite element case studies showed exterior beams carry more live load moment than the first interior beams. One possible solution to this problem is to reduce the spacing between the exterior and the first interior beams, if equal prestressing for all beams is a desirable objective.

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APPENDIX A – GENERAL CHECKS ON ADINA PROGRAM

Case 6a, a 48 ft (14.6 m) wide bridge, with five AASHTO Type V beams, will be used as an example to verify the overall accuracy of the ADINA program output. The simple span bridge has a span length, L, of 96 ft (29.3 m), and a beam spacing, S, of 10 ft (3.05 m) (see Fig. A1). Beam depth is 63 in. (1.6 m).

Non-composite and composite properties are shown below. The moment at midspan for an HS-25 truck, positioned as shown in Fig. A2, can be easily calculated by the principles of statics and is:

$$M_{lane} = 1800 \text{ kip-ft } (2441 \text{ kN-m})$$

The resultant axial forces and bending moments for each basic beam, as obtained from the ADINA program, are also shown below. The bottom fiber stress at the midpoint of the flange can be calculated using the familiar beam formula:

$$f_b = P/A + M/S_b \quad \text{(A1)}$$

For example, for Beam B2 under Load Case 1, P = 211 kips (938 kN) and M = 7585 kip-in. (857 kN-m). Therefore:

$$f_2 = 1000\,[(211/1013) + (7585/16308)] = 673 \text{ psi } (4.64 \text{ MPa})$$

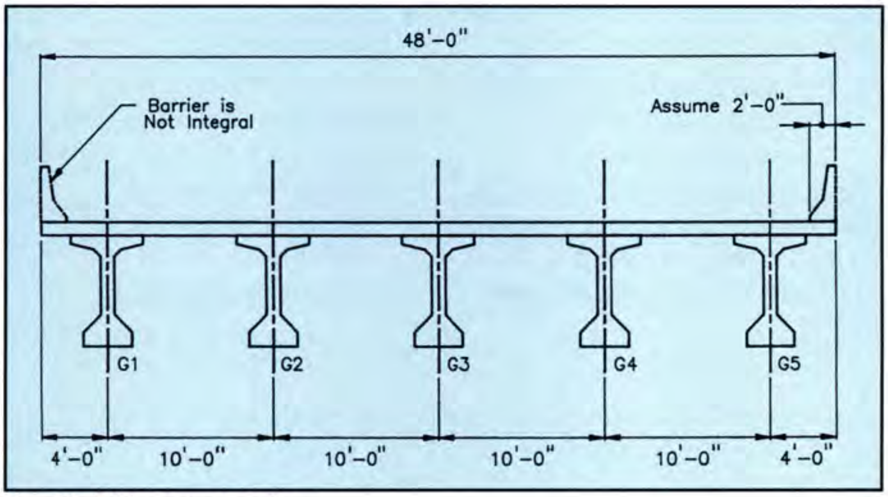

Fig. A1. Bridge cross section.

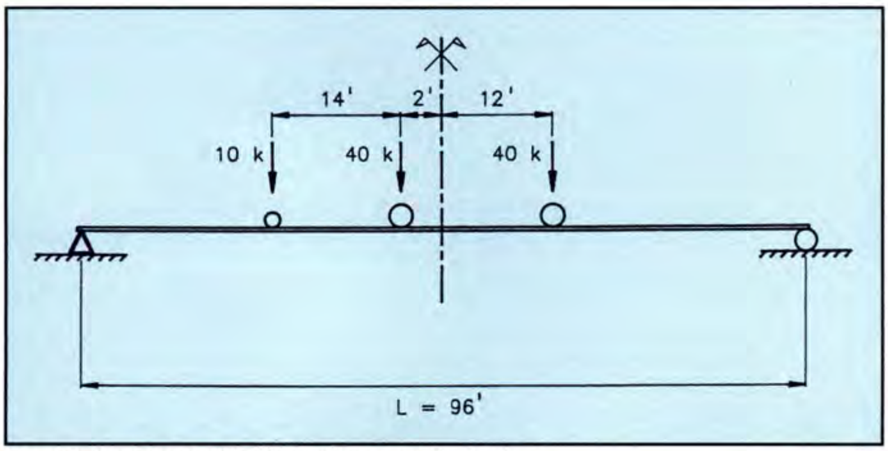

Fig. A2. Longitudinal truck position over the bridge.

Table A1. ADINA program results for midspan section (calculated bottom stress $f_b = P/A + M/S_b$).

Basic Beam B1	B2	B3	B4	B5	Average f_b
Load Case 1: Truck positioned as close to right barrier as possible.					
M = 8715 in.-kips	7585	6505	5141	3445	
P = 217.9 kips	211.0	194.7	160.3	111.4	
f_b = (750) psi	(673)	(591)	(473)	(321)	562 psi
Load Case 2: All trucks displaced transversely by 2 ft (0.6 m).					
M = 7320 in.-kips	6991	6579	5767	4685	
P = 186.7 kips	201.0	199.9	174.8	133.6	
f_b = (633) psi	(627)	(601)	(526)	(419)	561 psi

Note: 1 psi = 6.895 kPa; 1 kip = 4.448 kN; 1 in.-kip = 0.113 kN-m.

Other bottom stresses can be similarly calculated and are found to be:

f_1 = 750 psi (5.17 MPa)

f_3 = 591 psi (4.08 MPa)

f_4 = 473 psi (3.26 MPa)

f_5 = 321 psi (3.26 MPa)

The average stress is therefore:

f_{avg} = (750 + 673 + 591 + 473 + 321)/5
= 562 psi (3.87 MPa)

Using simple statics and making a cut through the whole midspan section of the bridge, one can also estimate the average bottom fiber stress of the bridge from Eq. (10) as:

f_{avg} = [3 lanes x 1800 ft-kips x12000/ (5 beams x 23,017 cu in.)
= 563 psi (3.88 MPa)

The above value is very close to the one derived from the ADINA program results [i.e., 562 psi (3.87 MPa)]. As an additional check, Eq. (8) can be used:

M_c = 7585 kip-in. + 211.0 kips (31.04 + 0.6 x 9)
= 15,274 kip-in. (1726 kN-m)

This latter value compares well with the composite beam moment:

M_c = 0.673 ksi x 23,017
= 15,490 kip-in. (1750 kN-m)

Major Geometric, Section, and Material Properties in Finite Element Analysis for Case 6a

- Bridge section shape:
  - AASHTO Type V
  - Span L = 96 ft (27.4 m)
- Number of loaded design lanes: 3
  - Beam spacing, S = 10 ft (3.05 m)
- Loading per lane: HS-25
  - Moment at midspan per lane: 1800 ft-kips (2441 kN-m)
- Number of basic beams: 5
  - Out-to-out bridge width: 48 ft (14.6 m)
- Effective slab thickness: 9 in. (229 mm)
  - E_{slab} = 4067 ksi (28 MPa)
  - Orthotropy factor: D_y = 2.37
- Properties of basic beam:
  - A = 1013 sq in. (0.65 m^2)
  - S_b = 16,308 cu in. (0.27 m^3)
  - y_t = 31.04 in. (788 mm)
  - E_{beam} = 5250 ksi (36 MPa)
  - St. Venant J = 35,592 $in.^4$ (1.48 x 10^{10} mm^4)
- Composite beam:
  - Section modulus at bottom fiber: S_{bc} = 23,017 cu in. (0.377 m^3)
- Midspan diaphragm (non-composite with slab):
  - Assumed 10 x 36 in. (254 x 914 mm)

APPENDIX B – NOTATION

A = cross-sectional area

A_o = area enclosed by centerlines of element

b = beam width or effective slab width

D.F. = distribution factor

D_y = orthotropy factor

d = beam depth

E_1, E_2 = Young's moduli of elasticity

e_g = distance between the centers of gravity of beam and deck

f = tensile stress due to flexure

f_b = combined axial and bending stress at beam bottom fiber

G = shear modulus

I = moment of inertia

J = St. Venant torsional constant

K_g = longitudinal stiffness parameter = $n(1 + Ae_g^2)$

k_1 = a constant defined by $2.5N_b^{0.2}$ ($k_1 \geq 1.5$)

k_2 = a constant defined in Eq. (2)

L = span length

M_b = bending moment at center of gravity of beam

M_b' = beam moment referenced to a slab plane

M_c = composite bending moment

m_{lane} = midspan moment

M_{slab} = slab moment

m = multiple presence factor

N_b = number of beams

N_L = number of design lanes

n = modular ratio between beam and deck materials

P = axial force at center of gravity of beam

S = average beam spacing

s = element length

S_b = non-composite section modulus at bottom fiber

S_{bc} = composite section modulus at bottom fiber

t = element thickness

t_s = slab thickness

W_c = roadway width

y_t = distance from beam centroid to top fiber

μ = Poisson's ratio